Project Name: ***Dynamic Visualization Tools for Multivariable Calculus***, developed with support from NSF–DUE–CCLI grant #0736968

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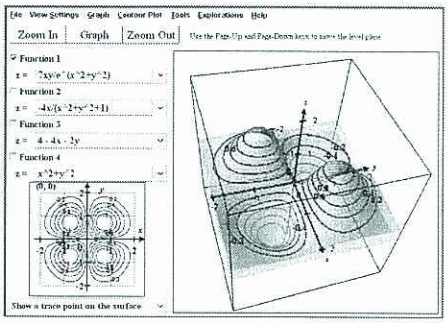
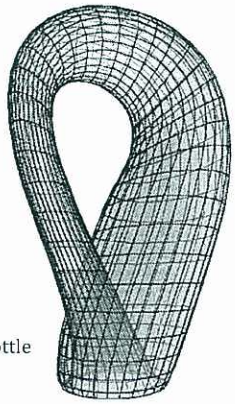
Project Website: http://web.monroecc.edu/calcNSF/



Relating a contour plot with the corresponding surface

Dynamic Visualization Tools for Multivariable Calculus is in the third year of a 3-year NSF grant that runs from Jan. 1, 2008 through Dec. 31, 2010. To date, at least 30 professors from 24 different institutions (including 2 high schools) have used the main project applet in their classes and have provided feedback (or plan to do so). This has involved well over 500 students. So far half of these professors have also chosen to assign their students to complete at least one of the guided exploration assessments that were developed for assessing students and the project. (See more on these below.)


Klein Bottle

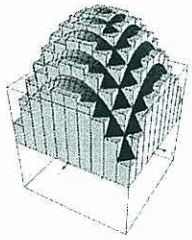
Project Description:

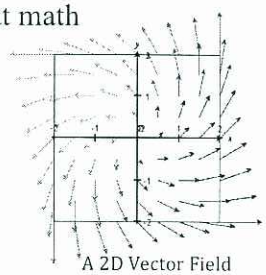
Calculus is very geometric in nature. It describes motion and change. Static diagrams and chalk drawings often do not show a complete picture of the actual interactions and relationships it describes. Motion is needed. For this reason, students have difficulty visualizing (and fully understanding) many of the concepts of calculus. This is an even greater issue for students learning multivariable calculus where most of the concepts are three-dimensional in nature. To address these difficulties, Seeburger (PI) has developed a **multivariable calculus exploration and visualization applet** in Java that allows him to demonstrate the concepts of multivariable calculus visually and dynamically in his lectures. Students are also required to use this application as part of their homework assignments in the course.

The main goal of this project is to develop students' geometric intuition about the concepts covered in multivariable calculus, helping them understand the concepts more deeply and make connections between concepts visually in a way that has been difficult without such software.

To accomplish this goal, the project includes four parts:

1. Creating a **multivariable calculus exploration and visualization applet** using Java (called CalcPlot3D) and publishing it on a website: http://web.monroecc.edu/calcNSF/.
   See the PDF documents listed on the main webpage for more details on the current features of this applet and also for a **guided tutorial**. Link directly to the CalcPlot3D applet at: http://web.monroecc.edu/manila/webfiles/calcNSF/JavaCode/CalcPlot3D.htm.
2. Creating a series of focused applets that demonstrate and explore particular 3D calculus concepts in a more dedicated way.
3. Developing a series of guided exploration/assessments to be used by students to explore calculus concepts visually on their own. Student success is measured by pre-tests/post-tests.
4. Dissemination of these materials through presentations and poster sessions at math conferences and through other publications.

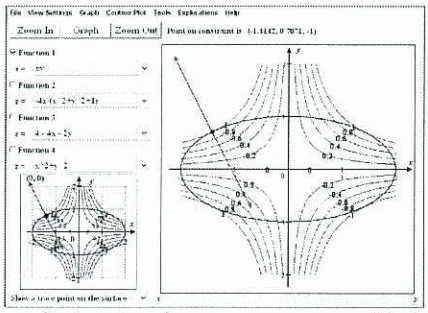

Approximating a Double Integral

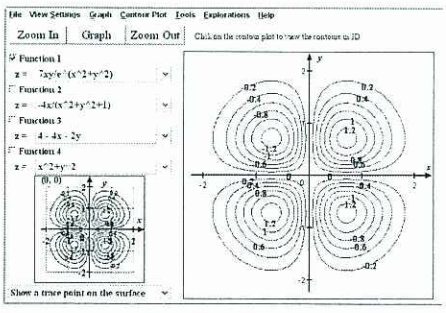

A 2D Vector Field

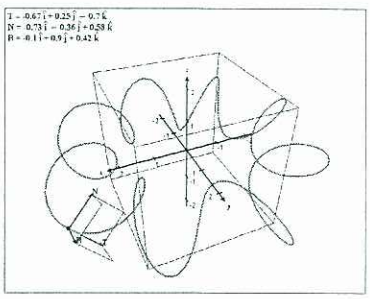
Intellectual Merit:

This project provides dynamic visualization tools that enhance the teaching and learning of multivariable calculus. The visualization applets can be used in a number of ways:

- Instructors can use them to visually demonstrate concepts and verify results during lectures.


See gradient vectors change as you move a point along a constraint curve for Lagrange multiplier optimization.

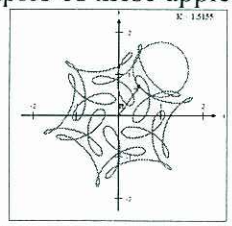

A Contour Plot

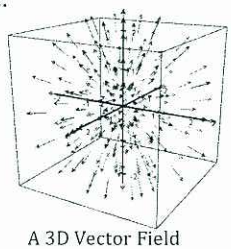

Animating the TNB-Frame

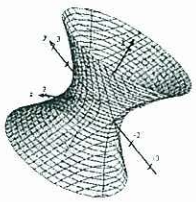
- Students can use them to explore the concepts visually outside of class, either using a guided activity or on their own.
- Instructors can use the main applet (CalcPlot3D) to create colorful graphs for visual aids (color overheads), worksheets, notes/handouts, or tests. 3D graphs and 2D contour plots or vector fields can be copied from the applet and pasted into a word processor like Microsoft Word. The images included in this document were all created using CalcPlot3D.
- Instructors can use CalcPlot3D to create lecture demonstrations containing particular functions they specify and/or guided explorations for their own students using a scripting feature that is being integrated with this applet.

The guided activities created for this project provide a means for instructors to get their students to use these applets to actively explore and 'play' with the calculus concepts. Research indicates that when we play and have fun with concepts as we learn them, we learn these concepts more completely. The interactive visual nature of these explorations also provides deeper understanding of the geometric properties and interrelationships of the concepts.

Paul Seeburger, the Principal Investigator (PI) for this grant project, has much experience developing applets to bring calculus concepts to life. He has created 100+ Java applets supporting five major calculus textbooks (Anton, Thomas, Varberg, Salas, and Hughes-Hallett). These applets essentially make textbook figures come to life. See examples of these applets at: www.monroecc.edu/wusers/pseeburger/.


Osculating Circle on a Plane Curve

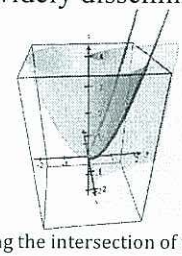

A 3D Vector Field


A Hyperboloid

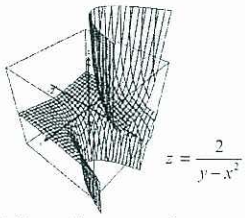
Broader Impacts:

This project provides reliable visualization tools for educators to use to enhance their teaching in calculus and also in various Physics/Engineering classes. It is designed to promote student exploration

and discovery, providing a way to truly 'see' how the concepts work in motion and living color. The applets and support materials are being published and widely disseminated through the web and conference presentations.



Visualizing the intersection of two surfaces



A discontinuous surface

Presentations on this Project:

1. **2011 JMM** (Joint Math Meetings), **Jan. 2011**: Paul has been approved to present a 4-hour minicourse on the project. See the description online using the link below in #2.
2. **MathFest 2010, Aug. 2010**: Paul has been approved to present the same 4-hour minicourse. See Minicourse #6 at: http://www.maa.org/mathfest/minicourses.cfm.
3. **ICTCM 2010, Mar. 2010**: Paul will present two regular sessions related to his project and present a paper on visualizing surface intersections using the CalcPlot3D applet.
4. **2010 JMM** (Joint Math Meetings), **Jan. 2010**: Paul presented two contributed paper sessions on his project and also presented a 2-hour poster session on the project.
5. **2009 AMATYC, Nov. 2009**: Paul presented once on the project (as well as on other applets he has written for single variable calculus) and he also presented a 2-hour poster session on the project. (See the proceedings link below.)
6. **2009 Fall MAA Seaway Section Meeting, Oct. 2009**: Paul presented a one-hour hands-on computer workshop on this project, leading the group through a tutorial on the applet that visually explores a variety of topics from multivariable calculus.
7. **Mathematics Colloquium at SUNY Geneseo, Sept. 24, 2009**: Paul gave an hour-long presentation on the project to close to 100 students and a few faculty.
8. **MathFest 2009, Aug. 2009**: Paul presented two contributed paper sessions and a 4-hour minicourse on his project. Its description is the same as the one shown above for #2.
9. **2009 Spring MAA Seaway Section Meeting, Apr. 2009**: Paul gave a half-hour presentation on the project.

10. **2009 JMM, Jan. 2009**: Paul presented one paper on the project and another that included the project, and he also presented a 2-hour poster session on the project.
11. **2008 AMATYC, Nov. 2008**: Paul presented a 2-hour poster session on the project.
12. **2008 Fall MAA Seaway Section Meeting/NYSMATYC Region 1 Conference, Oct. 2008**: Paul presented on a series of applets he created to help students visualize all levels of calculus, including a presentation of the new CalcPlot3D multivariable calculus exploration applet.
13. **2008 NYSMATYC Conference, Apr. 2008**: Paul presented the same talk as in #12.
14. **2008 ICTCM Conference, Feb. 2008**: Paul presented on a series of applets he created to help students visualize all levels of calculus. This talk included an announcement of the NSF grant project and showed the prototype applet that has become CalcPlot3D. (See the proceedings link below.)

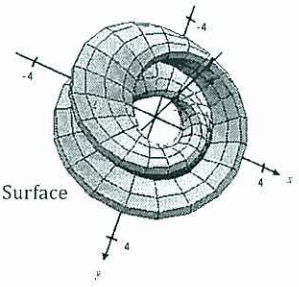
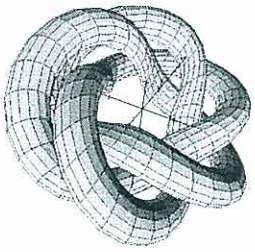


Figure 8 Klein Surface



A Knot (parametric surface)

Publications on the Project (more are planned):

1. The project has been submitted to the Resources section of the MathDL, and Paul also intends to submit at least one article this spring to Loci, the online journal associated with the MathDL.
2. An article on using CalcPlot3D to visualize intersections of surfaces has been accepted to be published in the newsletter of the Math Intensive Committee of AMATYC.
3. Conference Proceedings of AMATYC 2009: See Paul's poster session and Session 35 at: http://www.amatyc.org/Events/conferences/2009LV/proceedings.html.
4. Conference Proceedings of ICTCM 2008: See the presentation on a series of calculus applets by Paul Seeburger at: http://archives.math.utk.edu/ICTCM/i/20/S100.html. Note Figures 10-11 show the applet that has become CalcPlot3D.

For project evaluation results, please see the **Project Evaluation** webpage at:

http://web.monroecc.edu/calcNSF/Evaluation.