AN INTERACTIVE, ONLINE, SINGLE-VARIABLE CALCULUS TEXT

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1. Background: Project CALC We describe here an online textbook for first-year calculus, the second edition of *Calculus: Modeling and Application.*[1] The principles on which the book is based are the same as those the first two authors developed in Project CALC more than a decade ago, but the online version has many interactive features that could not be in a print-based text. The book will be published by the Mathematical Association of America in 2010, but through the 2009-10 academic year, it is freely available to all at http://www.math.duke.edu/education/calculustext. We are eager to have teachers and students use any or all of it and provide feedback.

Project CALC: Calculus As a Laboratory Course was funded by the National Science Foundation (NSF) from 1988 to 1995 (DUE-8953961, DUE-9153272), and the first edition textbook and lab materials were published in 1996 by Houghton Mifflin. Some online lab modules were later developed as part of the Connected Curriculum Project, also funded by NSF (DUE-9352889) from 1993 to 2001. In 1991, Project CALC won the EDUCOM Higher Education Software Award as *Best Mathematics Curriculum Innovation*, and in 1993 it was cited by Project Kaleidoscope as *A Program that Works*.

The second edition text shares with Project CALC the following characteristics: hands-on learning activities, guided discovery learning, real-world applications as motivators for mathematics, writing exercises (for both learning and assessment), high expectations of students, encouragement of teamwork, learning to use available tools intelligently, and emphasis on students checking their own work. These features were selected on the basis of research on educational strategies that lead to durable and transferable learning, as well as on modeling what students could be expected to encounter once they leave the academic world.

2. What's in the Book? Here is a brief summary of each chapter, including some of our reasons for structuring the book as we have.

Chapter 1: Relationships. The central question of the introductory chapter is "What is a function?". Our objective is to separate this concept from other relationships between varying quantities and especially to separate it from "formula". Our purpose is to replace some of the inappropriate ideas students typically bring from secondary mathematics

[1] This work has been supported by National Science Foundation Grant No. 0231083. Any opinions, findings, and conclusions or recommendations expressed in this article are those of the authors and do not necessarily reflect the views of the National Science Foundation.

with a healthy regard for the mathematical concept that will be the foundation for the rest of the course. We also take up the algebra of functions and pose some problems for which the solutions are functions or classes of functions (e.g., symmetry, additivity).

Chapter 2: Models of Growth: Rates of Change. Here we establish some basic reasons for studying calculus, especially to be to able to solve differential equations. Our primary example is the natural population growth equation, the simplest ODE to solve, and an immediate reason for moving beyond polynomials. We introduce difference quotients, derivatives, slope fields, initial value problems, solutions (which are, of course, functions or families of functions), exponential and logarithmic functions, and logarithmic plotting. The primary tool for understanding the derivative is zooming in on locally linear functions, and the primary formula is "slope equals rise over run."

Chapter 3: Initial Value Problems. This short chapter builds on Chapter 2, introducing Newton's Law of Cooling (exponential decay) to solve a murder mystery, then studying falling objects without air resistance (polynomial solutions).

Chapter 4: Differential Calculus and its Uses. This is the heart of the first-semester course, consolidating what has been learned about derivatives to take up optimization, concavity, Newton's Method (as an exercise in local linearity), and the basic formulas for differentiation. The product rule is introduced to study the growth rate of energy consumption, the chain rule to study reflection and refraction, and implicit differentiation to calculate derivatives of the logarithmic functions and of general powers. Zooming in is connected to the concept of differentials and to Leibniz notation.

Chapter 5: Modeling with Differential Equations. Here we return to falling bodies (e.g., raindrops, skydivers) and introduce air resistance proportional to the velocity or its square. The latter requires (at this point) numerical solutions, and we take up Euler's Method as another "slope equals rise over run" application. We introduce periodic motion (with second-order ODE's, harking back to Chapter 3, where we derived position from constant acceleration), along with the basic trigonometric functions and their inverses and derivatives. This chapter concludes the first semester, and at the end of the chapter we summarize the derivative calculations.

Chapter 6: Antidifferentiation. At the start of the second semester, we turn our derivative summary inside out and catalog the functions for which we can now find antiderivatives – a necessary step if we're going to solve differential equations. We expand our tool kit with the simplest case of partial fractions to solve the Verhulst (logistic) model of population growth and explore how Verhulst, writing in 1840, could predict the US population in 1940.

Chapter 7: The Fundamental Theorem of Calculus. The big moment everyone has been waiting for – we introduce the integral as an averaging process, e.g., finding average temperature over a day or a year, and then relate that to area under a curve. We approach the FTC by exploring the linkage between speedometer and odometer, and then we "derive" the theorem by solving a differential equation – given the derivative, what's the function? – a question for which we already know one kind of answer. The partial sums of the left-hand rectangular approximations to area are, in fact, the Euler approximations to the solution of the differential equation, and this establishes the connection between antidifferentiation and area. Given this connection, it makes sense to introduce the indefinite integral as a notation for antidifferentiation.

Chapter 8: Integral Calculus and its Uses. This is the second-semester analog of Chapter 4. We start with a problem of fundamental physical importance, moments and centers of mass, to reinforce the idea of integration as averaging. We develop numerical methods through Simpson's Rule (as a weighted average of the trapezoidal and midpoint rules), so that no definite integral need remain unevaluated when one is working at a computer. Then we address the basic rules for integration by hand: algebraic and trigonometric substitutions and integration by parts. We close with an elementary look at Fourier analysis, using an electrocardiogram as an example.

Chapter 9: Probability and Integration. Our model problem in this chapter is reliability theory – how long do things last? The simplest model is the exponential distribution, which leads naturally into improper integrals. After eight chapters of limiting behaviors, we introduce the standard notation for limits (but not the ε-δ definition, which belongs in a later course). We also take up other probability distributions (e.g., the normal) for which finding a mean or standard deviation may involve proper or improper integrals that can't be evaluated in closed form. This leads to defining some functions (e.g., the error function) by their integral representations.

Chapter 10: Polynomial and Series Representations of Functions. Our emphasis is on representation of important functions, whether approximately by polynomials (perhaps very long polynomials) or by "infinitely long" polynomials. We start with the easy ones – exponential and trigonometric – and work up to the error function, using substitutions, differentiations, integrations. As a practical application, we note that one can evaluate the error function (even on a calculator) fast enough to graph it by using a relatively short polynomial. The primary tools for testing convergence are the alternating series test (AST) and the ratio test (RT) – and often they are the only tools needed. The first is geometrically obvious, and the second we obtain by comparing the tail of a series to that of a geometric series. Both come with error estimates. For power series, only the RT is needed unless there is a finite radius of convergence – and then the AST or comparison with, say, a harmonic series will usually do the trick.

3. The Second Edition Textbook Our goals for this edition are to (a) make the text flexible, hyperlinked, interactive, richly illustrated, and available at low cost, and (b) demonstrate the feasibility of an online textbook. In our redesign and redevelopment we encountered issues of page design, navigation, directory and file structure, sources for illustrations, nature and implementation of the built-in interactivity, technical requirements, and effective presentation of mathematics online. Here are brief comments on how we dealt with some of those issues.

The basic page design was provided for us by an in-house designer at MAA. Our navigation is based on a Table of Contents window for each chapter that remains open beside the main text window. Notes, comments, and checkpoints open in pop-up windows that can be closed when they are no longer needed. Each main page has forward and back buttons, as well as a link to the Contents page. The front page and every Contents page links to the Index, which contains all the mathematical terms in the book, each linked back to the page on which that term is defined.

Our illustrations are of two kinds – mathematical graphs or diagrams and photographs or other illustrations. For the first kind, we construct a mathematically correct graph in a computer algebra system and edit it in a graphic tool. For the second, we find public domain or otherwise free photos or we take our own digital pictures and then crop, resize, enhance as necessary.

Our interactions use built-in pop-up numeric and graphing calculators, as well as files from which a student can print simple graph layouts, graph paper (for hand drawing), or slope fields for sketching solutions of ODEs. We have some embedded Flash applets for carrying out experiments, and we have many prepared computer algebra (CAS) files in Maple®, Mathcad®, and Mathematica® that each get students started on an assigned task, but that will not completely solve the problem without student thought and inputs.

Our main text pages are constructed in XHTML, with most of the formulas presented in MathML. We also use ASCIIMathML for some of our formulas, including all the ones in pop-up pages, which are ordinary HTML pages.

To support our use of MathML across platforms, we require that the user have or install the Firefox browser and Mozilla's recommended (STIX) MathML fonts. To support ASCIIMathML and our control code for pop-ups and other interactions, we require that the user enable javascript. And to support our CAS activities, we require one of Maple®, Mathcad®, or Mathematica®. Our embedded applets require the Flash® player, which is included with all modern browsers, but it's also available as a free download.

4. Classroom Testing Our textbook has been classroom-tested at Hood College since 2006 under the guidance of the third author, who is the lead teacher for calculus. (In the 2008-09 academic year it is also being used at four other colleges.) Hood is a private, coeducational, liberal arts college with about 1200 undergraduate students, including a significant number of commuter students.

The online text has been used for Calculus I since fall 2006 and for Calculus II since spring 2007. On average, there are three sections of Calculus I in the fall with 70 students, 2 or 3 instructors and 4 teaching assistants (TAs), as well as one section of Calculus II with 15 students, one instructor and one TA. In the spring, about 45 students continue on to two sections of Calculus II, taught by 2 instructors and 3 TAs. There is also a spring section of Calculus I with about 20 students, one instructor and one TA.

The challenges of using this book include getting students to accept an online text, learning how to use the text in class, convincing students to actually read the book, and coping with editing that was ongoing while the course was in progress. On the asset side of the ledger are the direct links to technology and to outside information, the checkpoints and activities with (slightly) "hidden" answers, and the opportunity to have a direct impact on the emerging edition.

A typical class day at Hood includes varied activities that might be any mix of discussion of the text (sometimes via lecture), working on a lab in pairs or on a project in somewhat larger groups, working on individual worksheets (possibly consulting with a neighbor), or writing a group report. Over the course of the year, the Hood classes covered most of the text but omitted periodic motion and circular functions in the first semester, as well as Fourier representations and some probability theory in the second semester. Parts of the omitted material were covered instead in labs, projects, or worksheets, either from the first edition (and eventually to be in the second edition) or of local design.

Here is our advice to instructors considering adoption of this online text:

- Take time to get familiar with the structure of the book and how the topics build on each other. Keep in mind that students who have seen calculus before may find this organization unfamiliar.
- Be ready and willing to talk with students about why your class has the structure it has and why you are using an online text. Acknowledge the benefits and the drawbacks.
- Have the students read a bit about math education research, e.g., Smith, D. A., "Thinking about Learning, Learning about Thinking" (in A. W. Roberts, Ed., *Calculus: The Dynamics of Change*, MAA Notes No. 39, 1996; available online at http://www.math.duke.edu/~das/essays/thinking/).
- Spend time in class having your students read the text. Have them do the activities as they read, and perhaps interrupt periodically to have a class discussion.
- Exploration takes time, and there's no substitute for experience – the students' experience, that is. Build in time for exploration.
- Letting the class explore means you don't have complete control over what will happen next. Be flexible.
- Find ways to find out what your students really know. Think outside the (exam) box.
- Work the projects ahead of time!
- As you teach, keep track of what you do when. Some topics in the text are introduced at a surface level and revisited as tools become available.

Acknowledgments Our project evaluator, Jack Bookman, has provided very helpful formative evaluation. Our Flash applets (including the graphing calculator) were developed by Douglas Ensley and Barbara Kaskosz. ASCIIMathML was developed by Peter Jipsen, who also created our numeric calculator. Lake Bookman converted our Maple worksheets to Mathematica. Our SVG graphics are being developed by Troy Henderson. Some of our javascripts were created by Kyle Siegrist, who also assisted us with presentation of mathematics in MathML. Our development of WeBWorK exercises has been greatly assisted by Sam Hathaway.