VERIFYING SURFACE INTERSECTION CURVES VISUALLY

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In multivariable calculus, we ask students to determine the parametric equations for the curve of intersection of two surfaces. This includes finding the intersection of two planes (a three-dimensional line), and the intersection of a pair of surfaces including spheres, paraboloids, hyperboloids, cylindrical surfaces, and planes. Using a freely available online multivariable calculus applet called CalcPlot3D, a pair of surfaces can be graphed along with the intersection curve (entered parametrically), and the intersection curve can be verified visually.

CalcPlot3D is part of an NSF-funded grant project called *Dynamic Visualization Tools for Multivariable Calculus* (DUE- CCLI #0736968).
See http://web.monroecc.edu/calcNSF/.

This lesson includes two examples to demonstrate in class and a worksheet for students to complete as homework. Students are asked to use the CalcPlot3D applet to graph the surfaces in their exercises along with the space curves they obtain to represent the surface intersections. The applet allows the graphs to be printed and includes a date/time stamp and the name of the computer used for the exploration.

As you will see, the applet allows students (and instructors) to choose which coordinate system they wish to use to graph the surfaces. For example, for graphing the surface, $x^2 + y^2 = 4$, cylindrical coordinates can be used, or students could solve the equation for x or for y and graph the two functions they obtain together to form the cylindrical surface. As another example, the sphere, $x^2 + y^2 + z^2 = 10$, can be graphed using spherical coordinates ($\rho = \sqrt{10}$), or students can solve the equation for z and graph the two hemispheres they obtain in this way together to see the whole sphere.

When presenting this concept in class, you might use the following examples. Usually my students need very little additional instruction in how to use the applet once they have seen me do a couple of these examples in class, but here I have spelled out the first example clearly in a step-by-step fashion.

1. Determine the vector-valued function that will trace the intersection of the surfaces defined by the equations below using the parameter t.

$$z = x^2 + y^2 \quad \text{and} \quad y = x^2$$

Here we can let $x = t$. Then $y = t^2$ and $z =$ ___________________.

So the vector-valued function we obtain that traces out the intersection is:

$$\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + (t^2 + t^4)\,\mathbf{k}$$

Now let's verify this in CalcPlot3D visually!

a. Open the CalcPlot3D applet from my website: http://web.monroecc.edu/calcNSF/. The applet link is in the upper right side of the webpage. Once the applet is loaded and active, enter the first function listed above ($z = x^2 + y^2$) in **Function 1** and press Enter (or click on the **Graph** button). The surface plot of this paraboloid should appear in the plot window.

b. Now, to enter the second function, either right-click on the z = to the left of **Function 2**, or use the **Format Surfaces** option on the **View Settings** menu to choose the function type, $y = f(x, z)$. This will set the function to 0 by default. Then enter **x^2** in the textbox beside **Function 2** and press Enter (or use the **Graph** button).

c. To extend the second surface farther up the paraboloid, we can change the z-axis limits in the **Format Axes** dialog.

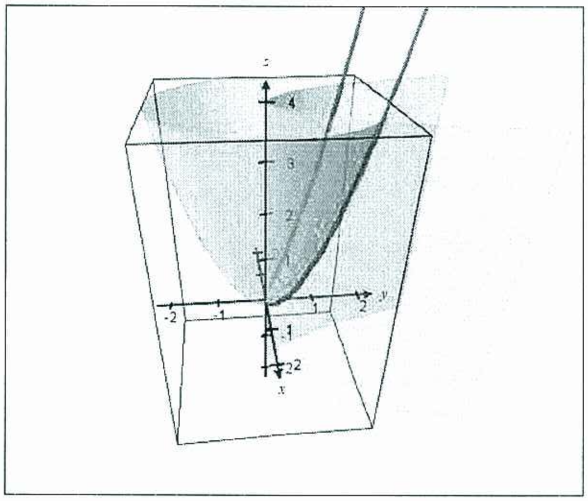

Figure 1: Surface Intersection in Example 1

Choose **Format Axes** from the **View Settings** menu (or press the **A** key). Then set **z-max** to **4**. This will automatically change the **zClip-top** value to 8. Let's change this value to **4** also.

d. Now press the **OK** button to see these new settings take effect and then cancel/close the **Format Dialog**.

e. Next we need to graph the space curve to see how well it fits the intersection of the surfaces. Select **Add a Space Curve** from the **Graph** menu.

f. The **Space Curve Dialog** should appear to the upper right. Enter the three parametric equations we obtained (each in terms of t). Then enter a range of -2 to 2. If you press Enter on the second value, it should produce the curve on the plot. If it does not appear, click the **Graph** button.

g. Finally rotate the graph to see if it looks like we found the correct intersection curve.

h. Make the surfaces semi-transparent using the option on the View Settings menu or by typing Ctrl-T to get a clearer view of the intersection curve.

2. Determine the vector-valued function that will trace the intersection of the surfaces defined by the equations below using the parameter $\boldsymbol{y = 2\sin t}$.

$$2x^2 + y^2 = 4 \quad \text{and} \quad z = x^2 + y^2$$

Here we should recognize that we can parameterize the ellipse (1st equation) with,

$\boldsymbol{x = \sqrt{2}\sin t}$ and $\boldsymbol{y = 2\sin t}$.

Once we know x and y, the second equation makes it easy to determine z in terms of t, in this example.

Simplifying the expression, we obtain, $z = 2 + 2\sin^2 t$.

So $\mathbf{r}(t) = \sqrt{2}\sin t\,\mathbf{i} + 2\sin t\,\mathbf{j} + (2 + 2\sin^2 t)\,\mathbf{k}$

Now graph to verify the result. To obtain the first surface, solve the equation for y and graph both equations to obtain the entire surface.

Once they are graphed it is helpful to change the coloring of the paraboloid surface so that it is easier to see the intersection. To do this, choose **Opaque – reversed color** for the paraboloid Function from the **Format Surfaces** submenu of the **View Settings** menu.

Now graph the curve and check the result visually. What should the range of values be for t?

You will enter:

x = sqrt(2)*sin(t)
y = 2sin(t)
z = 2 + 2(sin(t))^2