# INTEGRATING SPREADSHEETS, VISUALIZATION TOOLS AND COMPUTATIONAL KNOWLEDGE ENGINES IN A LIBERAL ARTS CALCULUS COURSE

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## 1 Quantitative Reasoning, Calculus, and the Liberal Arts

Franklin College recently implemented a new Liberal Arts Core curriculum which is to be completed by all students. The curriculum is designed to help students acquire skills needed to function in the modern world, including critical thinking and technological skills. Every student must demonstrate competencies in these areas through (among other means) completion of a quantitative reasoning course. Currently, three courses satisfy this requirement: Quantitative Reasoning (a general liberal arts mathematics course), Calculus, and Discrete Mathematics I. Roughly one-third of all students satisfy the Liberal Arts Core quantitative reasoning requirement with Calculus, including all majors in science, mathematics, and business.

As one of the appointed quantitative reasoning courses, Calculus at Franklin College is tasked with several strategic student learning outcomes:

- Prepare students for the technical use of Calculus in subsequent courses in mathematics, chemistry, physics, and economics.
- Impart a baseline of technological literacy to students, particularly the use of computer software to solve problems and work with data.
- In particular, prepare students to use spreadsheet software to model data in a variety of representations and extract information from their models.
- Sharpen student skills in written and oral communication, especially as regards technical communication.

Subsequent courses in the Liberal Arts Core assume student competency in each of the last three areas. For example, students going on to take the college's course in Public Speaking are expected to be able to construct a convincing oral argument using visual data analysis.

Most college-level Calculus courses orient themselves toward the first goal above and give at least some attention to the remaining three. Our Calculus course must substantively address all four points. In order to do this effectively, we have developed learning activities for Calculus that address some combination of these four objectives.

## 2 Technology and Pedagogy

The learning activities we have devised for Calculus are predicated on the use of three pieces of software:

- **Microsoft Excel**, for spreadsheet use and analysis of data;
- **Winplot**, a graphing tool for visualizing functions and data; and
- **Wolfram|Alpha**, a computational knowledge engine for assisting in algebraic computations and mining for quantitative information.

Each of these tools is either free (Wolfram|Alpha, Winplot) or nearly ubiquitous (Excel) and easily accessible to all students. Moreover, the activities described below are not dependent upon any single instantiation of these tools; the spreadsheet activities, for example, could quite easily be done using any spreadsheet software having basic data fitting and plotting capabilities.

We now describe the learning activities mentioned above and how the technology is used in each one. We have posted several examples of these activities on the internet, and the links to these are aggregated at the following website:

`http://sites.google.com/site/gashtalbertictcm/`

### 2.1 Microsoft Excel and Calculus Writing Projects

Written projects play a prominent role in the Calculus course. They are effective tools for completing the many different learning objectives outlined above. Students self-select into groups or are placed into groups, and each group is required to submit a group response to the project. The project output requires the use of some sort of spreadsheet software, usually Microsoft Excel, in order to communicate effectvely the mathematics involved. Moreover, students are encouraged, if not required, to utilize their spreadsheet or charts in their written exposition. Written projects also involve a reflection component in which each student in the group submits an anonymous evaluation of themselves, his/her team members and the project as a whole. There are typically four projects in a semester.

There is also a heavy emphasis on good writing. Unlike many standard mathematics activities where other calculus students or the instructor is the intended audience for the final product, students are usually responding to the author of a fictitious letter. The authors of these letters are not always mathematically literate, let alone experts in calculus or the notation used in a calculus class. Some of the fictitious letter writers that have appeared are an old grandmother, an eighth-grade boy, a financial investment banker and the owner of a candy store. With such a wide variety of potential audiences, students are forced to think critically about the tone, readability and content of their writing.

In addition to tone and readability, students are graded on their use of correct grammar. This standard is held not only on the groups' written products, but also on the reflections submitted by each individual student. Though the standard may vary from course to course, there is always a linear relationship between the number of grammatical errors and the penalty for making the errors. Because the penalties are stiff, students quickly realize that communicating mathematics does not mean that grammar, politeness and paragraph structure are no longer important.

The written projects tend to focus on the same topics. The first project involves the relationship between functions. This first assignment may also be a student's first assignment with Excel, so an emphasis is placed on creating good graphics. The second assignment usually lets the students work with limits; it is often on this project that students learn to use the spreadsheet formula tools to emulate limit behavior with actual numbers. The third assignment generally focuses on a topic from derivatives, usually related rate problems and optimization, and the fourth project asks students to explore integration and area underneath a curve. (Due to the use of spreadsheet software, students can evaluate areas underneath curves that they would be unable to do without the software.)

Permission was granted by a group of three former MAT 135 students to share some of their work from the written project *A mutual fund that makes money in 2009*. They began their written response with the following introduction:

> "Dear Mr. Steven Yusef:
>
> We would first like to thank you for the opportunity to work with you and your company IndyBank. Hopefully our analysis of this data can clearly answer the question you are posing..."

This demonstrates that the students are paying attention to the writer of the letter. Their response does not immediately begin with mathematics; rather, they make an introduction to their audience. The tone of the introductions varies from written project to written project.

Because the group's use of Excel created some hidden, yet significant rounding errors, the students arrived at answers they suspected were incorrect. It is in this regard that the students had to intersect their use of technology with their mathematics content knowledge. This is illustrated in the following excerpt from their letter:

> "We feel confident as a group that our mathematical techniques [are mathematically sound]. Although, upon further investigation we have came to the conclusion that this representation of your data doesn't work..."

After finishing the written product, the students wrote a reflection in which they analyzed their participation and the participation of their colleagues. Two excerpts, in their unedited form, are given here:

> "Wes was workhorse of our group... He also found the antiderivative from our first equation."
>
> "Evan... proofread both Wes's and my work. He was also there to answer questions if we had any. He also helped with analyzing the graph and its overall look."

These examples are indicative of the work done by most groups on these projects. It is evident that written exposition and effective and thoughtful use of technology is required to be successful in the completion of the written projects. This group also included a well-organized and professional-looking chart with their written letter to the banker in this project.

## 2.2 Winplot and Discovery Learning-Based Group Activities

Another important component of the Calculus course is the use of visualization tools to represent functions graphically and to draw conclusions from graphs. Graphing calculators are allowed but not preferred, due to their proprietary nature, limited abilities to interoperate with computer software, and expense. We have been using Winplot, a free Microsoft Windows-based graphing tool, for several years in Calculus and other courses. Being free software, Winplot is accessible to all students; we have installed it on virtually every public Windows machine on campus and require students with computers running Windows to download and install it on their machines. It is simple to learn and use, has an active development process and user community, and contains many powerful graphing features. Students are given an assignment during the first week of class to watch several training videos for Winplot and prepare to use Winplot during class activities by a set date. From that date forward, Winplot makes regular appearances during class instruction, group activities, and written assignments.

A course activity involving Winplot to learn core course material by discovery is shown in the document *Exploring maximum and minimum values*. This discovery learning exercise is used to follow up a brief lecture on absolute and relative extrema of functions. A common stumbling block for students in this topic is visualizing different combinations of relative and absolute extreme values on the same graph. In past attempts to teach this material, students were either shown graphs on the whiteboard having different extrema combinations or given specific functions to plot on the computer or a graphing calculator. In both cases, students see the combinations but do not internalize them, leading to issues later on in the topics of derivative tests and optimization.

The activity attempts to build students' conceptual knowledge of such combinations by having them construct those combinations themselves, thereby imparting an internalized experience of this graphical information before moving on to an analytical treatment with derivatives. In the activity, students set up a fourth-degree polynomial in Winplot with parameters for the coefficients. They then open up "sliders" to allow them to change the coefficients independently until several different graphs, each with different combinations of relative and absolute extrema, appear. They record the coefficients for later reference. Having built examples of several different combinations of extrema, they are then asked to look over all their examples and extract patterns. From there, students are conceptually prepared to move on to the more analytical topics of critical numbers and derivative tests.

This activity and others like it using Winplot have proven to be effective with students in introductory Calculus. Many of these are visually-oriented learners, and having free and simple visualization software to use addresses their learning style. Additionally, the use of interactive computer software elements, such as the slider functionality in Winplot, addresses learning characteristics specific to the "Millennial" generation, such as the tendency toward simulation activities and immediate feedback ([1]).

## 2.3 Wolfram|Alpha and Conceptual Knowledge Activities

The most recent technology in our list is Wolfram|Alpha, a web-based "computational knowledge engine" developed by Wolfram Research, makers of *Mathematica*. Wolfram|Alpha allows users to enter in a term or string of terms. Then, rather than search the internet for web pages pertaining to the query, Wolfram|Alpha will attempt to compute all information it can using its internal data. In particular, Wolfram|Alpha can produce mathematical information about functions, including plots, derivatives, definite and indefinite integrals, and limits, and in many calculations it is possible to show the steps that were used to get to the results.

Wolfram|Alpha's ability to handle and explicate algebraic computations, along with its natural-language entry of query terms and free accessibility, makes it an attractive substitute for a computer algebra system for simple settings such as basic calculus. The document *Discovering derivative rules* shows another guided discovery learning activity in which students develop some foundational derivative rules, without lecturing on the professor's part except to debrief student activities, using Wolfram|Alpha to manage some of the limits and algebra.

In this activity, students use the standard limit-based definition of the derivative to set up and calculate derivatives of various functions, starting with linear functions and progressing through $y = x^2$, $y = x^3$, $y = x^4$, $y = x^{10}$, and $y = x^{25}$. As the algebra of evaluating each of these functions at $x + h$ and then simplifying the difference quotient and taking the limit ratchets up in computational complexity, Wolfram|Alpha gradually takes over. We do not use Wolfram|Alpha to *hide* the algebra but rather to *do* it, with all steps shown, so that students can look on the results and reason through why their derivatives are turning out the way they are. Hence, students see that the derivative of $y = x^n$ (for positive integers $n$) is $y' = nx^{n-1}$ because, when $(x+h)^n$ is expanded and $x^n$ subtracted off, the only term left with a single factor of $h$ after cancelling is $nx^{n-1}$; all other terms "die off" when the limit is taken.

In prior treatments of the Power Rule, students would have to contend with the Binomial Theorem to see this fact. With Wolfram|Alpha one can simply watch the algebra take place and get straight to the calculus concept. As many students are unfamiliar or weak with relatively advanced high school algebra results such as the Binomial Theorem, including that theorem often obscured more than it revealed. Having students use Wolfram|Alpha instead circumvents algebra that, while important, is a means to an end rather than an end in itself, so that students can focus on the end: an understanding and knowledge of the Power Rule.

Since we started approaching the Power Rule via this activity rather than a lecture in which no explanation of the Power Rule is given at all (on the one hand) or a full-blown proof of the Power Rule using the Binomial Theorem is shown via lecture (on the other), students' conceptual understanding has improved markedly and their mistakes with the Power Rule reduced. For example, upon completing this activity, students are quite keenly aware of the differences between getting the derivative of $y = x^n$ and the derivative of $y = e^x$ — the algebra for each plays out completely differently in Wolfram|Alpha — and we have seen far fewer instances of students finding the derivative of $y = e^x$ as $y' = xe^{x-1}$.

# 3 Conclusion

The use of written and discovery learning projects in Calculus — powered by spreadsheets, visualization tools and computational knowledge engines — successfully promotes the Liberal Arts Core standards for use of technology, effective communication and fluency in various representations of data. The tools utilized here are either free or nearly ubiquitous, making the technology affordable and easy to implement. Moreover, the learning curve for all these technologies make integration into the calculus course quite reasonable. Each of these technologies is flexible and can be used by both students during projects, homework or classroom activities and instructors during lectures.

There are several different avenues for further study. It would be interesting and worthwhile to ascertain the pedagogical gains of effectively utilizing technology in the calculus curriculum. Given the newness of Wolfram|Alpha, it would be worthwhile to develop and share a host of classroom activities utilizing Wolfram|Alpha, and effective implementation of it in a testing environment is certainly among the chief areas of intrigue. Finally, and not to be overlooked, effective and efficient ways to grade written exposition in a mathematics class would also make an inviting path for additional study.

# References

[1] D. Oblinger, Understanding the new students: Boomers, Gen-Xers, Millennials. *EDUCAUSE Review* **38**(4):37–47, July/August 2003.