A PEER-LEARNING APPROACH TO CONCEPTUAL UNDERSTANDING WITH ICLICKER

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ABSTRACT. This paper is an introduction to the work I have been involved in for the past few years developing good questions to use in conjunction with the iclicker classroom response system to create peer-learning opportunities that promote conceptual understanding in college algebra.

The use of personal response systems (iclicker, PRS, H-ITT, TurningPoint, etc.) has proliferated rapidly through universities across the nation in recent years. Although the spread of clicker use may have occurred with more speed and enthusiasm in other areas, the mathematics classroom may well have the most to gain from the carefully reasoned use of these systems. Even the most basic use of a clicker system to check students' fundamental manipulative skills can represent a significant boon to both the teacher and the learner of mathematics. The biggest teaching and learning payoffs, however, emerge from joining the clickers together with peer-learning opportunities that challenge students' intuition and promote conceptual understanding.

Understanding mathematical relationships translates into the ability to solve problems. When we find some problem easy to solve, usually this is because we have a clear understanding of the underlying mathematical relationships. We have an opportunity to perceive the limits of our understanding when we consider problems that challenge our intuition. If we act to push our understanding beyond its current limits, we will solve problems that arise from more diverse and sophisticated scenarios. This is the philosophical basis of the teaching strategy described here. Although the context for this discussion is a freshman level course in College Algebra, the techniques have also been applied (by others) to the calculus sequence and linear algebra. What follows is first a description of the activities of the course and later a discussion of a few of the most engaging questions.

Before each lecture students read (or watch a video of) the material from the text which will be covered in the coming lecture. The students will then answer two or three warm-up questions and submit their questions about the reading via an on-line homework system. At the beginning of the lecture several exercises (which include the warm-up questions when necessary) will be posed by the lecturer and answered by each student via iclicker. These answers are collected by the lecturer's computer, which then anonymously shows the results of the tally. The students and lecturer together work out the answers to these exercises. This provides feedback to the students regarding how well they understood the reading assignment and how well they understand how to work the manipulative exercises.

Next, following a brief lecture including answers to the most commonly submitted reading questions, a sequence of conceptual multiple choice or true/false questions is posed and answered via the personal response system. The questions leading into the main discussion are chosen so that as much as possible the students will be able to answer and explain them entirely based on their previous knowledge of mathematics. The main discussion question is first answered by the students. After the students' first reactions are polled, time is allotted for discussion between pairs of students regarding why certain answers should be correct and others incorrect. Next, the answers are polled again, frequently with a noticeable shift toward the correct response. Finally, the instructor guides a student discussion of which answer is most correct, and why. This process helps students to understand the value of their previous knowledge of mathematics, and often the questions are formatted to make a connection between that knowledge and their real-world experience. The students later complete a discussion write-up, a brief explanation of the answers to the discussion questions. They receive detailed feedback on these write-ups from their instructor.

We start with the topic of linear relationships - perhaps the most important of all the concepts included in the standard college algebra curriculum.

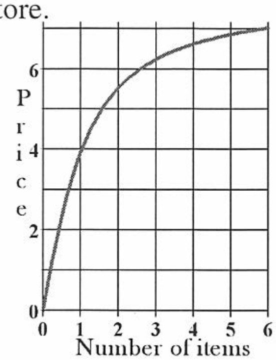
Problem. Imagine that there is a rope drawn tightly around the equator of the earth. Suppose we increase the length of this rope by 60 feet. The new rope is held in a circular shape centered about the earth, off of the ground. Then the tallest of the following that can walk beneath the rope without touching it is:

A) An amoeba
B) An Ant
C) A Dog
D) Me (the student)

This question is difficult for students because the answer goes against our natural intuition. The change in circumference is very small relative to the circumference of the original loop, so it seems as though the change in radius should also be very small. This intuition is reasonable, but the height of a person is also very small relative to the radius of the earth. Since the relationship between the circumference of the circular shape (the length of the rope) and the radius of the circular shape of the rope is linear ($C = 2\pi r$) only the difference in the size of the two circumferences matters. That means that whether the rope is around the earth or the moon or a tennis ball, increasing the length of the rope by 60 feet increases the radius and moves the rope to a height of almost 10 feet ($60/2\pi$ feet) above the surface. (The author originally encountered this problem in the GoodQuestions for Calculus project at Cornell University - see [1] or google GoodQuestions for details.)

Our next example concerns transformations of functions, and the sticky topic of multiple transformations.

Problem. Suppose this is a graph of the cost for buying some number of items in a store.



If there is a buy one get one (BOGO) free sale plus 3 dollars off of your total order, then we should

A) move the graph down 3, and then horizontally stretch by a factor of 2.
B) move the graph down by 3 and vertically stretch by 2.
C) move the graph left by 3 and horizontally stretch by a factor of 2.
D) Horizontally stretch the graph by 2 and then move it down 3.

This is one of my favorite problems because the peer-learning segment typically alerts the students to the second level of sophistication - namely, they start to discuss whether or not answers A and D are equivalent. In this case, since there is only one horizontal and one vertical transformation represented by the problem these answers are in fact equivalent, even though this is not immediately obvious and requires a bit of explanation. This is a nice topic to bring up, since the order in fact matters very much when choosing among multiple horizontal or multiple vertical transformations - a point which is uniformly overlooked by textbook authors for this course.

The topic of the last question included here is exponential growth. Many teachers and texts go to some length to explain that exponential functions grow faster than polynomial functions. Although there is a sense in which this is true, it can and does easily turn into a misconception, even for research mathematicians.

Problem. You come across an advertisement in a magazine for an investment opportunity that guarantees that your investments with their company will "Grow Exponentially Fast!". You invest \$5,000 bucks, since everybody knows exponential growth is really fast. Five years later, your investment is worth \$5,250. You should

A) definitely sue, since no exponential function could grow that slow.
B) still expect to get rich, but plan to live a really long time.
C) do nothing, since your investment will be worth way more within the next couple of years.

This problem is actually a take away from my own research, which concerns the exponential asymptotic decay rates of various probabilistic quantities. There are in fact a huge number of example of phenomena for which the growth or decay of some quantity is known to be exponential, but for which the size of the rate of growth or decay also matters very much.

As time allows, there are other interesting questions that we might consider concerning the topics of quadratic relationships, irrational numbers, the intermediate value theorem, arithmetic sequences, and other topics which frequently appear in college algebra courses. The author was also a participant in the Cornell GoodQuestions for Calculus study, and would be more than happy to respond to questions regarding how these teaching techniques are extended to the calculus sequence.

To find out more information about the Cornell GoodQuestions for Calculus project, google search for GoodQuestions (no space). Currently the Cornell project is the first hit on this search. See [1] for more on this study.

References

[1] Maria S. Terrell, Robyn L. Miller, and Everilis Santana-Vega. Can good questions and peer discusion improve calculus instruction? *Primus*, 16(3):193–203, 2006.