

SURFACE INTEGRATION COMPUTER ACTIVITIES IN MULTIVARIABLE CALCULUS

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Introduction

Students in our Multivariable Calculus course tend to find the Divergence Theorem to be quite challenging. To large extent, this challenge is due to the geometric complexity of boundaries of solids. Most solids require that the boundary surface be broken up into a number of smooth pieces before iterated integrals can be properly set up to calculate the flux. Often, different pieces will involve a mix of different coordinate systems (Cartesian, cylindrical, spherical).

On the other hand, prior to taking Multivariable Calculus, students' exposure to solid geometry tends to be minimal. Consequently, we found it beneficial to augment the course with computer activities designed to help students learn how to properly deal with these issues.

Computer Laboratory Activities

In a typical activity, a student is given a specific solid, and is asked to use a Mathcad template document to construct a parametric representation for each piece of the piecewise smooth boundary. Students interact with the activity module by specifying these parametrizations by selecting two variables of integration and the limits of iterated integrals.

Figure 1 shows the initial screen associated with one of the activities.

At the bottom of the screen, there are eighteen collapsed regions marked by horizontal lines preceded with small triangles. By clicking on a triangle, the student can expand the corresponding region to set up double integrals of the specific type.

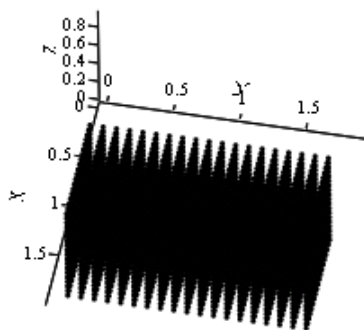
Click to hide the solid:

Lab - Surface Integrals

Click to hide the surface:

Each surface corresponds to the double integral with the given number (see below)

$S_0 = 0$ $S_1 = 0$ $S_2 = 0$ $S_3 = 0$ $S_4 = 0$ $S_5 = 0$ $S_6 = 0$ $S_7 = 0$



Problem 1.

Set up all surface integrals required to integrate on the boundary of the box $\{(x,y,z) \mid 1 \leq x \leq 2, 0 \leq y \leq 2, 0 \leq z \leq 1\}$

1. SURFACE INTEGRALS IN CARTESIAN COORDINATES

- ▶ 11. $dx \, dy; z=z(x,y)$ _____
- ▶ 12. $dy \, dx; z=z(x,y)$ _____
- ▶ 13. $dx \, dz; y=y(x,z)$ _____
- ▶ 14. $dz \, dx; y=y(x,z)$ _____
- ▶ 15. $dy \, dz; x=x(y,z)$ _____
- ▶ 16. $dz \, dy; x=x(y,z)$ _____

2. SURFACE INTEGRALS IN CYLINDRICAL COORDINATES

- ▶ 21. $dr \, dz; \theta=\text{const}$ _____
- ▶ 22. $dz \, dr; \theta=\text{const}$ _____
- ▶ 23. $dr \, d\theta; z=\text{const}$ _____
- ▶ 24. $d\theta \, dr; z=\text{const}$ _____
- ▶ 25. $dz \, d\theta; r=\text{const}$ _____
- ▶ 26. $d\theta \, dz; r=\text{const}$ _____

3. SURFACE INTEGRALS IN SPHERICAL COORDINATES

- ▶ 31. $drho \, dphi; \theta=\text{const}$ _____
- ▶ 32. $dphi \, drho; \theta=\text{const}$ _____
- ▶ 33. $drho \, d\theta; \phi=\text{const}$ _____
- ▶ 34. $d\theta \, drho; \phi=\text{const}$ _____
- ▶ 35. $dphi \, d\theta; \rho=\text{const}$ _____
- ▶ 36. $d\theta \, dphi; \rho=\text{const}$ _____

Figure 1. The initial screen of one of the Surface Integration activities.

For instance, the first region contains double integrals in Cartesian coordinates with respect to x (for the inside integral) and y (for the outside integral). There are five additional regions corresponding to double integrals in Cartesian coordinates, ($dydx$, $dxdz$, etc.). They are followed by six regions corresponding to cylindrical coordinates and six additional ones associated with spherical coordinates. Each region allows the student to set up three integrals of the given type.

Click to hide the solid:

Click to hide the surface:

Each surface corresponds to the double integral with the given number (see below)

Lab - Surface Integrals

MATH 312

$S_0 = 111$ $S_1 = 112$ $S_2 = 0$ $S_3 = 0$ $S_4 = 0$ $S_5 = 0$ $S_6 = 0$ $S_7 = 0$

Problem 1.

Set up all surface integrals required to integrate on the boundary of the box

$\{(x,y,z) \mid 1 \leq x \leq 2, 0 \leq y \leq 2, 0 \leq z \leq 1\}$

1. SURFACE INTEGRALS IN CARTESIAN COORDINATES

11. $dx \, dy; z=z(x,y)$

| | | |
|--|--|--|
| $\int_{yL_{111}=0}^{yU_{111}=2} \int_{xL_{111}(y)=1}^{xU_{111}(y)=2} \blacksquare \, dx \, dy$ <p style="text-align: center;">$z_{111}(x,y) \equiv 1$</p> | $\int_{yL_{112}=0}^{yU_{112}=2} \int_{xL_{112}(y)=1}^{xU_{112}(y)=2} \blacksquare \, dx \, dy$ <p style="text-align: center;">$z_{112}(x,y) \equiv 0$</p> | $\int_{yL_{113}=0}^{yU_{113}=0} \int_{xL_{113}(y)=0}^{xU_{113}(y)=0} \blacksquare \, dx \, dy$ <p style="text-align: center;">$z_{113}(x,y) \equiv 0$</p> |
|--|--|--|

11. $dx \, dy; z=z(x,y)$

Figure 2. The screen displaying two of the six boundary surfaces of the solid box.

The area of the document directly above the eighteen regions contains the description of the solid whose boundary the student is describing as well as an interactive 3D view (which can be rotated and scaled) of this solid along with any surfaces resulting from the double integrals the student defined below. Each surface appears in a different color.

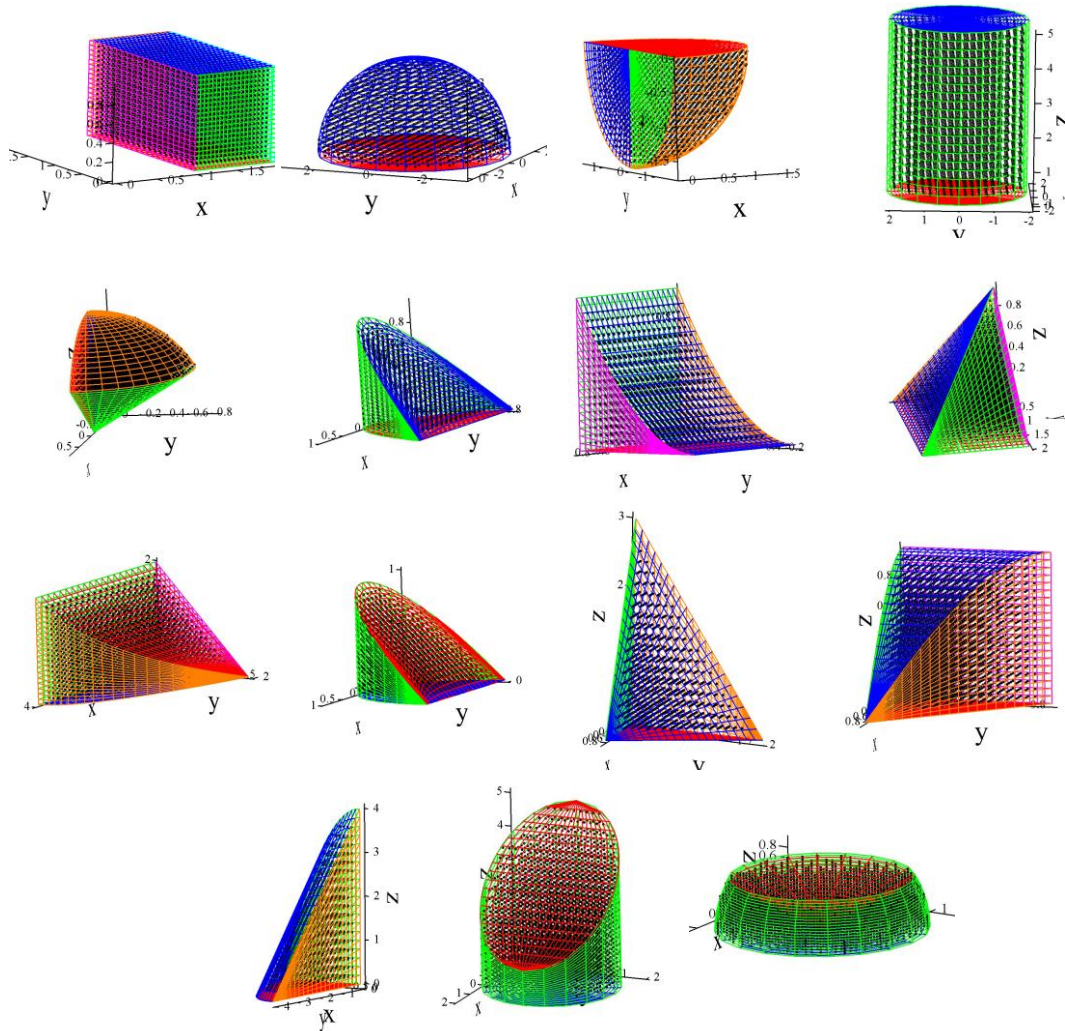


Figure 3.
Solids used in conjunction with the surface integration laboratory activities.

In order to make it possible to associate each surface being displayed with the corresponding double integral, a legend is included at the top of the document. Each double integral is assigned a three-digit number: the first digit corresponds to the coordinate system (1=Cartesian, 2=cylindrical, 3=spherical). The second digit designates the variables of integration and their order (e.g., for the Cartesian group, 1= $dx dy$, 2= $dy dx$, 3= $dx dz$, etc.). Finally, the third digit points to the specific double integral among the three contained within each collapsed group. For instance, 123 corresponds to the third integral with respect to $dy dx$. Individual surfaces and the solid itself can be hidden by clicking the corresponding checkboxes at the top of the screen.

Figure 2 contains a partial screen where two of the six boundary surfaces of the solid box have been defined.

The resulting surfaces are immediately displayed in 3D context providing the students with a lot more meaningful feedback than would be afforded by a "right/wrong" type of a grading scheme. Students often have to correct their early attempts before they find a proper boundary surface for the given solid.

Figure 3 displays 15 solids which are currently used in conjunction with the lab activities described here. Each student (or student team) is required to complete a certain number of basic activities (typically, the four shown in the top row of Figure 3) and a certain number of additional activities chosen among the remaining ones.

Conclusions

The majority of students to whom we assign this lab activity complete it within the allocated time (usually 85 minutes), although some (about 20-30%) require a time extension to do so.

Although we have not carried out a formal study of the effectiveness of these modules, it is safe to say that after this lab, students appear to have a substantially better mastery of the geometry of solids vs. geometry of the solids' boundaries. As a result, they have a much better chance of successfully solving divergence theorem problems.

In the future, we intend to pursue implementing similar activities on a web-based platform.