

CREATING CALCULUS DEMOS WITH GEOGEBRA 4

Barbara K. D'Ambrosia
Carl R. Spitznagel
John Carroll University
Department of Mathematics and Computer Science
Cleveland, OH 44118
bdambrosia@jcu.edu
spitz@jcu.edu

Introduction

For several years, we have been using GeoGebra to develop dynamic classroom demonstrations for use in teaching calculus. We have found GeoGebra to be far preferable to other software packages that we had formerly used for this purpose, including Derive, The Geometer's Sketchpad[®], and the TI-Nspire[™] CAS Computer Software. Some of our reasons for liking GeoGebra so much are:

- GeoGebra provides a tight connection between the algebraic and geometric representations of mathematical objects. For example, a function in GeoGebra is at once both an algebraic object and a geometric one, and can therefore be manipulated in either way. This connection helps to make GeoGebra an ideal environment for exploring concepts of calculus.
- Version 4 of GeoGebra, released in the fall of 2011, includes a number of new commands for working with functions. For instance, the process of illustrating and computing a Riemann sum is now a snap, as we will show below.
- GeoGebra 4 has simplified the process of creating both stand-alone Java applets and simple web pages with embedded applets. Since the interactive demos that you create with GeoGebra can easily be placed on the web, you can provide your students with free, readily available access to those demos for continued exploration outside of class.
- GeoGebra is *free*! Distributed by its international development team under the GNU General Public License, GeoGebra will cost you nothing to use or even to install on every computer on campus! But remember that your students will not need GeoGebra to interact with the GeoGebra applets you create; they will need only the most common of today's computer tools: an internet connection and Java. To get started with GeoGebra, visit the GeoGebra web site, at <http://www.geogebra.org>.

In this paper we give step-by-step instructions for creating several interactive calculus demos. We hope these inspire you to create some of your own.

Riemann Sums

Follow these directions to create a GeoGebra applet to demonstrate the convergence of Riemann sums to a definite integral.

- Create a function, like $f(x) = 2 - \frac{1}{2}(x+1)^2$, in the input bar.
- Define numbers a and b by typing $a = -1$ and $b = 0$ in the input bar. Later we will insert input boxes to allow the user to change these values.
- Create corresponding points A and B on the x -axis by typing $A = (a, 0)$ and $B = (b, 0)$ in the input bar. Right-click on the point A and open the Object Properties box for this point; add the caption “ a ” and change the Show Label option to “Caption.” Similarly, change the caption for point B .

- Use the “Slider” tool  to create a slider named n for the number of rectangles that will be drawn. The slider type can be either “Number” or “Integer.” Either way, the minimum should be 1, the maximum should be a relatively large number (like 200), and the increment should be 1. In the “slider” tab of the dialog box, set the width of the slider, and uncheck the “Fixed” property so that you can reposition the slider later. You can change all of these settings later in the Object Properties box for the slider, if you decide you need to.

- Create a slider named $position$ which specifies the relative location of the sample points in each subinterval. Use a minimum value of 0 and a maximum value of 1, and an appropriate increment value, like 0.1.

- Use the “RectangleSum” command to create the Riemann Sum and corresponding picture, by typing this expression in the input bar:

$$\text{RiemannSum} = \text{RectangleSum}[f, a, b, n, position]$$

- Use the Insert Text tool  to create a text object that explains the use of the position slider: “Use this slider to set the relative position of the sample points in each subinterval.” Create a text object that explains the use of the n slider: “Use this slider to set the number of subintervals.”

- Click the “Insert Input Box” tool  to create input boxes for the values of a and b , and the function f . Include the appropriate caption, and choose the linked object from the drop down menu. After creating the objects, right click on their captions, open the Object Properties, and use the “Style” tab to set an appropriate length for the input box. A length of 2 or 3 should be sufficient for a and b .

- Create a text object to show the Riemann sum. In the text creation dialog box, type the phrase “Riemann sum =” (no quote marks), and then select the RiemannSum variable from the “Objects” drop down menu. This creates a field that holds the value of the RiemannSum variable.
- Use the special action tool  to create a check box to show the value of the Riemann sum. Select the Riemann sum text object from the drop down menu.
- Format colors, font sizes, line thicknesses and point sizes as desired, and move the sliders, input boxes, and text fields to appropriate locations. Determine which objects’ labels to show. (In the worksheet below, we’ve chosen not to show the label for the *position* slider.)
- Set a , b , n , and $position$ to the values you would like the user to see on first opening the worksheet or applet. Adjust the toolbar and window size. Then save the worksheet and export it as a webpage. (See the section below on Creating Applets for the Web.)

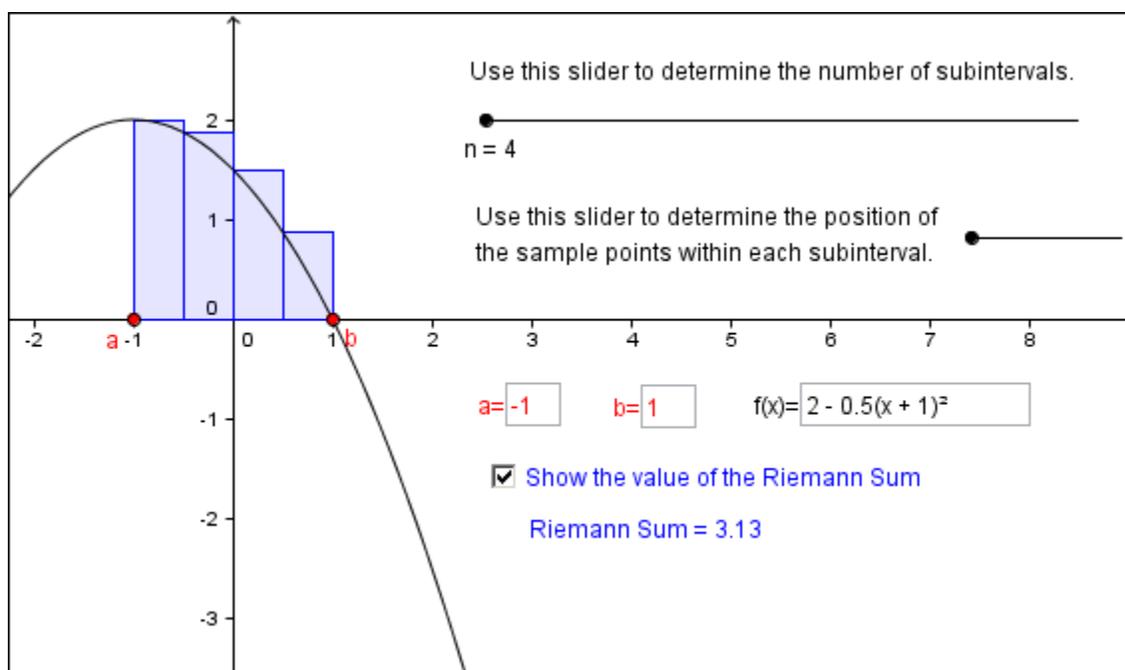


Figure 1: The finished Riemann sum worksheet

Optional Enhancement for Riemann Sums

We can make a more obvious connection between the value of the *position* variable and the special sample points that calculus textbooks usually direct students to use. Create

three separate text objects, with the phrases “left endpoints,” “right endpoints,” and “midpoints.” Drag these to their relative locations under the *position* slider. Then in the Object Properties for each text object, select the “Advanced” tab and enter values in the “Conditions to show” field: $position = 0$ for left endpoints, $position = 0.5$ for right endpoints, and $position = 1$ for left endpoints. (Of course, you need to make sure that the increment for *position* allows $position = 0.5$ to occur.)

Graph of the Derivative Function

Follow these directions to create a GeoGebra applet to sketch the graph of $f'(x)$ from a given graph of $f(x)$.

- In the Options menu, turn “Point Capturing” off.
- Create a function, like $f(x) = \frac{1}{4}x^3 - 2x + 1$, in the input bar.
- Use the point tool  to create a point on the x -axis. In the object properties for this point, change the caption to x , and change the “Show Label” option to “caption.”
- Create the corresponding point P on the graph of $f(x)$ by typing $P = (x(A), f(x(A)))$ in the input bar. Use the special line tool  for tangents to construct the tangent line to the graph of $f(x)$ at P .
- Use the measurement tool  for slope to measure the slope of the tangent line. The default name for this variable should be m . If it’s not, use the object properties for the slope and change the name to m . Right-click on the slope, and deselect “Show Object.”
- Plot the corresponding point Q on the graph of $f'(x)$ by typing $Q = (x(A), m)$ in the input bar. As you use the mouse to drag the point labeled x , notice that the point P moves along the graph of $f(x)$, and the point Q moves along the graph of $f'(x)$.
- Right-click on the point Q to turn traces on (or off). To erase existing traces, use the View menu to refresh the view.
- Use the text tool  to insert a text object showing the coordinates of P . In the text creation dialog box, type the phrase “P =” (no quote marks), and then select P from the “Objects” drop down menu. This creates a field that holds the value of the variable P .

- Similarly, create text objects for the slope of $f'(x)$ and the coordinates of Q .
- Use the special action tool  to create a check box to show the tangent line at P . Select the tangent line from the drop down menu.
- Similarly, create check boxes to show:
 - the slope of the tangent line – select the text object with the slope from the drop down menu.
 - the point Q and its coordinates – select both Q and the text object with its coordinates from the drop down menu.
- Click the special action tool  to create an input box for the function f . Type “ $f(x) =$ ” as the caption and choose $f(x)$ from the “Linked Object” drop down menu. After creating the input box, right click on its caption, open the Object Properties, and use the “Style” tab to set an appropriate length for the input box.
- Format colors, font sizes, line thicknesses and point sizes as desired, and move the input box, check boxes, and text fields to appropriate locations.
- Move the point labeled x , turn the check boxes on or off, and turn the traces for Q on or off, so that the worksheet appears as you want the user to first view it. Adjust the toolbar and window size. Then save the worksheet and export it as a webpage. (See the section below on Creating Applets for the Web.)

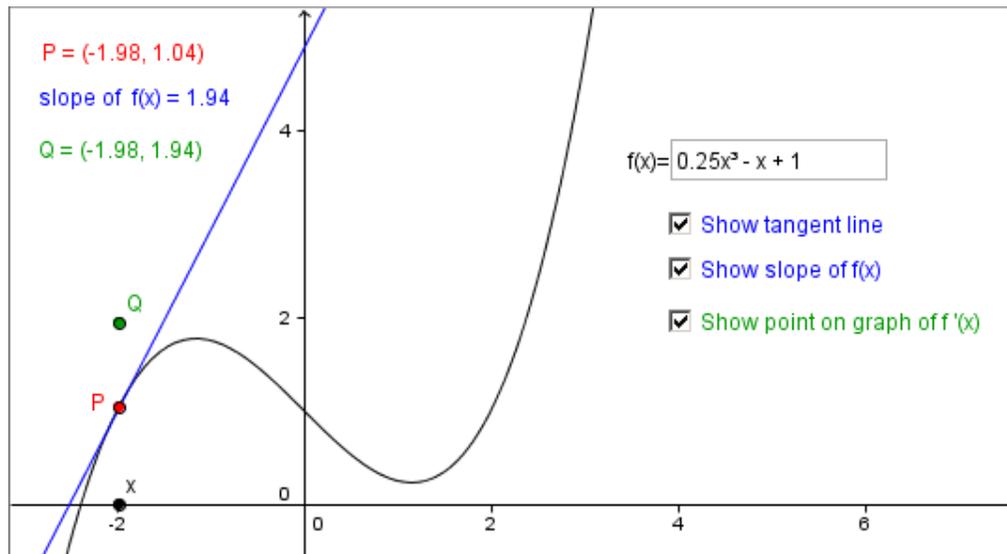


Figure 2: Completed worksheet for the graph of the derivative

Optional Enhancement for Graph of the Derivative Function

Type $g(x) = \text{Derivative}[f]$ in the input bar to get the graph of $f'(x)$. Change this graph to a different color, and insert a check box to show/hide it. This allows the user to see the point Q moving along the graph of $f'(x)$.

Creating a New Tool: Newton's Method

You can create a new tool in GeoGebra to iterate the steps of a construction. We demonstrate this via Newton's Method.

- In the Options menu, turn "Point Capturing" off.
- Create a function, like $f(x) = \frac{1}{4}x^3 - x + 1$ in the input bar.
- Use the point tool  to create a point A on the x -axis.
- Create the corresponding point P on the graph of $f(x)$ by typing $P = (x(A), f(x(A)))$ in the input bar.
- Use the special line tool  for tangents to construct the tangent line to the graph of $f(x)$ at P .
- Use the point tool for intersections  to construct the intersection of the tangent line and the x -axis.
- From the Tools menu, select "Create New Tool."
- In the dialog box, under "Output Objects", use the drop down list to select the tangent line, the point P , and the x -intercept. Then click "Next."

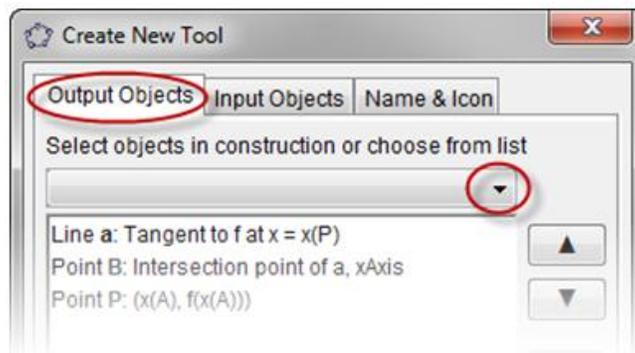


Figure 3: Create New Tool - Select the Output Objects

- The function f should automatically appear in the “Input Objects” tab. Select the initial estimate A from the drop down list. Then click “Next.”

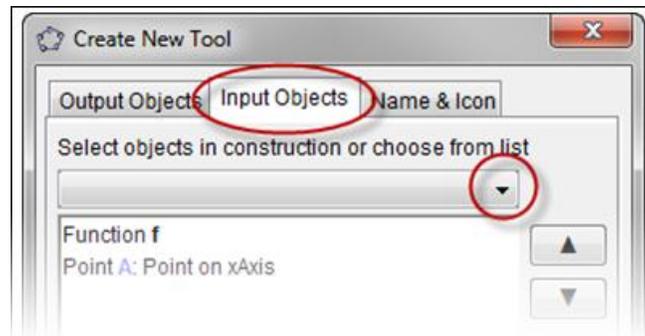


Figure 4: Create New Tool - Select the Input Objects

- In the “Name & Icon” tab, type the name of the tool: “Newton’s Method.” The command name (for use if you type the command in the input bar) appears automatically. Change this if you like. Add tool help directions, and change the icon if you have an appropriate image file stored on your computer. Make sure the “Show in Toolbar” box is checked. Then click “Finish.”
- Practice using your new Newton’s Method tool.
- Format colors, font sizes, line thicknesses and point sizes as desired. Delete all of the tangent lines and points on the graph, leaving just the graph of the function and the initial point A . Adjust the toolbar and window size. Then save the worksheet and export it as a web page. (See the section below on Creating Applets for the Web.)

Optional Enhancement for Newton’s Method

Click the special action tool  to create an input box for the function f . Type the caption “ $f(x)=$ ” and choose $f(x)$ from the “Linked Object” drop down menu. After creating the input box, right click on the caption, open the Object Properties, and use the “Style” tab to set an appropriate length for the input box.

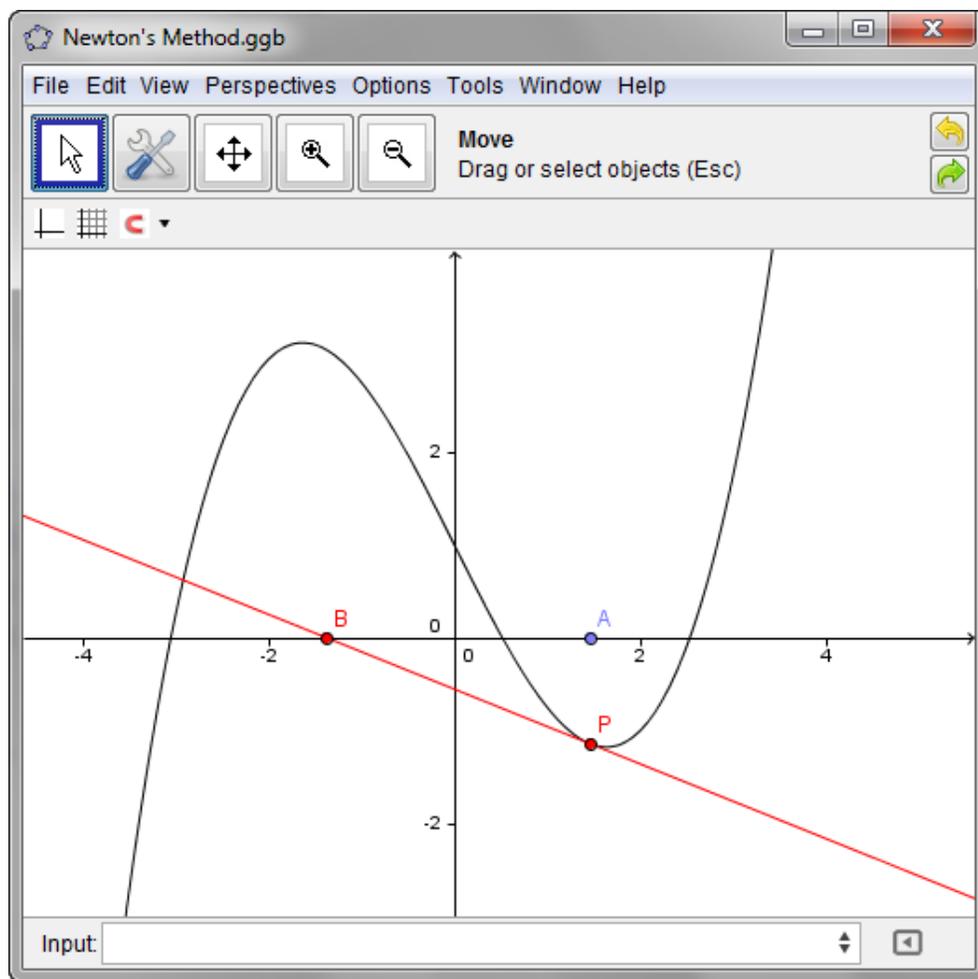


Figure 5: Finished Newton's method worksheet

Creating Applets for the Web

Before saving a GeoGebra worksheet as a web page, adjust the format to what you want the users to see. For example, you might want to use the View menu to hide the algebra portion of the worksheet. Use the Tools menu to customize the toolbar, giving users only the tools they need for that particular activity. Finally, resize the window to the desired size of the applet that will be embedded in the web page. If you create a web page using a full screen GeoGebra worksheet, students will have to scroll to see parts of it in the resulting web page.

Once the worksheet has the desired format, follow these steps to create a web page containing the GeoGebra worksheet as an applet.

- Choose “Export ... Dynamic Worksheet as Webpage” from the File menu. Click the “Export as Webpage” tab in the resulting dialog box.
- Type a title for the web page in the Title field, and type directions and explanations in the fields for Text above and below the construction.
- Click the “Advanced” tab (below the author information field), and select the desired features under Functionality and User Interface.
- Make sure that “Include jar files” is unselected.
- Click “Export.”
- Use an html editor if necessary to format and/or change the text on the web page.
- Put the resulting web page on your web server or somewhere else where students can access it. We frequently embed these pages directly in Blackboard; presumably you can embed pages in your course management system as well.

Conclusion

As the examples in this paper show, the ease and versatility of GeoGebra make it an ideal tool for creating dynamic worksheets for both classroom demonstration and student exploration. GeoGebra’s close connection between the algebraic and geometric representations of functions helps students visualize concepts from calculus from multiple perspectives. Moreover, since it is extremely simple to make dynamic Java applets available on the web with GeoGebra, you can easily make it possible for your students to continue interacting with mathematical concepts beyond the classroom.

For additional examples of creating calculus demos using GeoGebra we invite you to visit our web site, at <http://www.jcu.edu/math/ICTCM2012>, and/or see the paper listed in the reference section below.

Reference

[1] D’Ambrosia, Barbara K. and Carl R. Spitznagel. “GeoGebra: Dynamic Mathematics Made Easy.” *Proceedings of the 22nd Annual International Conference on Technology in Collegiate Mathematics* (2011).