USING A MATH PLACEMENT EXAM TO DEVELOP A PERSONALIZED PRECALCULUS PROGRAM

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1. Introduction. In about 2005, the Department of Mathematics began experimenting with a mathematics assessment test for incoming freshmen who were to be placed in Calculus I or Precalculus. At the time, the student's high school GPR and SAT scores were primarily used by advisers to make the placement decision. This early test was administered as a paper test to students, and had all multiple choice questions. The results of this test were subsequently compared with final grades in these courses. The results were compelling in their predictability of student success (i.e. grades of A, B, C). The College of Engineering, our biggest client college, was impressed and wanted to use it as a placement test. By 2007, we had a fully online placement test, called the Math Placement Exam (MPE). Students were given permission to take this test as soon as they were admitted to Texas A&M University. Most took it at home. The motivation for this early date was that the student should perform better while were completing their senior year in high school, and therefore be more mathematically adept. The goals of the test were to accurately place students in the "right" math class, and to accurately predict success of students in Calculus I or Precalculus.

Eventually, we hoped to use the test to provide remedial help in a form that students could use online and prior to matriculation. This is the principal topic of this report. What are we doing? What are the effects of what we are doing? Our project was funded through a grant from the NSF[1].

2. Remediation and Placement. It is instructive to briefly review what does not seem to work for remediation. The first is to require students to take a full three or four-hour course in Precalculus. The forcible use of the Precalculus course option should not be taken lightly. Here are just a few reasons.

- Students believe they already know the material (they have certainly already seen most of it).
- Students in Calculus I and II inform Precalculus students it is a waste of time.
- For engineering students taking precalculus there has been a high drop rate (which implies a transfer to a nonSTEM major) and a low success rate.
- Students successful in Precalculus (i.e. grade of A, B, C) still have a relatively low success rate in Calculus I. Indeed, unless a student receives a grade of A or B in Precalculus, they are overwhelmingly not successful in the Calculus sequence.
- Requiring a student to take Precalculus sets them off sequence in the rigorous and demanding engineering degree plan sequence.

Criteria for placement, such as school rank and SAT/ACT exam score, are also not effective. For example, the correlation of the SATM (math part of SAT) score with final grades in Calculus I was extremely low (Pearson $r \approx 0.2$). It was similar for Precalculus. For class rank, the correlation coefficient was even less, nearly zero. Apparently, these criteria had never been checked as to validity for placement. Moreover, this was not noticed until we began the analysis of the MPE with Calculus I grade connectivity. By way of background, let us illustrate the mathematics background of our incoming students in Table 2.1. It shows the numbers of high school units[2] of the various math courses of enrolled students in Calculus I.

Math Course	Total Math Units	Advanced Math	ALG 1	ALG 2	GEOM	SAT Quantitative
Calc I	5.0	2.0	1.0	1.0	1.0	647.4
Precalc	4.7	1.7	1.0	1.0	1.0	573.4

Table 2.1

[1] NSF-DUE 856767

[2] A high school unit is normally a full academic year course.

3. The Math Placement Exam. Our MPE consisting of 33 questions is given online to all incoming STEM majors, with 90 minutes allowed. The test is substantially algebra and precalculus type questions. According to discrimination analysis, the range of difficulty is from easy to medium. There are no difficult questions on the test. The score of 22/33 for admission to calculus was originally recommended with enthusiastic support from all Colleges, and is now mandatory. This means students cannot enter the course without the minimum score, unless an override is given by a dean. The mandatory nature of placement based on the MPE has somewhat changed the dynamics of the test.

Subscales. There are seven subscales of problems on the test: Graphing, exponential, functions, problem solving, polynomials, trigonometry, and power rules. These are the "big seven" topics that most mathematics instructors perceive as the tools a student needs for success in Calculus I. How they measure up and correlate with final grades is shown in Table 3.1.

Subscale	Number of Problems	Average	Relative average	Correlation with grades
Exponential	4	3.10	77.6%	0.18
Polynomial	6	5.23	87.2%	0.22
Graphing	5	4.00	79.9%	0.15
Functions	5	3.77	75.3%	0.23
Trigonometry	5	4.16	83.2%	0.23
Power Rules	4	3.46	86.5%	0.20
Problem Solving	4	3.61	90.4%	0.11
Totals	**33**	**27.34**	**82.8%**	

Table 3.1

Using the relative average column, it is evident the overall average on the placement exam was about 83%. However, the lowest relative average of 75.3% was on functions. As is well known a key element of learning calculus is dexterity with functions. Note: In the state of Texas, the State of Texas Assessments of Academic Readiness (STAAR™) exams are given for Algebra II, and other courses. Like the TAMU MPE, the Algebra II STAAR also tests facility with functions. See: http://www.tea.state.tx.us/staar/rpt/sum/yr12/. It is shown in this report that it is precisely on rational functions that students performed the worse This indicates that functions are not only difficult to learn but take much time and repetition to master. Moreover, our schools may not be up to the task.

A table of success rates in Calculus I versus the MPE~~. It~~ is shown in Table 3.2.

MPE Score	MPE cummulative Count	Success Count	Probability of Success
33	48	44	0.917
32	106	100	0.943
31	133	126	0.947
30	173	155	0.896
29	170	149	0.876
28	161	132	0.820
27	136	116	0.853
26	141	117	0.830
25	122	98	0.803
24	112	83	0.741
23	97	64	0.660
22	113	80	0.708
21	3	3	1.000
20	1	1	1.000
19	4	1	0.250
18	0	0	0.000
17	1	1	1.000
16	3	2	0.667
15	1	1	1.000
14	1	1	1.000

Table 3.2

With this summary from Fall 2012 data, we are led to the questions of what can be done to improve Calculus I success and how can we use our MPE to help. Note, our MPE is the best single predictor of course grade in Calculus I we have.

4. The Personalized Precalculus Program (PPP). The PPP is an online remediation program offered in the summer semester prior to matriculation. Beginning in the summer 2013, the charge to students is $125 for the complete session. The program will eventually be self-sustaining. It is normally about six weeks in length. The criteria for admission to the PPP are given below. In this criterion, G is the student's MPE score.

- If $G \geq 22$, the student enrolls in calculus.
- If $14 \leq G \leq 21$, the student qualifies for the summer PPP remediation.[3]
- If $G < 14$, the student enrolls in pre-calculus.

The nature of the PPP, relatively simple in it structure, has the following components.

[3] This was changed from 16 from the previous two years, as it was hoped to enlist more students to the program, and to help even those students that will be placed into Precalculus.

1. PPP students are placed into cohorts of up to 20 students. There are about eight cohorts normally, beginning in a staggered fashion with weekly start dates.
2. Each cohort is assigned a ("live" online) tutor.
3. Each tutor meets with the students three times per week in a synchronous, online session. Each session is 2 hours in length. Each online session is conducted via Centra®, a powerful online conferencing tool that houses an electronic whiteboard, has the capability to put students in small groups, and allows for program and file sharing. All the synchronous, online sessions are recorded for future viewing.
4. WebAssign® handles all grading of the Personalized Study Plan (PSP), and the tutor has access to students' responses within the PSP.
5. Students access materials on WebAssign® commensurate with their MPE performance.
6. The WebAssign® PSP is modular in nature and consists of text, quizzes, instructional videos. and a complete online text.
7. To complete the PPP, students must complete the PSP.
8. WebAssign® monitors student progress through each PSP module.

The nature and skill set of the tutors is extremely important. Students value their tutors highly. There are some findings relevant. Tutor feedback shows they are well trained on necessary video conferencing technology. Participant feedback is very largely positive and the most helpful aspect is online tutoring. Tutors must be chosen carefully – and paid well. The summer is ideal to run the program as a number of our tutors have regular jobs during the traditional academic terms. Most of the tutors are either high school or junior college mathematics teachers.

The Just-In-Time Program (JIT). JIT, a program for weaker students, is face-to-face. It is a companion tutorial program to the PPP. We have found it to be rather valuable, though staffing of this program requires care. Key points are:

- Pick your acknowledged best instructor, one trusted by the students.
- Meet 90 minutes per week.
- Cover needed algebra skills and the calculus of the week.

The results are stunning. The MPE scores of the students who attended JIT and those who did not were statistically similar. However, the differences between the average GPR was almost 0.6! This was in spite of the fact that the correlation between MPE and GPR was similar in both groups.

5. Results of the PPP. Overall, the program is successful, but needs attentive management. This is not a program that runs itself. There is much communication with prospective students, some with parents, and much with the tutors.

Here is some data:

- Summer 2010 – over 75 students

- Summer 2011 – over 200 students
- Summer 2012 – over 100 students
- For students with scores in range 16-21 enrolling in and completing the PPP, their average increase in MPE score is 5-6 points.
- Subsequent performance in Calculus I was slightly better than peer group (students with same scores).

Challenges to the PPP. Student participation has been a challenge. In summer 2011, more than 600 students were eligible for the program. They are those with scores in the 16-21 (out of 33). Only 240 expressed an interest, with only about 210 enrolling. Finally, only 90 fully completed the program by retaking the placement exam. There is a lot of data about this and it is sometimes difficult to define successfully completing the program. That is, do they finish the PPP or retake the MPE and then drop out, having been admitted to calculus? There are other possibilities.

Getting students to actively participate in the program can be difficult. This is a summer online program taken from home by students. We have treated it more-or-less like a course with the exception that we hire exceptional tutors. So, our challenge continues to be keeping those students in the program engaged. Many high school students work or travel during the summer.

To determine which students benefit most from our program will require a very finely honed statistical analysis. This cannot yet be done as we do not have enough data. At this point we are using our best sense of what is correct, all based on years of online teaching and classroom experience. It is thought there is a "motivation" factor or "maturity" factor involved. But these are difficult to measure. Our statistician has some ideas that will be implemented this cycle.

We have the full online record of the student's performance via WebAssign. As well, we have attendance records for the tutoring sessions. This information will be used to get a profile of which students are doing well, which students are doing poorly, and which complete all the tasks required.

To interpreting which of the online media have the greatest attraction, we have generated a great deal of media and materials. We will employ WebAssign statistical data to measure which of the videos have the greatest impact. This will help guide which new materials to develop.

There is more than just a knowledge of Precalculus topics necessary for a student to be successful. Three less tangible factors are relevant: *persistence, self-efficacy,* and *motivation*. See, Boelkins, et.al. 1998, Bressoud, et.al, submitted, Niemi, et.al, 2003, and Triesman, U., 1992. These are complex topics, and all four algebraic knowledge plus the others must be considered to achieve better predictability of success.

In summary, we have developed a turn-key online system for pre-matriculation remediation of students having a Calculus I trajectory. It can, however, be applied to virtually any entry level

course. The key is a reliable placement exam with the intent to inform the remediation program via the selection of appropriate learning modules.

References.

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