TEACHING APPLIED CALCULUS UTILIZING MATLAB

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Abstract

MATLAB is integrated within or alongside a Calculus curriculum to enhance and motivate understanding of the Mathematics, as applied directly to the Sciences (e.g Physical, Life, Social, Economics/Finance). This fusion of Math, Science, and Technology occurs within individual lab modules that can parallel or sequentially follow a traditional Calculus lecture. Each lab consists of a specific case study that combines i) a particular principle of Calculus, ii) its natural presence within a distinct area of Science, iii) implementation of Technology via a computing tool/language, MATLAB, and iv) subsequent questions and outputs, as well as related observations, analysis, and conclusions.

Key Terms: *Applied Calculus; MATLAB; STEM*

Introduction

Students taking a Calculus course traditionally experience lectures that present derivations and processes generally confined to mathematical expressions. As a stronger foundation of student understanding is established, such lectures and related problems venture further at times, engaging the mathematics in word problems with applications in the Sciences and other 'real world situations'. However, anecdotal evidence and curriculum searches indicate that such application problems tend to merely skim the surface, and generally do not extend to deeper or broader possibilities, such as those demonstrated in this paper. As a result, students are not exposed to significantly richer applications until, and if, they engage in graduate study, or come across specific applications during post-college experience in industry.

This paradigm possesses inherent shortcomings in several aspects. Perhaps foremast is the absence of a more natural pedagogical 'bridging', in that student exposure to Applications of Calculus transitions, from a relatively rudimentary experience, often times limited to the "Exercises" section of a Calculus text, abruptly and dramatically to addressing substantially more complex syntheses of Calculus, Science, and Technology within research or industry.   The lack of bridging presents a loss for all

involved: students, research, and industry. Note that other curriculum, within and outside of Mathematics, very often provide a smoother and measured transition between levels of advancement, facilitating student mastery, enthusiasm, and ultimate contribution.

More intensive student engagement (beyond textbook exercises) with the principles and their applications offer an opportunity for a deeper understanding and appreciation of the underlying Mathematics, as well as a more intuitive feel for those principles. This situation naturally suggests and fosters better-prepared students for work in research and industry. Furthermore it provides a more interesting experience for students, naturally providing motivation to engage further with the subject matter, possibly to more challenging aspects of Calculus, e.g. modeling in Applied Mathematics or related underpinnings inherent to aspects of Real Analysis within Pure Mathematics. The benefit of advancements across various fields becomes obvious.

Of course a variety of approaches can be utilized to address the issue and provide an educational experience that goes beyond the more usual and limited approach of 'textbook and lecture' that is standard fare in college curricula. One model has been implemented at the New Jersey Center for Science, Technology, and Mathematics Education at Kean University in Union, NJ, where initially one to three page lab modules were introduced in parallel to a Calculus curriculum. In order to enhance the learning experience, some of those modules were expanded to 'chapter versions', with increased detail regarding related Science, Calculus, and Technology content. Subsequently, the 'chapter version' model was used to develop additional labs on wholly new topics. That approach, which implements the 'chapter versions', is being investigated for implementation at Rutgers University in Newark, NJ with a focus on applications in Business, Economics, and Finance. The model of the 'chapter version' lab and one related implementation within Physics are discussed in this paper.

Approach

The Applications of Calculus course can be offered as i) a Calculus-based 'lab experience' that parallels a traditional Calculus lecture/course, ii) a lab embedded within a traditional Calculus course, or iii) a lab that runs serially beyond a traditional Calculus course. Students are expected to have aptitude for related subject matter, as pre-determined by testing and entrance requirements. However, students are not expected to have had previous exposure to the specific subject matter in Science or Technology, nor Calculus, beyond what has been learned in the corresponding lecture.

Because Calculus concepts are more broadly covered within the lecture counterpart of the lab course, the lab focuses primarily on just key aspects and applications of the related Calculus. As there is no counterpart for the Technology component, the initial lab class is entirely devoted to the fundamentals

of using MATLAB. Subsequent classes introduce additional aspects of MATLAB, as required for respective lab modules, thereby gradually exposing related Technology concepts and implementation as the course unfolds. This approach fosters progressively enhanced programming skills for students. In each lab, respective Science content is introduced (or reviewed depending on students' background) and inherent relationships to Calculus and Technology are examined.

We consider here i) a general instance, or method, of such application and then ii) a specific example, via aspects of a particular lab, to gain a better perspective regarding implementation. Note that just 'subsets' of the "particular lab" were selected in an attempt to strike a balance between maintaining brevity and providing a thorough picture of the methodology and process. One can then extrapolate how that particular lab, and other labs, achieve the goal of exposing students to, and engaging them in, the synthesis of Calculus, Science, and Technology.

Method

Basic components of the paradigm for any one lab module include:

i)   one particular concept within a traditional Calculus curriculum

ii) its natural presence in a particular area of Science and related applications

iii) implementation of a computing tool/language, in this case MATLAB, to perform related calculations and enhance understanding

iv) associated questions and outputs to facilitate and stimulate related comprehension

Outline

Lab Overview:

Principles regarding the Science and Calculus inherent to the lab are indentified and reviewed, as are related applications.

Pre-lab:

A series of questions, associated with the lab's specific aspects of Calculus and Science, is presented and designed to i) enhance deeper understanding of both, ii) stimulate further inquiry within the lab, and iii) facilitate directed inquiry regarding implementation of MATLAB.

Hypothesis:

'More specific' questions are raised, regarding the Science aspect and possibly related Calculus, i) establishing the overall goal of the lab and ii) helping to further guide the student in the subsequent *Activity* and its latter *Analysis*.

Activity:

Pointed questions are posed regarding the Technology (MATLAB) required to advance through the lab. Specific questions might address required commands, syntax, and algorithms. Ultimately this section requires the creation and execution of code. Depending on the specific lab and its goals, various forms of support material are provided, e.g. sample code, sample algorithms, and/or 'code comments' which suggest or prompt required code. In some labs, students are asked to provide analytic mathematical derivations as well. Analytic results can then be compared with MATLAB results.

Results:

Output is obtained from the execution of the developed code and presented in the form of tables and/or graphics which address the lab's Science inquiry and answer questions posed in the *Pre-lab* and *Hypothesis* sections.

Analysis:

Questions are presented relating to i) concerns previously raised in the *Hypothesis* section and, depending on the lab, the *Pre-lab* section as well, and ii) possibly other concerns regarding Calculus, Science, and/or Technology inherent to the lab.

Report:

The framework for the lab report is established and includes i) answers to questions raised in the *Pre-lab* and *Hypothesis* sections, ii) code developed within the *Activity* section (and Calculus derivations if applicable), iii) requested *Results* e.g. tables and graphics, and iv) related observations, analysis, and conclusions.

Example of Implementation

Lab Overview:

For a Physics-related lab regarding Center of Mass, the *Lab Overview* identified 'real world' applications and concerns regarding humans, vehicles, building structures, and objects in nature. References were made to the underlying Physics regarding center of mass for one-, two-, and three-dimensional systems and their dependency on i) mass of the object(s) and ii) related position of the particle(s) comprising the object or system. Corresponding mathematical relationships were introduced and reviewed, and tied in directly with prior Calculus lectures.

For one-dimensional systems, consideration was given to point-mass systems. Analytic solutions involved summations, where center of mass $\bar{x}$ is described as

$$\bar{x} = \frac{\sum_{i=1}^{n} M_i \, x_i}{\sum_{i=1}^{n} M_i}$$

with M_i and x_i corresponding to the mass and linear position of each respective particle.

Systems of continuous-mass density were also considered, where density could be constant throughout or vary across the object. Analytic solutions for center of mass involved definite integrals for one-dimensional objects of continuous-mass density, with center of mass $\bar{x}$ described as

$$\bar{x} = \frac{\int_a^b x\delta(x)dx}{\int_a^b \delta(x)dx}$$

where x is variable linear distance, $\delta(x)$ is density as function of linear distance x, and a and b are limits of linear distance x.

For two- and three-dimensional systems, consideration was given to just continuous-mass systems of constant density. Analytic solutions for center of mass involved integrals in these cases as well. For two-dimensional systems of constant density (mass per area in this case), center of mass $(\bar{x}, \bar{y})$ is described in integral form as

$$\bar{x} = \frac{\int x\delta L_x(x)dx}{M}, \qquad \bar{y} = \frac{\int y\delta L_y(y)dy}{M}$$

where x and y are respective variable linear distances across the two-dimensional body; $L_x(x)$ and $L_y(y)$ are the variable lengths of slices through the object, perpendicular to the respective axis; density δ is constant; and M is mass of the object. Three-dimensional systems were discussed as well. Further discussion addressed additional Science-related aspects of the problem.

Pre-lab:

Several developmental questions were posed to i) stimulate conceptual consideration of specific fundamental shapes and corresponding mathematical relationships, including related derivations of analytic solutions, and ii) facilitate discernment between constant and variable terms within the corresponding integral expressions. Beyond a deeper understanding of the Science and Calculus involved, portions of related answers would help form required MATLAB code. Questions included

1. *Consider an isosceles triangle of constant density. Should the center of mass be closer to the base, peak, or neither? Why or why not?*
2. *With respect to the following equation*

$$\frac{\int_a^b y\delta L_y(y)dy}{M}$$

   a) *Which terms would be constants? Why?*
   b) *Which terms would be variables? Why?*
   c) *Of the variable terms, which one(s) would be independent variables and which would be dependent variables? Why?*

Hypothesis:

Questions were posed regarding specific facets of the Science and Calculus aspects of the lab, further setting the stage for the subsequent *Activity*. One of the basic questions was:

Consider a specific two-dimensional symmetric shape of constant density, such as the isosceles triangle. If that shape is divided into two in such a way that the dividing line

i) crosses the triangle's <u>center of mass</u> and

ii) runs perpendicular to the triangle's axis of symmetry

should both resulting pieces have the same mass? Why or why not?

The question has an intuitive answer intended to help the student relate the combined Science, Calculus, and Technology aspects within the *Activity* section and in the latter *Analysis*.

Activity:

MATLAB plays the main role in this section (although analytic mathematics come into play as well) with initial questions, such as two listed here, geared at identifying functions, syntax, and numeric integration that will help in answering questions posed in the *Hypothesis* section.

1) *Symbolic Integration:*

    a) *Using MATLAB's symbolic integration tools, solve for $\bar{y}$ from the Pre-lab section. Use the requested "simplified definite integral form" of $\bar{y}$ as input for your MATLAB code.*

    It may help to first consider the following questions.

        i)   *What command is used to perform symbolic integration in MATLAB?*

        ii) *What syntax is used for an indefinite integral?*

        iii) *What syntax is used for a definite integral?*

        iv) *Do any other MATLAB commands need to be executed before executing the command for symbolic integration?*

    b) *Now analytically solve the "simplified definite integral form" of $\bar{y}$ from the Prelab section.*

    c) *Does your solution from the MATLAB symbolic integration agree with your solution of analytic integration?*

2) *Consider the triangle described in the Pre-lab section, in light of the question posed in the Hypothesis section (with the line that cuts through the center of mass and runs perpendicular to the y-axis). Calculate the mass and area of*

    a) *the total triangle*

    b) *the portion of triangle above the line*

c) *the portion of triangle below the line*

d) *the portion of triangle 'on' the line*

by numerically integrating for related area and mass. Use grid elements of length 0.2 for one run and then 0.1 for a second run.

MATLAB code is created based on such questions but also within a framework guided by comment statements. The following example illustrates that framework, in this case teaching students a numeric method for integration. (Often, simpler implementations of MATLAB are utilized, in the sense that symbolic functions are more commonly used, as opposed to numeric methods e.g. numeric integration.) Note that the students will then have solved related integrations three ways within the lab: analytically, symbolically with MATLAB, and by numeric methods that utilize MATLAB.

Example: *Iteratively loop through a pre-defined uniform grid, evaluating and tallying area/mass at each differential element along the way. Recall that for uniform density, area directly correlates to mass within the two dimensional shape.*

* NOTE that the comments and expected code are provided <u>here</u>, whereas <u>students</u> were provided with <u>just comments in the actual lab (i.e., no code)</u> and asked to develop related code statements.

Create uniform grid:

```
% Start loop over x-grid elements
for i=1:xnum;
% Start loop over y-grid elements
for j=1:ynum;
% Set current value of x to be number held in current element of x-grid array
x=xgrid(i);
% Set current value of y to be number held in current element of y-grid array
y=ygrid(j);
```

Sweep across grid in horizontal and vertical directions, evaluating at each step:

```
% CREATE CONDITIONAL STATEMENTS:
% If value for grid element indicates it is part of triangle AND above y-bar
    if ((y<=6-3*abs(x))& (y>2));
    % Increment triangle's total area and area above y-bar
    AreaTotal=AreaTotal+dA;
    AreaAbove=AreaAbove+dA;
    % Mark the grid element with a BLUE circle
    plot(x,y,'o b');
```
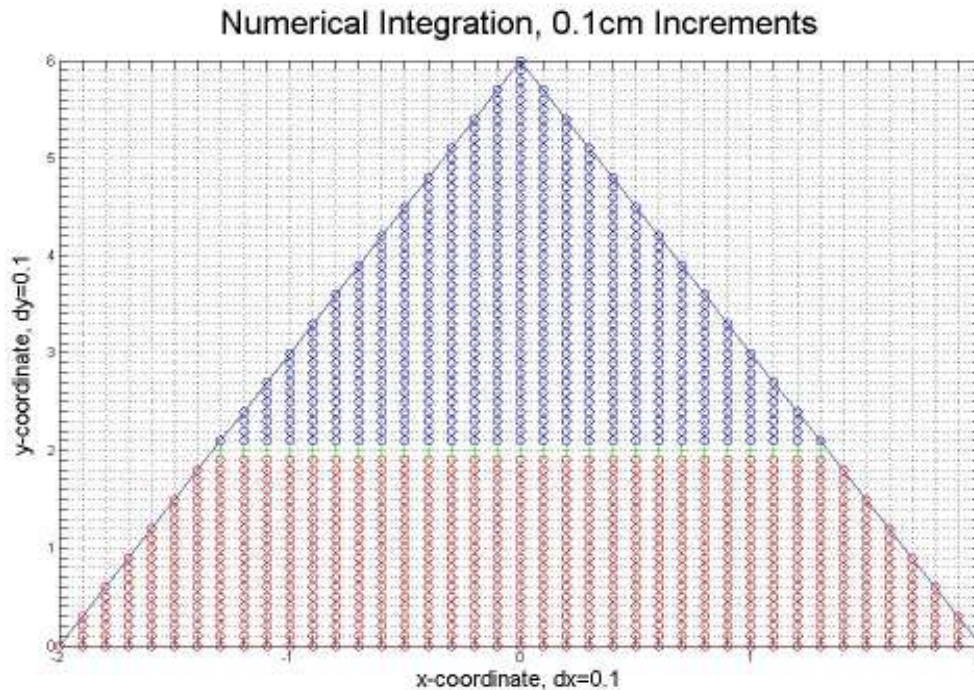
```matlab
% If value of grid element indicates it is part of triangle AND below y-bar
        elseif ((y<=6-3*abs(x))& (y<2));
        % Increment triangle's total area and area below y-bar
        AreaTotal=AreaTotal+dA;
        AreaBelow=AreaBelow+dA;
        % Mark the grid element with a RED circle
        plot(x,y,'o r');
% If value of grid element indicates it is part of triangle AND 'on' y-bar
        elseif ((y<=6-3*abs(x))& (y==2));
        %Increment triangle's total area and area 'on' y-bar
        AreaTotal=AreaTotal+dA;
        AreaOn=AreaOn+dA;
        % Mark the grid element with a GREEN "plus" sign
        plot(x,y,'+ g')
% End conditional evaluation statement and loops over y- and x-grid elements
        end
        end
        end
```

Results:

Tabular and graphic representations of results were created. The following MATLAB graphic correlates distribution of mass about, and 'on', a horizontal line drawn through the center of mass's y-coordinate. Visual results are examined in conjunction with corresponding numeric values of area, Science-related considerations, and limitations of the numeric approach.

Numerical Integration, 0.1cm Increments

y-coordinate, dy=0.1

x-coordinate, dx=0.1

Various other graphics were generated in relation to inquiries from previous sections of the lab.

Analysis:

Inquiries from previous sections of the lab are revisited, but now in light of MATLAB results, e.g.

> *In our numerical/computational integration, we calculated to determine an answer to the question: Does dividing the triangle horizontally across the center of mass' $\bar{y}$ coordinate yield two pieces of equal mass?". What did you find? Does it make sense? Why or why not?*

Closing Remarks and Conclusions

Much literature, both professional and 'commercial', underscores a need for educational advancements in the Science, Technology, Engineering, and Mathematics (STEM) disciplines, and combinations thereof, at secondary and collegiate levels. While achieving solid educational foundations in each of those areas is fundamental to reaching related goals, so is the ability to synthesize those disciplines for enhanced outcomes that benefit student comprehension and, ultimately, contributions those students will make as professionals to society. Within standard

undergraduate curricula though, relatively limited opportunities appear to exist that offer students that exposure to, and engagement in, the synthesis of those disciplines.

The model described within this paper however is one method of accomplishing the stated goals. Direct student feedback indicates so, although no formal measures (beyond typical course evaluations and volunteered or solicited feedback) have been made, nor related research conducted, to verify the extent. As discussed earlier and stated in this paper's title, the example presented here engages the student with various aspects of programming via MATLAB, e.g. both symbolic and numeric integration. Results are compared with those derived analytically using methods from the corresponding Calculus lecture.

It is worthwhile noting that the specific example discussed here is a subset of one of a number of lab modules. All of the labs are individually characterized by their own aspects and combination of Science, Calculus, and Technology. The use of Technology via MATLAB and its range of graphic possibilities introduces and enhances, to various degree depending on the specific lab, basic programming skills while leveraging resulting graphics that enable 'visual comprehension' of relationships between and within the Science and Calculus aspects of each lab. Other examples include i) a lab module dealing with Taylor Series expansion, where various orders of the expansion are depicted, demonstrating behavior of nth order curves, either 'peeling away' from or converging towards the exact solution (depending on perspective), and ii) a lab module dealing with Random Number Generators, where the condition of "striping" in pseudo-random number generation is illustrated. Note that these graphics represent a very small sample of MATLAB's capabilities.

Taylor Series Expansion for cos(x)

Lag Plot for LCG (with More Complex Striping)
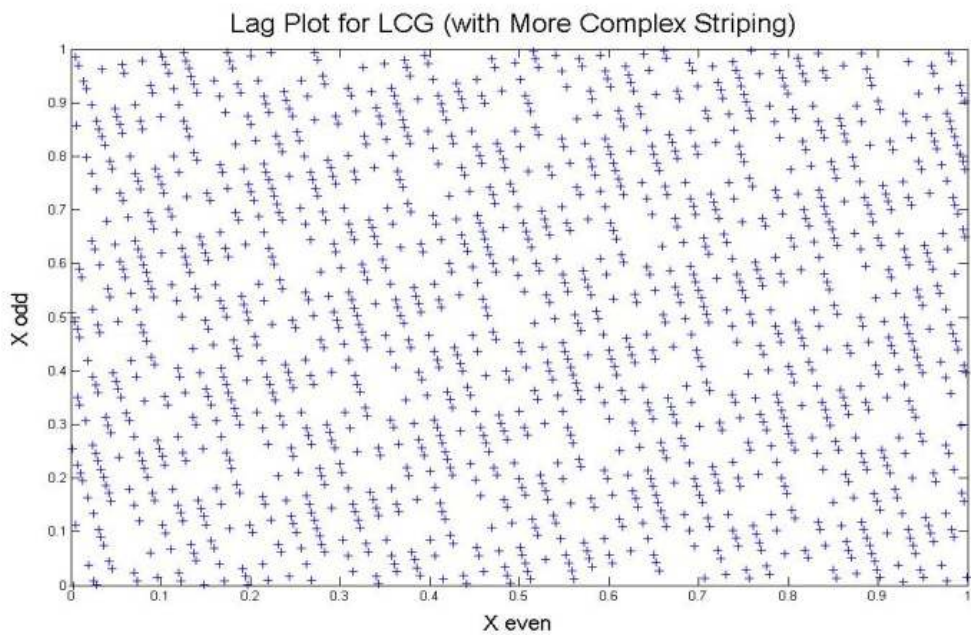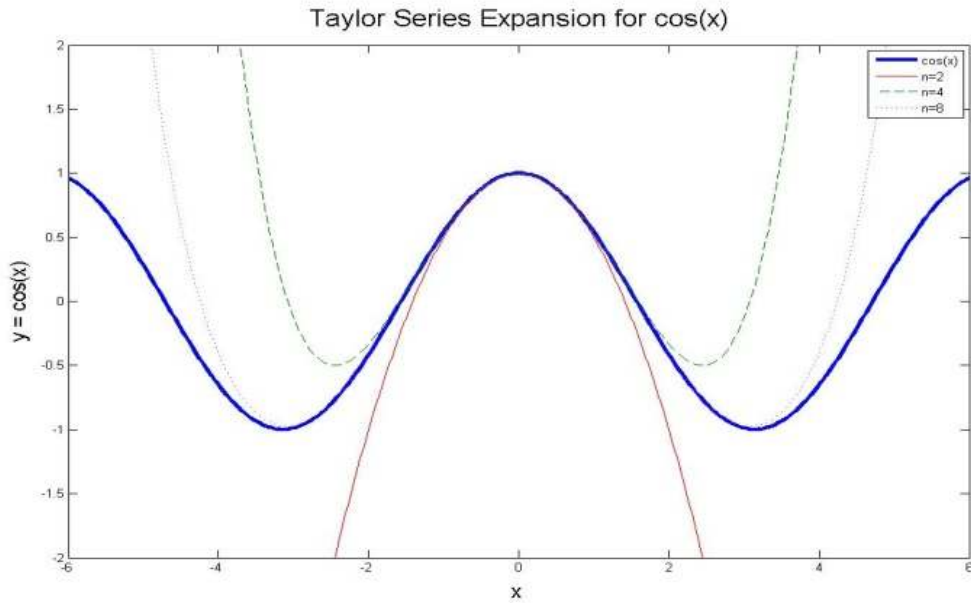
A common thread among all the labs is that, as each one progresses, the underlying Science, Calculus, and Technology are woven together through i) exposure, review, questions, and answers; ii) symbolic and/or numeric computation (via MATLAB) , iii) related results, including tabular and graphic representations; and iv) analysis and conclusions. In essence, all three disciplines work in concert to accomplish the stated goals.

Next Steps

It would be beneficial to scientifically investigate the effects of the described method to integrate Math, Science, and Technology on i) student understanding of related disciplines, i.e., specific areas in Calculus and Science; ii) student motivation, both to learn current subject matter and to further pursue related study and/or work within STEM; and iii) student understanding and interest in programming. Related challenges of such an investigation include running concurrent groups, one with the accompanying lab and one with just the Calculus lecture (i.e., sans lab). Other 'next steps' include expansion of applications/labs across the sciences, making the model available and beneficial for a wider audience.