STRATEGY FOR GRAPHING POLYNOMIALS & RATIONAL FUNCTIONS

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Almost all books in College Algebra, Pre-Calc. and Calculus, do not give the student a specific outline on how to graph polynomials and rational functions. Instead, domains, intercepts, limits, continuity and asymptotes are detailed separately, and the student is left bewildered in a mathematical maze trying to find a way out. This paper uses all of the individual graphing ingredients and weaves them in a step by step procedure, where the student can go through it mechanically and without a hitch.

An interactive (bullet format) outline follows with two examples to demonstrate the procedure.

Procedure:

1. State the domain.
2. Find the Y-intercepts (x=0), and the X-Intercepts (y=0) the easy one in particular. **You can use the synthetic division to find the rational zeros for the given polynomial function. Basically, if f(c)=0, then (x-c) is a factor of f(x).**
3. For rational functions **ONLY**, find the asymptotes.
4. Perform the sign analysis.
5. Graph the function.

Now if we elaborate on step (3) for rational functions, we have: vertical asymptotes, horizontal asymptotes, and oblique/slant asymptotes.

Asymptotes For Rational Functions

1. **Vertical Asymptotes:**
   Whatever makes the denominator zero is your vertical asymptote, as long as you do not have 0/0. Remember that 0/0 means that you have a hole in the graph.

2. **Horizontal & Slant asymptotes:**
   Are the limits of the rational function as $x \to \pm\infty$

<u>Horizontal & Slant asymptotes</u>

Consider the following rational function:

$$f(x) = \frac{a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0}{b_m x^m + b_{m-1}x^{m-1} + \cdots + b_1 x + b_0}$$

1. If the power of the numerator is the same as the power of the denominator (n=m), then the horizontal asymptote is y = the ratio of the leading coefficients of x, $y = \frac{a_n}{b_m}$
2. If the power of the numerator is less than the power of the denominator (n<m), then the horizontal asymptote is y=0.
3. If the power of the numerator is greater than the power of the denominator by one degree (n=m+1), then the slant asymptote is y= the quotient of the division.
   <u>Here the synthetic division can prove helpful when warranted.</u>
   Notice that for rational functions, it should be very obvious that <u>you cannot</u> have horizontal and slant asymptotes at the same time.

Using the Outlined Procedure Graph:

$$f(x) = (x-1)(x+2)(x-3)$$

1. Domain: $x \in (-\infty, \infty)$
2. Y-Intercept: x=0 → $(0,6)$
3. X-Intercepts: y=0→ $(1,0), (-2,0), (3,0)$
4. Sign Analysis:

$-\infty$ -2 1 3

$+\infty$

x				
$f(x)$	−	+	−	+

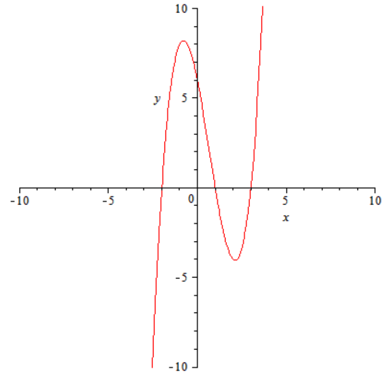

Fig. (1)

Using the Outlined Procedure Graph:

$$f(x) = \frac{2(x^2 - 1)}{(x+3)(x-2)}$$

1. Domain: $x \in (-\infty, -3) \cup (-3, 2) \cup (2, \infty)$
2. Y-Intercept: x=0 $\rightarrow \left(0, \frac{1}{3}\right)$
3. X-Intercepts: y=0 $\rightarrow (-1, 0), (1, 0)$
4. Asymptotes:

   $x \rightarrow \pm\infty, y \rightarrow 2;$ $\quad y = 2$ *is a Horizonatl Asymptote*

   $x \rightarrow -3, y \rightarrow \pm\infty;$ $\quad x = -3$ *is a Vertical Asymptote*

   $x \rightarrow 2, y \rightarrow \pm\infty;$ $\quad x = 2$ *is a Vertical Asymptote*

5. Sign Analysis:

$-\infty$ $\quad -3$ $\quad -1$ $\quad 1$ $\quad 2$

∞

x					
$f(x)$	+	−	+	−	+

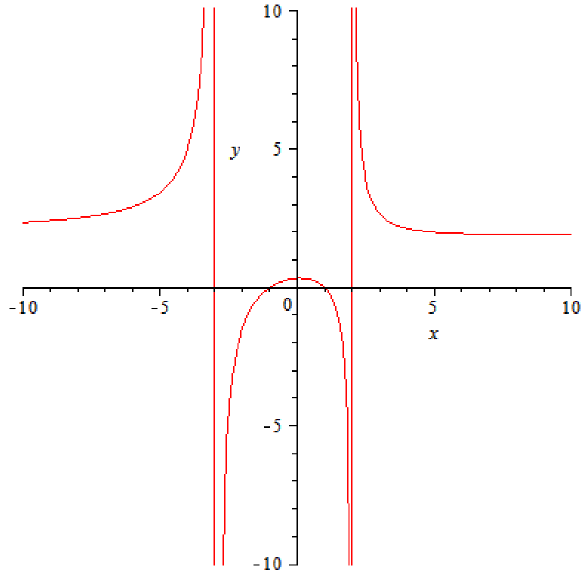

Fig.(2)