# THE EFFECTIVE USE OF ONLINE DIAGNOSTIC TOOLS TO HELP IDENTIFY APPROPRIATE LEARNING MODULES AS A FORM OF PROFESSIONAL DEVELOPMENT FOR MATHEMATICS TEACHERS


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## Introduction

According to the June 30, 2001 issue of the Science News, by about age 12, U.S. students who feel threatened by mathematics start to avoid math courses, do poorly in the few math classes they do take, and earn low scores on math-achievement tests. Some scientists have theorized that kids having little math aptitude in the first place justifiably dread grappling with numbers. That conclusion does not add up, at least for college students, according to a study in the June Journal of Experimental Psychology. On the contrary, student's intrusive worries about math temporarily disrupt mental processes needed for doing arithmetic and drag down math competence.

Math anxiety exerts this effect by making it difficult to hold new information in mind while simultaneously manipulating it. It soaks up working memory resources and makes it harder to learn mathematics, probably beginning in middle school.

Many teachers drill students in procedures that are soon forgotten. Students learn strict, seemingly infallible rules, ranging from the mechanics of addition to the intricacies of factoring algebraic expressions. In contrast, children should learn to apply mathematical rules fluently. Further, teachers often teach the mathematical procedures poorly. Students lack proficiency not from inadequate drill, but because too many teachers themselves do not understand the mathematical principles behind the rules. Most U.S. colleges and universities do not teach mathematical foundations. Therefore teachers only teach what they know (Hailikari, T., Nevgi, a., & Komulainen, E.).

The effective use of diagnostic tools, hands-on activities and computer games is the key to assist mathematics teachers in "what" and "how" they teach mathematics in the classrooms. The assessment, along with the use of hands-on activities and computer games suggested in this paper will help teachers in identifying their weak skill areas in various mathematics contents and helping them to achieve the proficiency needed in order to be a competent teacher in the mathematics classroom.

## National Crisis

Numerous national commissions recently have issued reports drawing attention to the crisis in U.S. student achievement in mathematics (Florida Coalition for Improving

Mathematics and Science Literacy website – www.flsummit.usf.edu/). "The workforce required for the U.S. to prosper as a nation is not being trained adequately… and the Commission believes the greatest impact will come from expanding programs to train teachers of science, mathematics, and technology" (*Commission on Implementation of United States Exploration Policy,* Aldridge Commission). "Thirty-four percent of math teachers (nationwide) lack a major or even a minor in the subject that they teach" (*Commission on the Future of the U.S. Aerospace Industry*, Walker Commission). Yet of the twenty fastest growing occupations projected through 2010, fifteen of them require substantial mathematics preparation (U.S. Bureau of Labor Statistics). Eight of the top ten degrees in demand in the job market are mathematics/science based: accounting, electrical engineering, mechanical engineering, economics/finance, computer engineering, and information sciences and systems (National Association of Colleges and Employers). And these degrees do not even address emerging fields such as genomics, computational biology, data mining or digital mapping. And students with higher-level mathematics skills earn more – up to double (Vanneman, A., Hamilton, L., et al. & National Center for Educational Statistics). To enable our students to participate and achieve in mathematics, we must build programs to address the areas of deficiency.

**Florida** is one of the lowest performing states in the percentage of young adults with a high school diploma. The state's 8th graders, particularly from low-income families, perform poorly on national assessments in math. Students from the 8th to 12th grades are not taking the upper-level math courses they need to succeed in rigorous college courses. Compared with best-performing states, small percentages of Florida high school students are enrolled in upper-level math (38%) compared to 59% for the top states. Compared with other states, the likelihood of 9th graders enrolling in college within four years is low primarily because the proportion of high school students graduating is among the lowest in the country (http://measuringup.highereducation.org). Yet students of all income levels who take rigorous mathematics and science courses in high school are more likely to go to college (Gonzales, P., Williams, T. et al.).

**Teacher quality counts**

It has been shown that nationwide low-income and minority students often have less experienced, less prepared teachers (Darling-Hammond, 1999). Poverty, large classes, and often a teacher with few opportunities for professional growth or technical assistance, combine to create a situation where students have little opportunity to succeed. However, the Federal *No Child Left Behind* Act requires that districts take action to make certain that there are not a higher percentage of unqualified teachers teaching poor and minority students than are teaching other students. Districts statewide often have high percentages of Not "Highly Qualified Teachers" (HQT) teaching in High Poverty classrooms (9.2% statewide).

NCLB requires that all teachers be certified in their field. This plan will not only help teachers master the necessary content for full certification but also provide them with real-world applications of the mathematics principles being learned. Therefore, if teacher quality is the single greatest predictor of student learning gains (Haycock, Education

Trust, 2001), then students in many elementary and secondary school classrooms are at a disadvantage in not having a teacher who is well prepared to teach mathematics subject matter or apply literacy strategies and skills to teach math content.

It is imperative that we find new ways to provide professional development support for these teachers. The need for staff development of mathematics teachers is evidenced by in part by (1) the demand for professional development in mathematics courses delivered through both traditional and online formats, and (2) the number of teachers pursuing alternative routes to professional certification.

From national and international studies, to statewide data, to local data, the need for increasing the mathematics skills of teachers is clear. Online professional development modules, available anytime and as-needed is one tool that is critically essential. Providing teachers and students with hands-on and computer-base interactive teaching and learning games is another tool that is crucial.

## Solutions: Utilizing technology to bring standards-based best practices to teachers

### I. Online Diagnostic Tools

***The Diagnostic Tool*** was developed to increase teachers' content knowledge of mathematical concepts and their ability to teach these concepts guided by recommendations of the National Council of Teachers of Mathematics, the National Council of Supervisors of Mathematics and the Third International Mathematics and Science Study. Lessons were aligned to the new Florida Next Generation Sunshine State Mathematics Standards.

***The online modules*** provided participants with a practical, hands-on, systematic approach to the teaching and learning of mathematics. Using an applied problem solving approach, these self-paced modules were designed to enable the participant to learn, re-learn, practice, and review a wide range of mathematics topics. The modules also helped improve learners' understanding of current mathematics applications by allowing them to solve real-world problems.

The content modules were developed and designed to accommodate learners with specific skills in grades K – 12. Learners took a diagnostic test that will help identify the needed skill areas. Learners could complete the diagnostic test and follow the systems guidance and complete the course modules all in the privacy of their home, computer lab, or classroom. In this way a well-rounded student will emerge.

Individuals who want to improve their mathematics skills, both basic and advanced, with topics ranging from basic arithmetic skills and computation to algebra, geometry, and trigonometry, or review topics in preparation for new teaching assignments have found the modules to be useful. The modules were designed to be taken at an individual pace with many opportunities for practice in the form of exercises and practical applications

with instant feedback. Each module contained a Pretest, the content, and a Posttest. Some modules also include separate practice exercises in addition to the practice presented within the content. Upon successful completion, participants were able to: understand and connect specific key ideas regarding mathematics operations and applications; the potential for introducing interdisciplinary mathematics scenarios to enhance student learning; and, understand and be able to teach mathematics applications with confidence.

Through the use of the diagnostic pretest, users will be able to determine the weak skill areas in which they need further instructions. Based on the pre-test results, users will receive an online prescription as to which modules they will need to complete in order to reach proficiency. In this way users will complete the modules that specifically address their deficiencies. Users will be able to complete as many modules as they want, however, they need not complete any modules that cover areas where they are proficient. In this way the program will react to individual content needs in mathematics, making the online assistance individualized, and standards aligned.

At the high school level, a secondary-level algebra course mapped to the new mathematics standards was developed to help mathematics teachers review and reinforce their basic skills. A forty-question pretest was designed with the purpose of benchmarking the teachers' mathematical skill level.

## II. Innovative Teaching Tools

***Calculus (Cakulus)*** is a beautiful subject with many dazzling applications. Dr. Tom Apostol, Professor of Mathematics, Emeritus, at Caltech and Director of *Project MATHEMATICS!* (that produces computer animated videos bringing exciting geometric methods to a wide audience) "was stunned to learn that many *classical* problems in calculus can be easily solved by an innovative visual approach that makes no use of formulas" (Apostol 2000). The new method was conceived in 1959 by Mamikon Mnatsakanian, Ph.D (http://www.mamikon.com/) who has developed his powerful geometric methods into a universal teaching tool for a wide audience, employing not only pictures, but also hands-on manipulatives, coloring books, games, puzzles and computer-based interactivities (http://www.its.caltech.edu/~mamikon/ ). He has taught these methods at UC Davis and in several schools in Northern California, ranging from Montessori elementary schools to inner-city ones. He has also presented them at many teacher conferences in California. Responses from both students and teachers are enthusiastic, because the methods are vivid and dynamic and don't require the algebraic formalism of trigonometry or calculus, not even calculations. Mamikon convinced Apostol that his methods have the potential to make a significant impact on mathematics education, especially if they are combined with visualization tools of modern technology. Since then over thirty joint papers were published on these innovative ideas in leading American Journals (see http://www.mamikon.com/ArticlesUS.html). They received numerous awards.

About a decade ago, while working as a school district mathematics specialist, Hui Fang Huang "Angie" Su, Ed.D (http://nova.edu/~shuifang/), creator of Project MIND – Math Is Not Difficult (http://projectmindmathisnotdifficult.com) came across Mamikon's work. She was surprised to find many similarities in their approaches. Impressed with Mamikon's work, Su invited Mamikon to be the keynote speaker at the annual Project MIND conference. Since then, they have collaborated and exchanged ideas. Project MIND has received numerous awards for its innovation and ability to help students become excited about math and improve their math skills in a short period of time. Students who were exposed to Project MIND strategies, especially at the elementary level, obtained impressive test results. This includes the learning disabled, the gifted, and teachers of all levels.

Recently, Mamikon and Su decided to combine their life-long work to help benefit teachers and students of all ages and levels bridging grade school mathematics with elementary replacement of the advanced calculus under the new Project called: ***Shapes and Numbers: from counting to calculus and beyond.***

**Part 1:** ***Shapes in Nature.*** Our first grasp of physical processes in Nature is visual through the shapes and their dynamic changes. This visual grasp is phenomenal so learning based on visual approaches is of fundamental importance and efficiency.

A new simple visual approach to analyzing complex natural shapes demonstrates how they are generated by basic ones, and how to "slice" the shapes naturally and reshape them into each other. The slicing is not traditional rectangular, but triangular and can be obtained in a physical manner: tangential to the curves, along the velocities. This leads to simple visual solutions of many classical problems in calculus with no formulas, equations or any special knowledge.

Examples are: triangular slicing of circles to rectangles, of any oval ring to a circle, cycloid to three circles, bicycle paths and tractrix to circular sectors, and many others including ellipse, parabola, higher power functions, pursuit curves, and exponentials. Children can even easily generalize and solve problems that calculus cannot do, taking their creativity beyond calculus! Now, for the first time ever, the fundamental knowledge and achievements of modern mathematics can be accessible to everyone: to visualize, truly understand and be able to appreciate the beauty and logic of natural shapes and their applications (http://www.its.caltech.edu/~mamikon/VisualCalc.html). Mamikon's method based on physical approach allows children to discover also important physical concepts.

**Part 2:** ***Numbers and Operations.*** Working with pre-kindergarten and kindergarten students challenges us to use familiar concepts while building new bridges to the unfamiliar and abstract. The best way to begin building the base-ten concept is to start with the everyday relationships, objects, and terminology that students know, such as fruits and vegetables, candies, classroom items, toys, animals, and people. Children develop relationships early on. They learn quickly to play and work with each other. Soon, children form "best friend" relationships that extend from classroom to play and

visa- versa. As children play within classroom areas, they often work in pairs, groups, and teams to play, build, create, and solve problems related to the tasks at hand. In the same way, we can build mathematical best friends to teach higher level concepts for solving mathematical problems. Children learn that 1 and 9, 2 and 8, 3 and 7, and 4 and 6 are "best friends." They also learn that 5 is best friend with its twin and that 0 and 10 are best friends.

The best friends' concept helps to easily perform additions of multi-digit numbers, subtraction, multiplication, division, fractions, and other number operations such as powers and roots. Besides best friends, we will introduce complementary numbers such as adding to powers of the base; for example, 98 and 2 add up to 100, which is square of 10.

The concept of *Best Friends* is easily extended to other numeration system. We utilize a rich delivery method to help hold students' interest in the learning process including the use of songs, arts, puzzles and brainteasers, mental calculation, and competition. The primary goal is to build connections between the *Best Friends* concept and the visual geometry concepts previously mentioned through the use of computer-based games and hands-on activities.

Mamikon and Su have extensive experiences in working with elementary-school age children, including children with autism. Su's research work with children with autism at yielded favorable results with the activities she designed for the study (see http://www.scirp.org/journal/ce/ Vol.1 no. 3 pp. 149-153). Mamikon has worked with children in K-8 at Montessori School in California. Historically Montessori schools were designed for children with disabilities, and they still have a portion of such children. Mamikon had a specific goal to design manipulatives and activities that will allow these children to discover the basic concepts of mathematics, including the ideas leading to calculus (Visual Calculus. See http://www.its.caltech.edu/~mamikon/calculus.html). He has also created computer based interactive educational games which simulate real life situations so the students, in seeking to win, discover important math concepts along the way (see http://www.its.caltech.edu/~mamikon/MamiGames.html). Mamikon and Su have designed and created unique hands-on and computer-based activities which allow students to discover basic mathematical concepts by themselves during engagement. The activities are designed to explore and facilitate learning a difficult subject in a playful manner.

Over the last 14 years, Caltech's *Project Mathematics!* has changed its direction from making standard mathematics videos to developing Mamikon's innovative approach, Visual Calculus, which Mamikon calls Cake-U-Lus for kids (like in cake). There are numerous scientific and popular journals and news articles worldwide written about Mamikon's contributions to mathematics and mathematics education. According to Tom Apostol, "using Mamikon's approach, we obtain a result that is not only simpler than the original treatment by Archimedes, but also more powerful because it can be generalized to higher powers." Mamikon's teaching experience shows that students from diverse groups, especially those historically underrepresented in mathematics, respond in a very

positive way to this geometric approach. It creates student's lovely attitude towards the so often "hated" Math.

Project MIND – Math Is Not Difficult was created by Su in 1985. Since its creation, the Project has impacted hundreds of classrooms throughout the United States. It has changed the way teachers teach and students learn mathematics. Su has received 85 awards and recognitions and more than 65 news articles, television interviews (including NBC World Nightly News), and magazines (including RedBook Magazine) have publicized her work with Project MIND. In 2009, she was the recipient of Florida Association of Mathematics Teacher Educators Mathematics Teacher Educator of the Year in recognition of her excellence in the field of education and in the areas of teaching, research and service.

**Part 3:** ***Educational Games using technology.*** Although, hands-on activities and games have proven to be effective means of lesson delivery, we believe these delivery methods have limitations. Hands-on activities/games don't give rise to as many possibilities as computationally facilitated ones. For example, with coloring books, you can produce a couple of objects, but a computer can produce an indefinite number. Computers also allow the students to check their results, which provide immediate assessment and feedback for students, parents, and teachers. Nevertheless, we believe that students should always start with hands-on activities in order to develop tangible experiences of their reality. The instructor brings them ready-made examples and students use their creativity to continue the learning process using their innate abilities through hands-on experiences and games. This practice is invaluable, as students can observe and experience how mathematics unfolds in front of their eyes.

There are tons of hands-on activities out there, but our activities are very specifically and uniquely designed so that the winning strategy (of our computer games) is related to a basic mathematical concept. Students will easily find the "perfect" strategy, which will allow them to discover the concept on their own. To quote the famous mathematician and educator George Polya, "The best way for a student to learn a concept is to discover it on his or her own." To the ordinary approach we add activities that will engage and enlighten young children. For example, Polya would say, "here are two numbers, find a common factor." That is not an engaging question. It may be appropriate in a university setting, but not for young children or beginning students. In contrast, our PolyStar game reaches the same goal in an engaging, playful manner. We suggest that you check the numbers 5 and 12 or 7 and 12 to watch the NSF logo appear in front of your eyes. The description of this game and other examples could be found at http://www.projectmindmathisnotdifficult.com/ (click on the PolyStars link).

Mamikon and Su have presented their mathematics education strategies in hundreds of training programs, K-12 schools, colleges, and universities, including Montessori schools and schools for children with autism throughout the United States. Historically Montessori schools were designed for children with disabilities, and they still have a portion of such children. Samples of the activities can be seen at the website www.mamikon.com by clicking the "Caltech" link.

This project is significant and systemic because it competes with calculus on an elementary level. It leads to simple visual solutions of the classical problems of calculus with no formulas, equations, or advanced mathematics. Russian popular magazine "Science and Society" called this on the cover page "A breakthrough in mathematics after 2,500 years". Now, for the first time ever, the fundamental achievements of modern mathematics will be accessible to everyone: to visualize, truly understand and be able to appreciate the beauty and logic of natural shapes and their applications. We can even generalize and solve problems that calculus cannot do! Because classical calculus problems relate to physical motion, students will learn important physics concepts as well. Montessori children were so attracted and engaged in the natural playful games and activities that they were calling him "Magic Mamikon" without realizing that they were learning advanced math.

The enthusiastic response of both students and teachers who have witnessed the use of these methods in classrooms at various levels suggests that they could have a great impact in teaching young students important mathematical ideas and motivating them to continue their studies at higher levels. These interactive materials will provide a valuable pedagogical tool that presents mathematics for what it is: understandable, exciting, fun, and eminently worthwhile. Here is what a group of 25 former students wrote in their evaluation of Mamikon's Visual Calculus course: "We learned with our eyes, our hands, our heads, and our hearts"!

Mamikon's Visual Calculus is like Proofs Without Words, or Aha's of Martin Gardner (http://en.wikipedia.org/wiki/Visual_calculus). Because the method is entirely visual and dynamic, there is no better way to present it, but through computer animations and interactive games. The use of computer also provides infinite options so necessary for developing students' learning experiences.

These games could help users learn or teach mathematics more effectively. Today, many teachers have used the games and diagnostic tool with their students as an organized instructional tool.

Currently, various student groups ranging from private to charter to public schools in grades k – 12 are utilizing the games and the learning modules to improve their mathematics skills.

## Bibliography

Apostol, T.M. (2000). A visual approach to calculus problems. Engineering & Science. Vol. 63. No. 3. Pp. 23-31. (http://www.its.caltech.edu/~mamikon/VisCal.pdf)

Darling-Hammond, L., (1999). Teacher quality and student achievement: a review of state policy evidence, *Center for the Study of Teaching and Policy*.

Gonzales, P., Williams, T., Jocelyn, L., Roey, S., Kastberg, D., & Brenwald, S., (2008). Highlights from TIMSS 2007: mathematics and science achievement of U.S. fourth- and eighth-grade students in an international context, *NCES.*

Haycock, Education Trust (2001). States can close the achievement gap by decades end, new education trust state and national achievement gap analysis. *The Education Trust.*

Hailikari, T., Nevgi, A., & Komulainen, E., (2008). Academic self-beliefs and prior knowledge as predictors of student achievement in mathematics: a structural model, *Educational Psychology, 28(1),* 59-71.

Mnatsakanian, M. http://www.mamikon.com/

Project MIND- Math Is Not Difficult. http://www.projectmindmathisnotdifficult.com.

Vanneman, A., Hamilton, L., Anderson, J., Rahman, T., & National Center for Education Statistics, ((2009). Achievement Gaps: How Black and White Students in Public Schools Perform in Mathematics and Reading on the National Assessment of Educational Progress. Statistical Analysis Report. NCES 2009-455.