

PODCASTING RICH PROBLEMS WITH TI-NSPIRE CAS

Michael Todd Edwards
Miami University, Department of Teacher Education
Oxford, Ohio 45056
edwardm2@muohio.edu

Asli Ozgun-Koca
Wayne State University, College of Education
Detroit, Michigan 48202
aokoca@wayne.edu

Michael Meagher
Brooklyn College, School of Education
Brooklyn, NY 11210
mmeagher@brooklyn.cuny.edu

I. Introduction

The capacity of TI-Nspire to link representations in multiple directions provides students with new pathways for exploring rich problems. In this paper, we highlight instructional advantages of representational options within NSpire through the exploration of the "Glass Rod Problem" (Haigh, 1981). Initially envisioned as a simulation task, the problem is well-suited for geometric and algebraic exploration.

II. Statement of the Rich Task

The "Glass Rod Problem" has a rich history (Budden, 1982; Haigh, 1981; Whittaker, 1990). Haigh (1981, p. 37) poses the problem in the following manner:

The Glass Rod Problem: A glass rod drops and breaks into three pieces. What is the probability that a triangle can be formed from the pieces? (p. 37)

Viewing the task as a simulation activity, we "mathematized" the problem, labeling breaks as X and Y; lengths of resulting pieces as a, b, and c; and endpoints of the rod as 0 and 1. This interpretation is illustrated in Fig. 1.

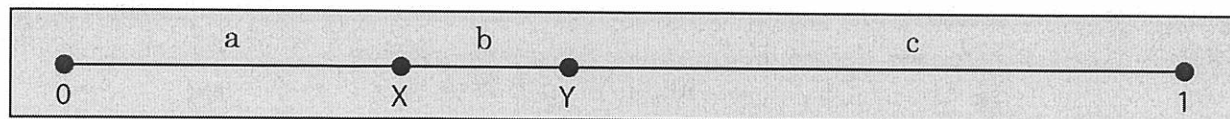


Figure 1: "Mathematized" sketch of the Glass Rod scenario.

While many solve the problem as a simulation (randomly generating X and Y for a large number of cases), "pure" geometric approaches are equally productive.

III. Simulation-Based Approach

Generating Random Points X and Y. Viewing the problem as a simulation task, students generate random breaks X and Y directly from the calculator application. The command `rand(500)`, depicted in Fig. 2 (left), generates 500 random values, each ranging from 0 to 1. The commands in Fig. 2 (left) define lists a, b, and c in accordance with the "mathematized" interpretation of the problem in Fig. 1.

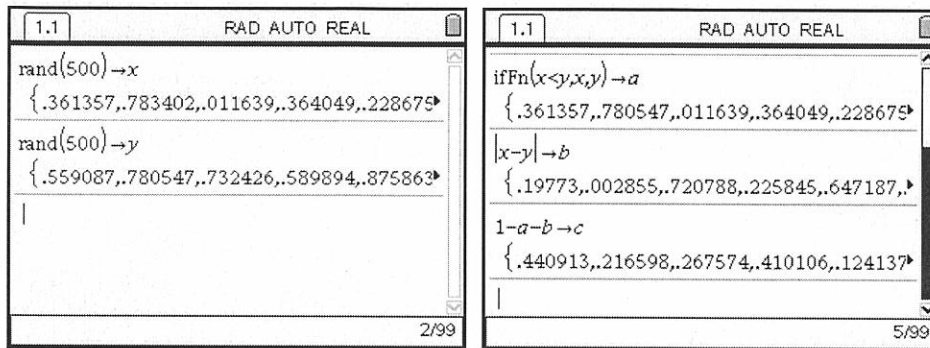


Figure 2: (Left) Random generation of points X and Y; (Right) Constructing lists a, b, and c from known values in lists x and y.

Next, considering entries of lists a, b, and c as lengths of segments, cases that satisfy the triangle inequality are identified as illustrated in Figure 3 (left).

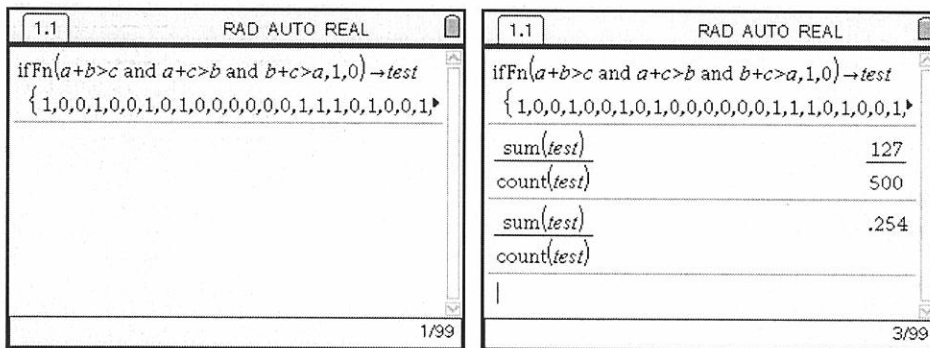


Figure 3: (Left) List test contains information regarding cases satisfying the triangle inequality; (Right) Experimental probability calculation.

Specifically, for all i such that $a[i]$, $b[i]$, and $c[i]$ satisfy the triangle inequality, $test[i]$ is assigned a value of 1 (i.e. true). In all other cases, $test[i]$ is assigned a value of 0 (i.e. false). Summing $test$ and dividing by the total number of entries provides an experimental probability for the given trial (see Fig. 3 (right)).

Plotting Points. Alternatively, lists may be plotted on a separate page using the Geometry & Graphs application. Lists xt and yt are defined as subsets of lists x and y in the following manner: when $(x[i], y[i])$ satisfies the triangle inequality, $xt[i]$ and $yt[i]$ are assigned the values $x[i]$ and $y[i]$, respectively, and 0 oth-

erwise (see Fig. 4 (left)). A scatterplot of ordered pairs $(pt[i], qt[i])$ - see Fig. 4 (right) - provides a visual interpretation of the solution set.

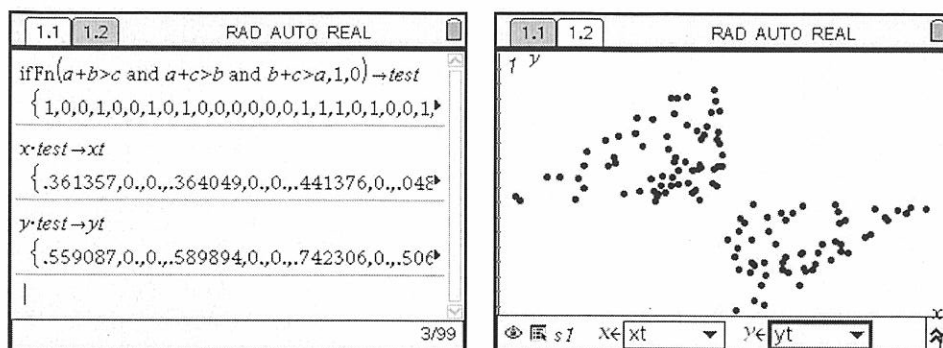


Figure 4: (Left) Lists xt and yt ; (Right) Plot of ordered pairs $(xt[i], yt[i])$ reveals a pattern regarding values of X and Y that yield triangles.

Graphical / Algebraic Representations. An algebraic argument is constructed by noting that two cases of X and Y exist - namely, $X \leq Y$ or $X > Y$. When $X \leq Y$, pieces have lengths X , $Y-X$, and $1-Y$. To form a triangle, the following inequalities hold: (1) $X + (Y-X) > (1-Y)$; (2) $X + (1-Y) > (Y-X)$; and (3) $(Y-X) + (1-Y) > X$. Algebraic simplification yields the following equivalent system: (1) $Y > 1/2$; (2) $Y < X + 1/2$; and (3) $X < 1/2$. Plotting this system of inequalities yields a triangular region with area $1/8$. (refer to Fig. 5). The alternative case, $X > Y$, also yields a triangular region with area $1/8$. Hence, the probabilities of the two cases equals $1/8 + 1/8 = 1/4$.

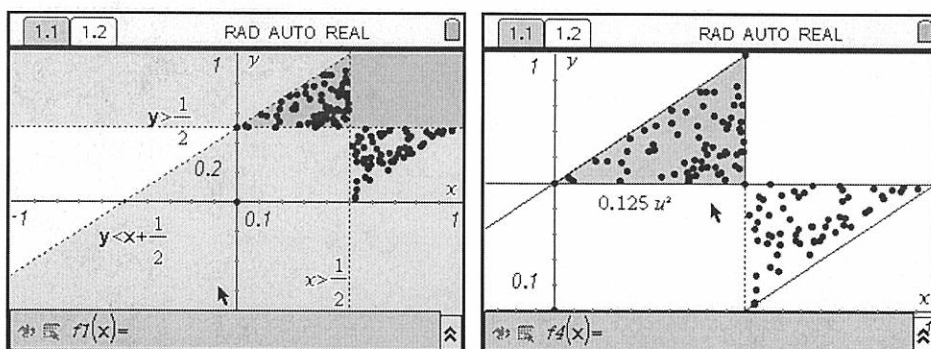


Figure 5: (Left) Plot of system of inequalities derived from triangle inequality; (Right) Triangular regions bounded by system has area $1/8$.

IV. Geometric Approach

Alternatively, a solution can be motivated using dynamic geometry tools. A sketch of the glass rod problem is depicted in Fig. 6.

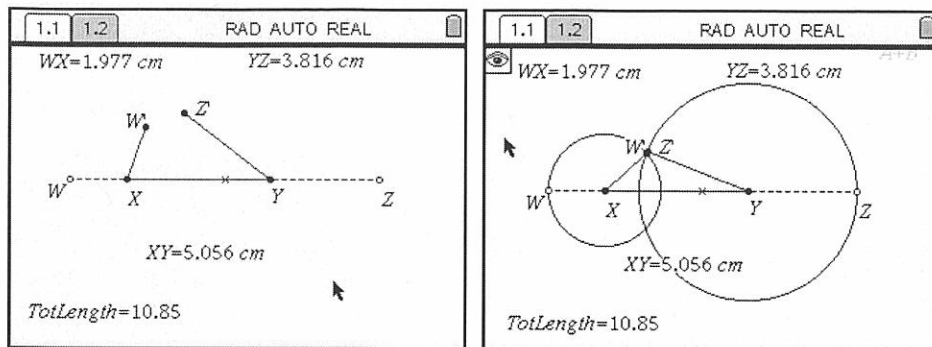


Figure 6: (Left) Sketch of rod; (Right) Students drag W' and Z' attempting to construct a triangle. This is possible when circles X and Y intersect.

In the sketch, Point W' is constructed on a circle with center X and radius XW. Similarly, Z' is constructed on a circle with center Y and radius YZ. Students experiment by dragging breaks X and Y along segment WZ. Students rotate W' and Z' in an effort to construct a triangle. As Fig. 6 (right) suggests, a triangle is constructible when circles X and Y intersect. Students soon realize that a side of a triangle can not be greater than half the perimeter (i.e. length WZ). When X and Y are on the same side of the cross (i.e. the midpoint of WZ), then one of the sides is greater than half the total perimeter. Assuming without loss of generality that $X < Y$ and $WZ = 1$, these observations lead directly to the system of inequalities (1) $X < 1/2$; (2) $Y - X < 1/2$; and (3) $Y > 1/2$, which in turn lead to a probability of $1/4$.

V. Constructing a Podcast

Purposes of Podcasts. We use podcasts as a vehicle to encourage students to explain mathematics concepts more thoughtfully. Future teachers may create lesson plans to accompany podcasts. In such cases, the podcasts may highlight a particular aspect of the lesson or use of a particular technology. For *any* student, podcasts may be used to highlight content that needs further explanation. Often, ideas are more easily conveyed with spoken descriptions as opposed to written explanations alone.

Suggested Tools/Sequence. To create Nspire podcasts, we follow these steps.

0. Download/Install Software. We use the following software: *Notepad*, *Windows Movie Maker*, *TI-Nspire Computer Link*, and *Audacity*. The first two titles are installed with Windows by default; the last two are freely downloadable.

1. Practice the Task Before Recording. Perform the desired Nspire activity several times without interruption (until the steps are routine).

2. Construct a Script. Using *Notepad*, type descriptions of intermediate steps. Think of steps in terms of images to be displayed (e.g. one image per step), labeling steps as "Step1", "Step2", etc. within the script.

3. Create Screenshots. Perform the activity from the beginning, taking screenshots with *Nspire Computer Link* software for each step. Name images according to script step (e.g. Screen001.tif corresponding to Step1).

4. Create Audio Files. Using Audacity, record spoken commentary/instructions for each step as .wav files. Name files according to script step (e.g. Audio001.wav corresponding to Step1 and Screen001.tif).

5. Combine Audio and Images. First, import audio files into *Windows Movie Maker*, dragging them into the application timeline (see Fig. 7, left). Next, drag screen images into the software, one at a time, in order. Fig. 7 (right) illustrates Screen001.tif dragged into the *Windows Movie Maker* video track. The length of time that a particular screen is displayed should be resized to fit the duration of the corresponding audio file. Fig. 8 illustrates screens 1-3 resized to fit audio 1-3.

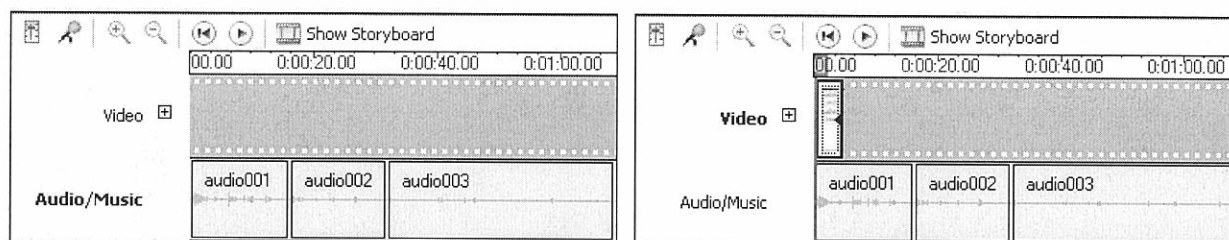


Figure 7: (Left) Audio dragged into *Windows Movie Maker*; (Right) Image dragged into video track of *Windows Movie Maker*.

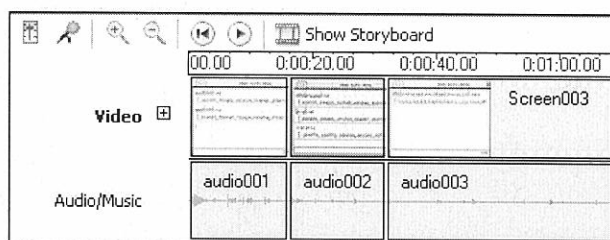


Figure 8: Screens 1-3 resized to fit audio files 1-3.

6. Save Video. Export the completed video from *Windows Movie Maker* selecting the appropriate option from the File menu (e.g. "Save Movie File").

References

- Budden, F. (1982). "Some Connected Problems on Probability." *Mathematical Gazette* 66(435), 4-10.
- Haigh, C. (1981). "The Glass Rod Problem." *Mathematical Gazette* 65(431), 37-38.
- Mathai, A. M. (1999). *An Introduction to Geometrical Probability*. Gordon and Breach Science Publishers. Montreal, Canada: CRC Press.
- Whittaker, J. (1990). "Random Triangles." *The American Mathematical Monthly* 97(3), 228-230.