ENGAGING STUDENTS IN MATH DISCUSSIONS

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Lecture is popular in teaching mathematics but it is not the best choice for all students. In giving a lecture, whether in a traditional face-to-face class or online using videos or PowerPoint's, the instructor assumes that the student understands the information as presented. However, that is not always the case. A student processes information based on her/his experience and knowledge: the student with a limited understanding of mathematics can have difficulty learning new concepts and methods, the student with limited experience with application and limited language skills can have difficulty with interpretations, and the student with limited analytical skills and experience can have difficulty with problem solving and analysis. While skills can be learned, they are learned through experience, practice, and correction: skills are not gained by watching others.

For example, a student who does not truly understand variables may give an interpretation of the derivative such as "the water is increasing": water cannot be "increasing" but the amount of water can be increasing, the height of water in a tank can be increasing, or the water level in a tank can be increasing. In addition, a student who does not understand variables may give an interpretation of the mean such as "the mean for the buildings" rather than "the average height of the buildings" or "the mean age for the buildings". Such a student may not understand the difference among these interpretations, and a lecture does not provide a student with the opportunity to explore the differences while a discussion can.

A lecture does not allow students to explore difference and similarities between formulas, to understand when using one formula may be more appropriate or easier than using another formula, or to understand how to use a formula. For example, let us consider the linear correlation coefficient formulas,

$$ r = \frac{\sum_{k=1}^{n} \left(\frac{x_k - \overline{x}}{s_x} \right) \left(\frac{y_k - \overline{y}}{s_y} \right)}{n - 1}, \tag{1} $$

$$ r = \frac{n \sum_{k=1}^{n} x_k y_k - \left(\sum_{k=1}^{n} x_k \right) \left(\sum_{k=1}^{n} y_k \right)}{\sqrt{n \sum_{k=1}^{n} x_k^2 - \left(\sum_{k=1}^{n} x_k \right)^2} \sqrt{n \sum_{k=1}^{n} y_k^2 - \left(\sum_{k=1}^{n} y_k \right)^2}}, \tag{2} $$

and

$$r = \frac{\sum_{k=1}^{n} x_k y_k - \dfrac{\left(\sum_{k=1}^{n} x_k\right)\left(\sum_{k=1}^{n} y_k\right)}{n}}{\sqrt{\sum_{k=1}^{n} x_k^2 - \dfrac{\left(\sum_{k=1}^{n} x_k\right)^2}{n}}\sqrt{\sum_{k=1}^{n} y_k^2 - \dfrac{\left(\sum_{k=1}^{n} y_k\right)^2}{n}}}. \tag{3}$$

Many introductory statistics textbooks include the linear correlation coefficient formula given in equation (1), yet students do not understand how to use this formula, especially those who are not familiar with summations. Students who do not understand summations frequently try to "distribute" the summation, not only over the product but also into the parentheses, thereby doing calculations that are the equivalent of

$$\frac{\left(\dfrac{\sum_{k=1}^{n} x_k - \overline{x}}{s_x}\right)\left(\dfrac{\sum_{k=1}^{n} y_k - \overline{y}}{s_y}\right)}{n-1} \tag{4}$$

This "interpretation" in expression (4), of course, is incorrect but a student who is not adept in working with summations or comfortable with mathematics does not understand this. In addition, Such a student is not does not understand that it is also incorrect to "distribute" the sum over the product alone,

$$\frac{\sum_{k=1}^{n}\left(\dfrac{x_k - \overline{x}}{s_x}\right)\sum_{k=1}^{n}\left(\dfrac{y_k - \overline{y}}{s_y}\right)}{n-1}. \tag{5}$$

The student who understands the relationship between a sum and a product knows that distributing a sum over a product, as in expression (5), is incorrect. However, many students taking introductory statistics courses do not understand that altering equation (1) to become the expression in (4) or (5) is incorrect and they do not understand how to properly use equation (1), using rounded or truncated values of the mean and standard deviation as well as rounding or truncating the intermediate values involved in the calculation.

Then, other texts and/or instructors may prefer to use the formulas given in equations (2) and/or (3). For a student who is uncomfortable working with fractions, using the formula given in equation (3) can be problematic and such a student frequently transforms the formula to produce the expression

$$\frac{\sum_{k=1}^{n} x_k y_k - \frac{\left(\sum_{k=1}^{n} x_k\right)}{n}\frac{\left(\sum_{k=1}^{n} y_k\right)}{n}}{\sqrt{\sum_{k=1}^{n} x_k^2 - \left(\frac{\sum_{k=1}^{n} x_k}{n}\right)^2}\sqrt{\sum_{k=1}^{n} y_k^2 - \left(\frac{\sum_{k=1}^{n} y_k}{n}\right)^2}}, \qquad (6)$$

dividing each sum in the product in the numerator by n and putting the division by n inside the square in the second part of each quantity in the square roots. Finally, if the similarity of the numerator and contents of the first square root in the denominator of the linear correlation coefficient formula in equation (2) to the numerator and denominator of the linear regression slope formula in equation (7) are not brought to the student's attention as well as the ease with which the necessary sums can be determined are not brought to the attention of the student then (s)he may miss it and, thereby, not take advantage of using this formula.

$$m = \frac{n\sum_{k=1}^{n} x_k y_k - \left(\sum_{k=1}^{n} x_k\right)\left(\sum_{k=1}^{n} y_k\right)}{n\sum_{k=1}^{n} x_k^2 - \left(\sum_{k=1}^{n} x_k\right)^2} \qquad (7)$$

Using the lecture, one can comment about these but using a discussion, the instructor can involve the student and help her/him to understand differences, similarities, do's and don'ts as wells as the how to's of working with formulas and interpreting results.

The discussion, in an online class or in a traditional face-to-face class, provides advantages for student learning. First, for the discussion, students are actively involved rather than passively listening and watching. Active participation allows students to contribute questions, including questions that an instructor may not have anticipated, alternate explanations that may be especially beneficial to other students since these explanations are in the student's own "language", alternate interpretations, and comparisons, especially those that an instructor may not have foreseen. Second, discussions can help students to recognize the importance of topics, make connections between topics, and recognize important information. Then, rather than being meaningless scribblings copied from a whiteboard or chalkboard, class notes that result from discussions can have significance and can contain meaningful information that is useful and comprehensible to the student. Through discussions, the instructor can determine how much and what the student understands and the extent to which the student understands course information, topics, and methods: discussion provides useful and vital feedback to the instructor that a lecture cannot. Discussions can be easily tailored to the needs of visual, auditory, and kinesthetic learners and, rather than being

talked at as with a lecture, discussion allows the instructor to speak with the students, interacting with the student as information about course topics is shared. Rather than the instructor being the "sage on the stage" as with a lecture, the discussion allows the instructor to be involved with the students in the exploration of course topics and methods and allows the instructor to help the student to integrate new topics and methods with those previously examined. As the instructor discusses course material with the student, the student can practice thinking using the mindset of the discipline as well as evaluate interpretations and consider applications, and by formulating and stating questions, problems, and interpretations, the student can experience the mindset of the discipline and learn to recognize and to use the language of the discipline.

Crucial to any discussion is the environment. For a discussion to be successful for both the instructor and the student, the instructor must be open to and respectful of student contributions. The instructor must be nonjudgmental and able to provide gentle correction and constructive criticism. The instructor must be willing and able to provide alternate explanations and interpretations as well as welcome, value, and encourage the explanations, interpretations, and questions contributed by the student. If a student explanation or interpretation is "off target" then the instructor must be able to acknowledge any correct points while steering the student back in the correct direction. By creating a positive learning environment in which participation is not forced and in which those who may try to command the discussion are not stifled but rather are guided to allow others to contribute as well, the instructor can provide the safe environment in which those students who usually watch and remain silent may choose to participate as well.

The developmental discussion, named by Norman Maier in 1952, is a problem-solving discussion that can be used to model the problem-solving process as well as used to help the student in data and problem analysis. In the developmental discussion, the group, facilitated by the instructor, breaks the problem, analysis, or proof into parts that are considered, examined, and worked by everyone together at the same time. This provides structure to the analysis as well as eliminates any feeling of disorganization from the discussion. This structure eliminates or, at least, attenuates feelings of frustration that many feel in the problem solving process and tends to eliminate wasteful or irrelevant comments. In addition, this group approach fosters the idea of each student's being involved in and contributing to the analysis while providing a bond with the instructor as a guide rather than a controller. Through the stages of the developmental discussion, the problem is formulated, hypotheses are suggested, relevant information and data are collected, and solutions and interpretations are determined and evaluated. As the problem is formulated, the student learns how to recognize and to voice a problem as well as to clarify a problem, determining what is known and what needs to be determined in addition to what information is relevant and what data are needed. It is through experiencing the problem-solving process that the student learns to devise, to recognize, and to test possible solutions. The developmental discussion enables the student to experience the problem-solving process as well as to be an active contributor to the analysis of a problem and to the determination of its solution.

During a lecture in which proofs are presented, the student is not an active participant in the proof process. However, using a developmental discussion, the student can experience the proof process, including the analysis of the statement, the determination of the given or conditional information, the connection of the given information to that which is to be established, and the process of writing the proof. One cannot learn to do a proof by watching an instructor do a proof or a student try to do a proof. Watching another do a proof neither helps the student to learn to break down the statement or to recognize the given or conditional information provided in the statement nor helps the student to connect determine how to connect this information to that which is to be proven or helps the student to understand how to use what (s)he knows to complete the proof. Through the developmental discussion, the student has the support of the instructor and other students as well as the guidance that can help the student to gain understanding of the proof process and to acquire the confidence to do proofs on her/his own.

The discussion provides a venue for analyzing formulas and methods, and learning to use course software, if any. As discussed at the beginning of this paper, the student may not have the mathematical skills to understand how to evaluate and to apply formulas. However, through discussion and especially with the contributions by others in the class, the student can gain perspective on how to apply methods, how to evaluate formulas, and when to use formulas. The student can contribute her/his misconceptions to help others to understand on what to focus and what not to do. While many instructors may prefer to provide a list of steps for using software, this list does not help the student to understand how to use the software or enable her/him to use the program on her/his own. With a proper introduction to and tour of the program environment as well as guided exploration of the use of the software, and the contribution of the natural first-time-user questions, the student can gain knowledge of the software sufficient for using the software to analyze and solve meaningful problems. Through discussion, new software can quickly develop into a tool and provide new skills rather than becoming a fresh source of discomfort in the course.

Today, in-class group work is very popular. However, the student cannot know how to work within a group unless group-work and in-group analysis are modeled. Lecture-based courses do not provide guidance on actively identifying, decoding, analyzing, and solving problems Therefore, when students are instructed to work in groups, they have little or no direction, they have little idea of how to perform necessary analysis or to formulate the underlying problem, and they have difficulty in determining a solution. However, if the group mindset is demonstrated and used as in the developmental discussion, students can have a basis for working together effectively as well as direction in determining, formulating and solving problems.

Correction, not a problem in a lecture, must be carefully performed during a discussion. Especially for online discussions and recorded discussions such as those done using *Elluminate Live!*, the discussions can be reviewed, reread and replayed, making the words that are used and the tone of voice, in the case of the recorded discussion and

discussions performed in the traditional face-to-face class settings, used in response to questions and in the correction of errors important to the continuation of the discussion and to the student's willingness to contribute to the discussion. If the student does not feel that her/his contributions are not valued and respected, if the student feels that (s)he is being judged, if the student feels that (s)he is being stifled, and if the student feels unwelcome then the student will not participate in the discussion. The student who feels put down tends to shut down within the course. The words that the instructor posts on a discussion board or says during class, whether recorded or not, cannot be retracted, and while an apology may suffice for some students, it may not repair damage done by a careless word or a negative reaction or interaction.

It is important to note that conducting discussions requires more preparation than giving a lecture. For a lecture, only the plan for the "deposit" of information need be considered. However, to "weave" a proper discussion, the instructor must carefully select problems that can be used in a group setting and these problems must require sufficient analysis and level of difficulty to involve the members of the class. The instructor must thoughtfully consider how to guide the discussion as well as consider the possible questions that may arise and alternate explanations that may be helpful to students. Since questioning is one of the best ways in which to begin a discussion, the instructor must cautiously reflect on the type of questions to ask, the wording to be used the questions, the amount of time to leave for students to reflect on questions, and how the questions posed can be used to achieve the overall goal, formulating, analyzing, and solving a problem. Creating the environment for and a supportive student-instructor relationship involving trust and respect are necessary for achieving discussion within the face-to-face classroom setting or online. While crafting discussions requires preparation, practice, and time, discussions are greatly beneficial to student-learning and transform the traditional and online class environments into a shared learning experience and a partnership for both the instructor and the student.

The discussion is a valuable tool in teaching and learning in every discipline. In mathematics, the discussion can be used to provide guidance in problem solving and proof writing as well as in understanding and using formulas. The discussion allows the student to experience analysis rather than watching the analysis performed by another. The discussion provides opportunities for concept exploration and for making connections between topics and methods. In addition, the discussion allows the instructor to become a guide and a support in learning and the student to become a participant rather than a watcher. Using the discussion, the instructor can evaluate student understanding and guide the student to higher knowledge and synthesis of concepts and methods while obtaining feedback on student learning and progress which can enable the instructor to adjust the course as needed. The discussion makes the course and the learning process a partnership between the instructor and the student, the instructor contributing guidance, knowledge and correction, and the student becoming an active participant open to new ideas, necessary correction, and making contributions.