The Impact of Technology and Dynamic Geometry Software on Fostering Discovery and Research at the Undergraduate Level, Some New Results.

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Abstract. Traditionally, most students at the undergraduate, not to mention at the secondary level, are not exposed to any research in mathematics due mostly to their lack of background. In this presentation we look at some conditions that seem to foster students' exploration and discovery in mathematics. In addition we will show a new result.

Most students at the undergraduate and secondary levels are exposed to very little, if any, research in mathematics. But, are these students, i.e., other than geniuses such as Gauss and Pascal, capable of doing some research and even finding new results in mathematics? Traditional wisdom points to the lack of students' mathematical background and an already crowded content in every mathematics course as the main reasons for why so little exposure to research is taking place. At the same time, I wonder if these reasons weigh so heavily on teachers at these levels that we seldom expose our students to basic "unstructured" problems that promote investigation and inquiry.

In 2001 I published an article in the proceedings of this conference that was later reprinted (Quesada, 2001a, b) on the fact that the amount of secondary students' discoveries in the previous decade seem to indicate that technology, in particular dynamic geometry software and graphing calculators, was empowering secondary students to find new mathematical results. The article included some interesting results found by students at this level. A synopsis of these results is included in table 1.

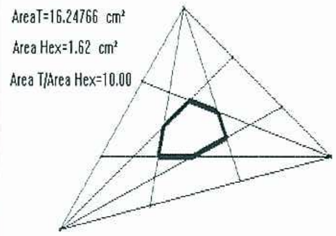
Later on, thinking about these students results and after talking with some of their teachers, I found two common threads supporting these discoveries. The first thread is that these students used interactive geometry software and graphing calculators. Clearly, these technologies allow students to navigate cumbersome calculations, and, when properly used, facilitate an inquiry-based approach that promotes exploration and discovery. The second thread, not surprisingly, is that most of the students involved have been challenged by their teachers with problems beyond the traditional book exercises. Making problem solving a central part of our everyday mathematics class has always been fundamental. However, the need for covering a syllabus while facilitating the students' learning of algorithmic processes, has often focused most of the students work on solving exercises rather than on true problem solving via inquiry in the Polya's sense (Polya, 1957). In a time when information and computer algebra systems are readily accessible, the lack of sufficient exposure to problem solving that, in my estimation, a good number of our college seniors, including preservice teachers, and graduate students seem to have, can hardly be justified. In our work with preservice and inservice teachers, we share with them the following questions that we could be asking ourselves to be sure that we are challenging our students on a daily basis:

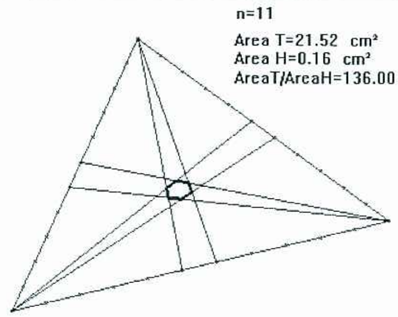
1. Do we ask our students to try to generalize their solutions?
2. How do we encourage our students to raise their own questions, and try to answer them?
3. How do we create extensions—for example, more challenging questions—to the activities students do?
4. Do we dare to ask our students truly challenging questions, questions for which we ourselves may not have an answer?
5. How do we reward students who accept these challenges?

The answer to these questions may help to foster students' exploration and discovery in mathematics. If we do not challenge our students or risk posing problems whose solutions we do not know, we will be hiding the true nature of mathematics, and we will miss the opportunity to be rewarded with their discoveries! (Quesada, 2009)

In the last few years, particularly while working with preservice teachers, I have used this approach, and although some students seem apprehensive about it, the use of teams to tackle unstructured problems that do not require an extensive mathematical background have helped to ease their lack of self-confidence. Students find extensions and in many cases their explorations yield known results, although some of them not that well known.

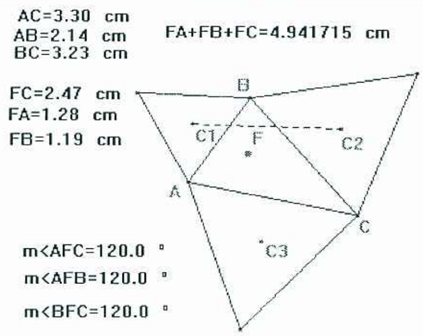
We share next a recent result found by one of my undergraduate students, Andrew Cooper. The limitation on size of the proceedings' articles prevents us from including the entire proof.

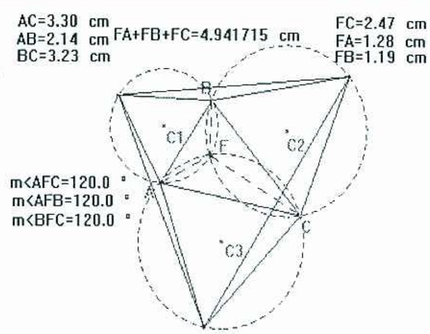




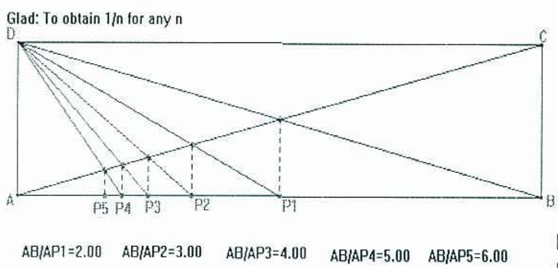
Ryan Morgan's result: For n odd, if the central n-section points of the sides of any triangle are connected to the opposite vertices, the ratio of the area of the original triangle to the area of the resulting hexagon is $(9n^2 - 1)/8$ to 1.

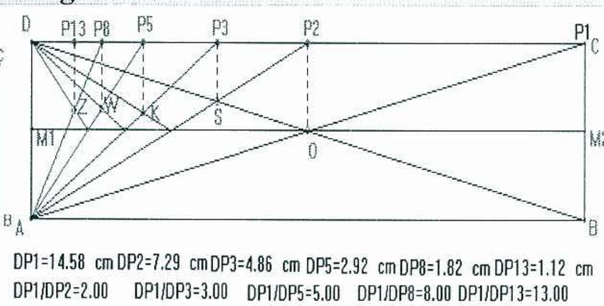
1. Marion Walter's theorem and the extension obtained by Ryan Morgan.



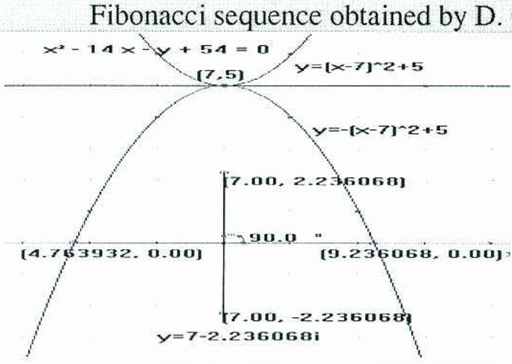


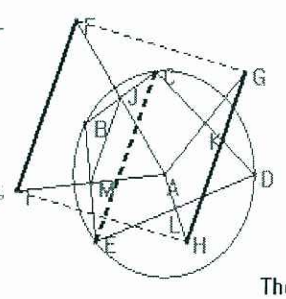
2. A different way of obtaining Fermat's point as a reflection of a vertex on the line connecting the external equilateral triangles centroids.





3. The GLaD construction to subdivide a segment in n parts without a compass, and the Fibonacci sequence obtained by D. Goldenheim & D. Litchfield





J and M midpoints of BC, BE, AF, and AI

FI || JM || CE

FI=2JM=CE

So |FI|=|CE| and FI || CE

Similarly, GH is conguent to CE, therefore |FI|=|GH| and FI || GH

Therefore FIHG is a parallelogram

4. A) Shaun Pieper's graphical representation of the imaginary solutions of a quadratic. B) Solution by Lori Sommar to a geometric problem.

Table 1

<u>The jagged boundary-edge problem</u>: Given an area of land bounded by two parallel lines and separated by a jagged-edge of arbitrary length into two properties, how can we find a straight segment from one line to the other that maintains the original areas of each property? Is there a solution segment of minimal length? Can the solution be extended to the general case of a jagged-edge between boundary lines that meet? (See figures 1 and 2)

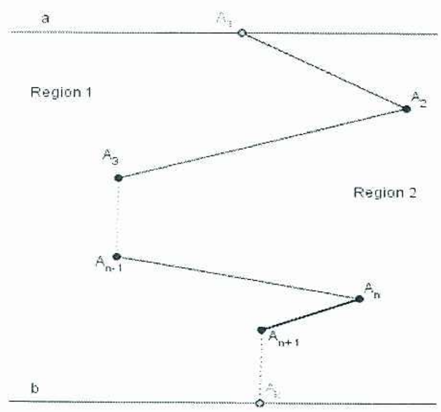


Figure 1

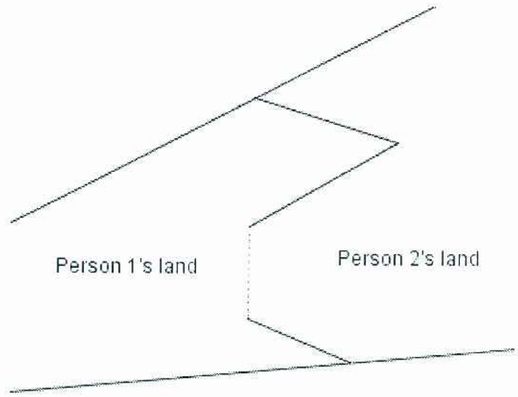


Figure 2

With some guidance, Andrew solved this problem geometrically. He also found a recursive algebraic solution based on the coordinates of the points that define the jagged edge. A sketch of his approach follows.

Lemma. Given a region bounded by two parallel lines and a single segment from one parallel to the other that splits the region into two, any other segment from one parallel to the other that passes through the midpoint of the given segment will preserve the areas of the regions on either side of the original segment.

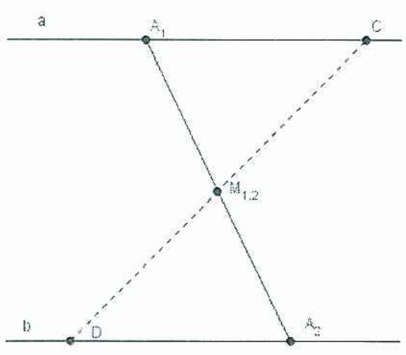


Figure 3

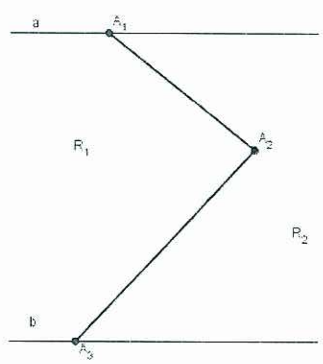


Figure 4

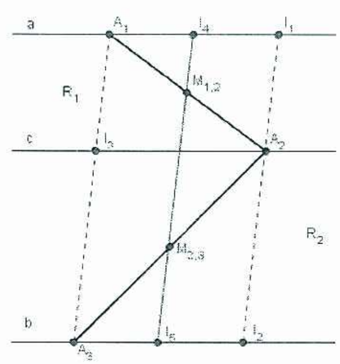


Figure 5

Figure 3 illustrates the lemma, while Figures 4 and 5 show how the lemma is extended to a two-segment boundary. In Figures 6 and 7 we see how the result can be extended using induction.

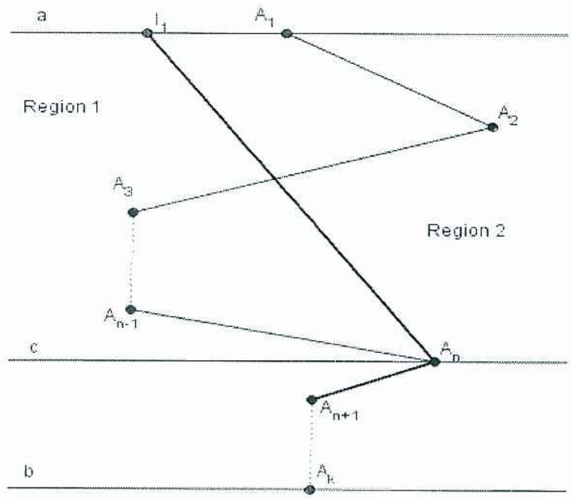


Figure 6

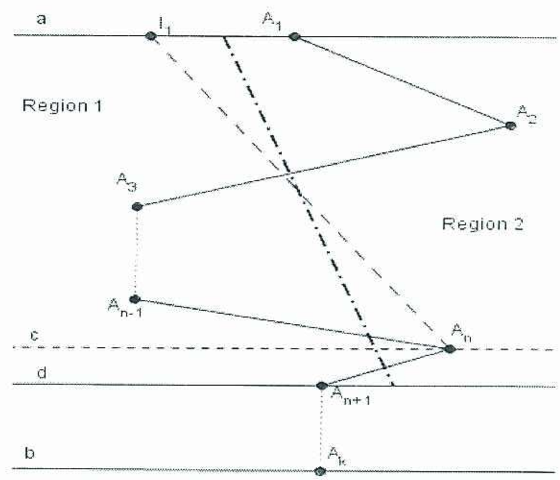


Figure 7

We have seen a geometrical solution to the jagged-edge problem finding the existence of a "*center point*" M such that any segment between the parallel boundaries passing through M is a solution. In particular the minimal solution is the perpendicular to the boundaries through M.

A recursive analytical solution was also found: For any jagged edge $A_1 A_2 \ldots A_k$ with coordinates (x_i, y_i) between two parallel boundary lines a and b, the coordinates of the *center point* $M_{12\ldots k}$ are given by:

$$\left(x_{M_{12\ldots k}}, y_{M_{12\ldots k}}\right) = \left(x_{M_{k-1,k}} + (y_{M_{12\ldots k}} - y_{M_{k-1,k}}) \frac{x_{M_{12\ldots k-1}} - x_{M_{k-1,k}}}{y_{M_{12\ldots k-1}} - y_{M_{k-1,k}}}, \frac{a+b}{2} \right).$$

The solution of the jagged-edge between non-parallel boundaries required more involved preparation and is been submitted for publication.

Conclusion. Nothing conveys the beauty of mathematics like solving problems. In a time when technology allows bypassing cumbersome calculations and when we need to increase the number of STEMM and in particular mathematics students, it is worth to entice some of our students by showing them why we love our subject.

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