DYNAMIC ALGEBRA USING TI-NSPIRE CAS

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Introduction

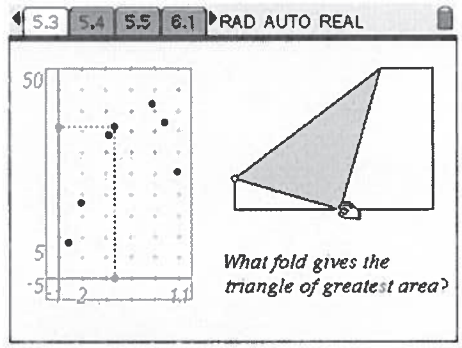
The term "dynamic geometry" has for many years been a popular descriptor for a generic software tool which supports construction and manipulation of geometric figures. More recently I understand that it has been copyright reserved for the popular Geometer's SketchPad package, and the term "Interactive Geometry" now assumes the more generic role of software-type descriptor. The essential component of such tools lies in the "live" nature of the constructs in the sense that a spreadsheet can be "live" - changes in one component lead to immediate changes in all dependent components. While we have come to expect this level of interactivity in our geometric and numerical environments, the vast majority of computer algebra software environments remain anything but "live". Not so TI-Nspire CAS. While the traditional "Calculator" environment of this tool remains a sequential history of past calculations, other working environments are dynamically linked, both within and across different representations. This brings with it new demands for ways in which such capabilities may best be harnessed as tools for teaching and learning, and this discussion offers opportunities to deepen familiarity with the available tools within TI-Nspire CAS. From the dynamically linked worlds of geometry, spreadsheet and calculator to the quietly under-utilized capabilities of Notes and the power of programming across these environments, we examine applications across the school and college years for students at all levels of instruction.

A Dynamic Approach to Algebra

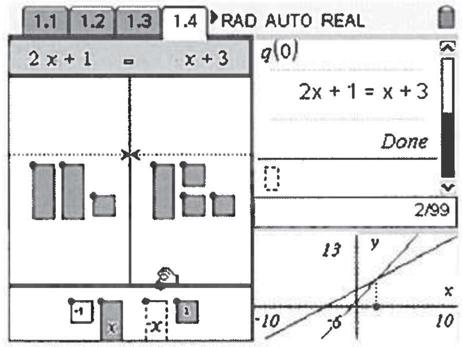
TI-Nspire technology, especially in its CAS (Computer Algebra System) form, offers great possibilities for enhancing and transforming the teaching and learning of algebra across the secondary school years. It may even be in the formative years when algebra is first being introduced, that the potential may be greatest.

Introduced without a great deal of forethought and caution, however, CAS stands to do at least as much damage as good in the early years of high school - a key problem with traditional algebra instruction lies in its focus on the syntax (the rules and manipulations with apparently meaningless symbols) at the expense of the semantics (the meaning and concepts behind the variables and functions being used). In their basic form, CAS tools are purely syntactic - they are manipulative tools, which blindly carry out the processes of algebra without recourse to any underlying significance.

A tool such as TI-Nspire offers much in this regard. It can be used to embody the manipulations, the variables and the functions within meaningful context - often underpinned by dynamic geometric models which can serve, not just to provide context, but also to provide measured data which can be graphed; this graphical model then forms the basis for verifying algebraic models constructed by the students.



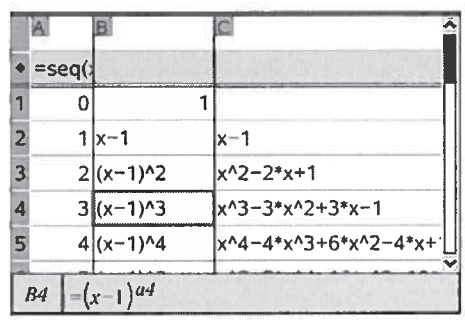
Within the geometric context, too, concrete models can be constructed which can strongly underpin student understanding and correct manipulation of the symbols of algebra.



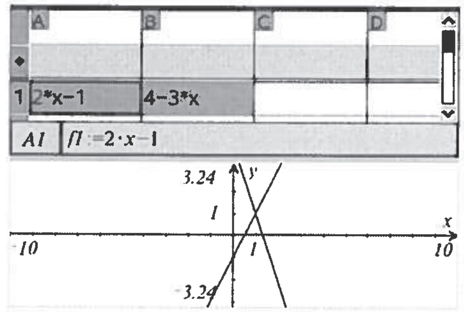
There lies within the Lists & Spreadsheet application, too, the potential for what might be termed dynamic algebra in which algebraic processes may be developed and verified in an interactive way. Some explorations are presented here: beginning with linear equation solving, simultaneous equation solving and completing the square, and moving on to applications to higher mathematics.

An Algebraic Spreadsheet

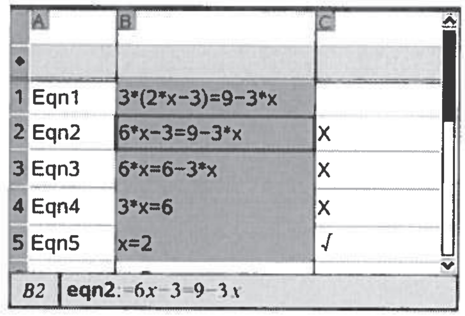
One of the unique features of TI-Nspire CAS lies in its Lists & Spreadsheet application, which offers a full algebraic working environment. Combining the power of the spreadsheet and its ability to expose the "inner workings" of a computation with computer algebra presents some interesting possibilities.



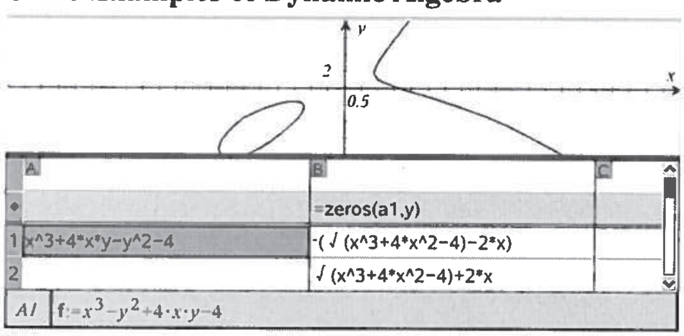
These possibilities are further extended when links between the spreadsheet and the Graphs & Geometry application are considered. It is easy to set up dynamic links between these two environments and allow students to explore the impact of each upon the other. For example, split a screen between L&S and G&G and define f1(x) = x and f2(x) = -x. Now go to the spreadsheet and enter functions into cells A1 and A2 as shown. Link these to the variables f1 and f2 using the VARS menu or just CTRL-L and you have a dynamic linkage.

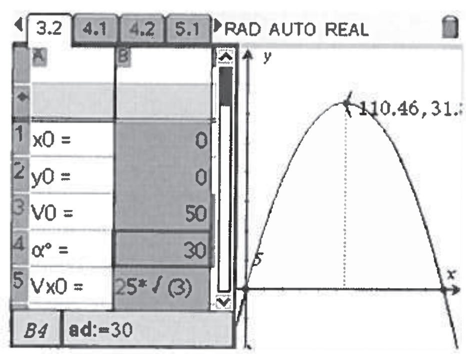


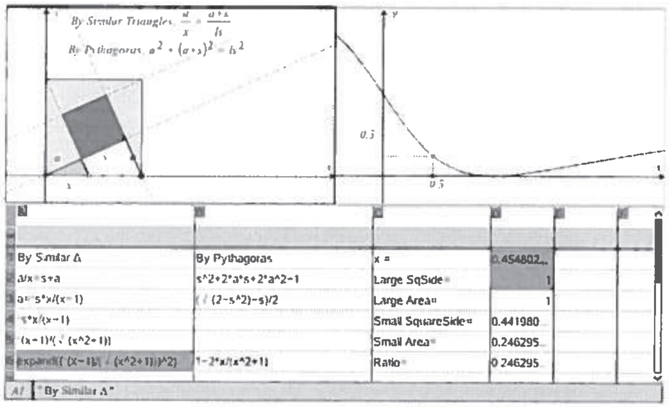
If we are genuinely to use computer algebra tools to support meaningful algebra learning, especially in the early years, then it becomes increasingly obvious that a major challenge lies in finding ways NOT to let the tool do all the work! Hence, an equation solving environment is shown in which students enter each of the steps and the algebraic tool checks their correctness on each line – powerful and immediate feedback by which sense can be made of the algebraic process.

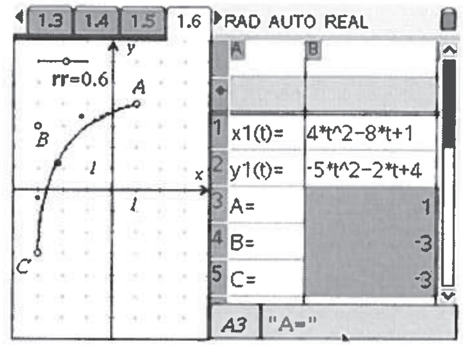


Some Examples of Dynamic Algebra





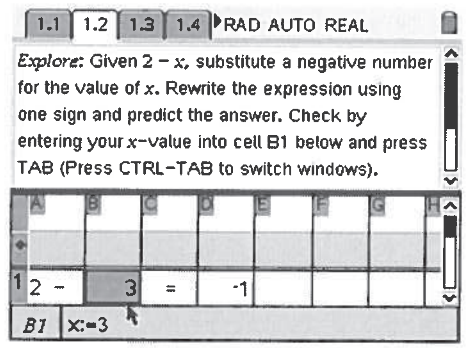




Spreadsheets and Magic

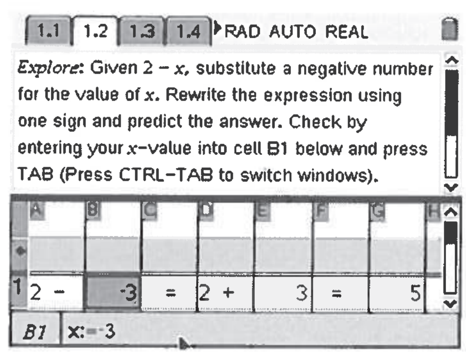
At the heart of the "magic" that can be used to make things appear and disappear in TI-Nspire lies the "when" command, which takes on some surprising power within both L&S and G&G environments. Its syntax is simple: **when(something happens, do me if the condition is true, [and optionally do me if false, or even do me if you don't know what to do]**.

For example, suppose we are interested in teaching students how to subtract positive and negative integers. If "x" represents the integer we wish to subtract, say from 2, then we could design a spreadsheet layout which might look like that opposite.



In cell A1, we have a text statement, "2 -" and in cell B1, the value for our variable "x" (currently 3). Notice that the cell is gray - the contents of this cell have been linked to the variable "x" using either CTRL-L or by pressing the VARS key and storing to "x".

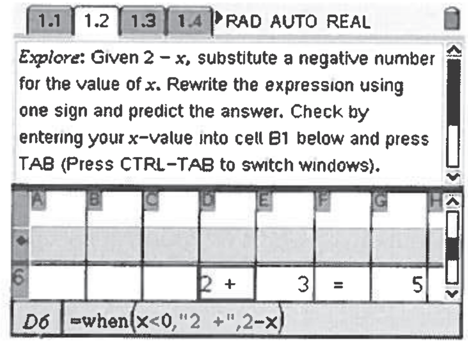
Cell C1 has " =" entered as text (always using "" when entering text) and cell D1 has the value of the expression "=2 - x" as required by the original question.



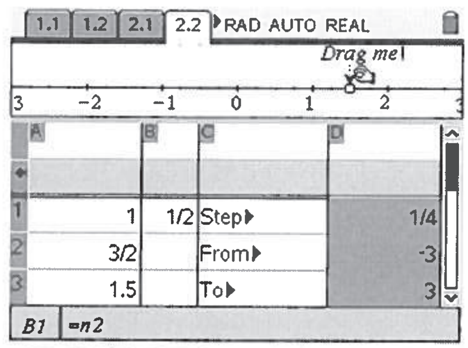
So far, so good - but what would we like to see happening here? For students to learn how to subtract a negative integer, I would like them to see an expression such as "2 - (-3) = 2 + 3 = 5".

Cells A1, B1 and C1 remain the same, but something has changed in cells D1 - G1. Let's have a look at what is happening in cell D1.

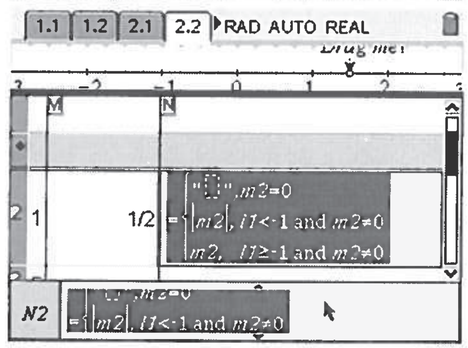
I chose to put these commands down in row 6 of the spreadsheet rather than in the actual cells D1 to G1: this prevents the user from accidentally erasing or changing these commands if and when they land in these cells; instead, in cell D1, I simply put "=D6" which will just display the contents of cell D6, as required.



This ability to control what is calculated and what is actually displayed in the spreadsheet is a very useful and versatile feature which can be used in a variety of contexts. For example, in the number line explorer, a point whose x value is linked to a variable is moved along a number line and different representations are displayed in the spreadsheet.



Here we are using a different form of the "when" command, which is just the simplest form for a piecewise function. In a piecewise function, we can define several different conditions and the results which follow from each.



It is important to note, before we leave our Lists & Spreadsheet behind, an important difference between the CAS (computer algebra system) and the non-CAS versions of this application. The CAS version will quite happily accept algebraic (symbolic) expressions in its cells, and quite happily act upon these. Thus, we can enter a valid command such as =solve(x^2-x-1=0,x) into cell A1, and the output will be the exact form for the solutions of this quadratic. Thus we can use CAS L&S to display algebraic expressions and equations, if desired. So many possibilities…