SHORTCHANGING OUR STUDENTS?
THE SHORTCOMINGS OF SHORT ANSWER MATH SOFTWARE

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Current math practice software overwhelmingly consists of problems to which the student enters only a short final answer. The typical program responds to any incorrect final answer with a stored solution. For a multiple-step math problem, such a solution often has little relation to the solution method actually used by the student, and so is unresponsive to whatever error or errors caused the incorrect final answer. Even if both solutions use the same method, the student learns of the error or errors only after completing the problem, and often fails to track them down, particularly if the steps of the two solutions are not precisely identical at least up to the first erroneous step.

Example 1 is from a major publisher.

Solve using substitution:

$$\begin{cases} 3x-10=-y \\ 2x-y=0 \end{cases}.$$

Working on paper, a student solves the **first** equation for y:

$$y=10-3x. \qquad (1)$$

The student then replaces y in the second equation by $10-3x$, but fails to distribute:

$$2x-10-3x=0.$$

$$x=-10.$$

Finally, the student replaces x by -10 in equation (1) above, and evaluates y:

$$y=10-3(-10)=40.$$

The student enters only an incorrect short final answer (solution set) into the program:

$\{(\square 10,40)\}$.

The program responds that the final answer above is incorrect, but then, ignorant of the student's method, shows a stored solution that begins by solving the **second** equation:

$$2x=y.$$

It then replaces y by $2x$ in the first equation, and solves for x:

$$3x-10=-2x.$$

$$5x=10.$$

$$x=2.$$

Finally, the program substitutes 2 for x in the second equation, and solves for y:

$$2(2)-y=0.$$

$$y=4.$$

Correct answer: $\{(2,4)\}$.

The stored solution above has no steps in common with the student's solution, and gives no help in finding the student's sign error. Indeed, this student could reasonably conclude that the only correct method is to begin by solving the second equation.

Example 2 is from another major publisher.

Solve for x:

$$\frac{1}{2}x - \frac{4}{3} = \frac{7}{9}.$$

On paper, Student A begins by multiplying both sides by 18 to clear the fractions, but then makes a sign error in transposing:

$$9x - 24 = 14.$$

$$9x = -10.$$

Student A enters only a short final answer into the program:

$$x = -10/9.$$

Student B, also working on paper, combines the fractions on the left and clears the fractions by cross-multiplying, but then also makes a sign error in transposing:

$$\frac{3x-8}{6} = \frac{7}{9}.$$

$$9(3x-8) = 6 \cdot 7.$$

$$27x - 72 = 42.$$

$$27x = -30.$$

Student B enters the same short final answer as Student A:

$$x = -10/9.$$

The program, ignorant of the students' methods, displays a stored solution, a condensed version of which is displayed here:

$$\frac{1}{2}x - \frac{4}{3} + \frac{4}{3} = \frac{7}{9} + \frac{4}{3}.$$

$$\frac{1}{2}x = \frac{7}{9} + \frac{12}{9} = \frac{19}{9}.$$

$$\frac{1}{2}x = \frac{19}{9}.$$

$$x = 38/9.$$

The program's stored solution never clears the fractions. It has no steps in common with either student's solution, and helps find neither of their transposing errors.

Responsible instructors invariably ask students to show all their steps. Seeing all the steps is essential for providing optimal feedback. The top-selling current math practice programs merely emulate math workbooks by showing just one method of solving each problem. Students require and deserve more and better help than can possibly be obtained from software using a short answer and stored solution approach.

Ideally, math practice software should emulate a good instructor. It should:

- Accept step-by-step solutions.
- Flag any incorrect step immediately.
- Review as appropriate after an incorrect step.
- Describe and suggest a reasonable next step after any correct step, regardless of solution method, whenever such help is needed or requested by a student.

To do this, software must necessarily compute, rather than store, suggested solutions. The key algorithms are:

- An "equivalency checker" to determine whether a newly entered step is correct, i.e., equivalent in the sense appropriate to that type of problem.
- A "suggested next step generator" applicable at any step of any correct partial solution to suggest a next step in that solution. Applied iteratively, such an algorithm can generate a suggested complete solution to any appropriate problem or a suggested continuation of any correct partial solution regardless of the solution method used to reach that intermediate step.

To the author's knowledge, no major American publisher currently offers such software. However, various individual math faculty have produced and self-published programs incorporating these types of algorithms.[1]

Example 3 is from ***xyAlgebra***, the author's completely free program.

Please solve this equation for c:

$$\frac{c}{5}-1=\frac{4}{5}-3c\,.$$

The student uses a correct but unexpected strategy of "collecting fractions:"

$$\frac{c}{5}-\frac{4}{5}=1-3c\,.$$

$$\frac{c-4}{5}=1-3c\,.$$

When attempting to "cross multiply," the student makes a mistake:

$$c-4=5\bullet 1-3c\,. \qquad (2)$$

The ***xyAlgebra*** equivalency checker detects the error. It opens a sub-window and generates an equation similar to the student's last correct equation:

Here is a suitable next step in solving the following equation for r:

$$\frac{r-2}{3}=2-5r\,.$$

Then it applies its suggested next step generator:

$$r-2=3(2-5r)\,. \qquad (3)$$

Hint: "Cross multiply." (a/b=c/d is equivalent to ad=bc.)

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[1] xyAlgebra (www.xyalgebra.org),
MathXpert (www.helpwithmath.com),
Math Teacher (www.mathkal.co.il).

Upon comparing equations (2) and (3) above and noting the hint, the student usually sees the nature of the error. Then ***xyAlgebra*** offers the student a chance to practice this operation on yet another similar equation:

Please enter a suitable next step in solving the following equation for ***t***:

$$\frac{t-3}{4} = 5 - 2t\,.$$

The student can enter anything equivalent, such as:

$$t - 3 = 20 - 8t\,,$$

or the student can press "escape" as soon as the point of this digression is clear. Then ***xyAlgebra*** closes the sub-window and returns to the original problem, which the student can usually continue correctly:

$$c - 4 = 5(1 - 3c)\,.$$

$$c - 4 = 5 - 15c\,.$$

$$16c = 9\,.$$

$$c = 9/16\,.$$

Such timely and targeted help requires software that accepts step-by-step solutions to all multiple-step problems. The three little-known programs cited in the footnote above are all reasonably good at emulating the intelligent help offered by a responsible instructor. The "short final answer only" format used by programs from major publishers cannot possibly provide comparable help because a short final answer contains inadequate information.

The reason that major publishers do not offer such intelligent step-by-step help appears to be the widespread failure of potential software adopters to insist that they would prefer this level of support for their students. The move from multiple-choice to short answer formats in the 1980s and early 1990s appeared to be driven by many math educators' repeated public comments about the limitations of the multiple choice format. Similar protests about the limitations of short answers are rarely heard today. Until more faculty voices are raised in support of accepting step-by-step solutions and responding intelligently to each step, major publishers are unlikely to produce such materials. The price of our failure is being paid by our students.