BUNGEE JUMPING AND MURDER INVESTIGATIONS USING MAPLE AND ONCOURSE IN DIFFERENTIAL EQUTIONS

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Abstract

Real life projects using the course management system, Oncourse, and the computer algebra system, Maple, were introduced to improve students' understanding of free fall, exerting a force, Newton's law of cooling, and Laplace transformations. Here we explain how technology helped students to solve problems about bungee jumping and to figure out the time of death of a murder victim.

Introduction

An introductory differential equations course was taught to a group of seven students, mainly pre-service high school teachers. One of the class assignments was a group project where students selected project topics, suitable for group work, from our textbook [4]. After forming groups, they submitted a one page written plan for their project approximately four weeks after the beginning of the semester. In the plan, students explained what they are going to do and what each member of the group would contribute to the project. Students used the course management system Oncourse to share Maple files and drafts of the project among group members and with the instructor. Additional Maple files were developed and posted on Oncourse to help students' understanding of different concepts throughout the semester, and those of particular value for the students' projects. The first written draft of the projects was due eight weeks after the beginning of the semester for feedback. Over the course of the project students were encouraged to submit several more drafts and were given suggestions for improvement. In a final written project, due twelve weeks after the beginning of the semester, students analyzed, discussed, and wrote proofs about the selected topic. Group members gave a group presentation on the final day of class using Microsoft Power Point, Microsoft Word, and/or Maple during their oral presentations. The written and oral parts of the project were equally weighted in calculating their group project grade.

Bungee Jumping

A group of three students used equations of Newton's Second Law of Motion, Hook's law, and methods of solving non-homogenous linear equations in answering several questions related to bungee jumping. This was a project related to Chapters 4 and 5 in the textbook [4] on modeling with higher order differential equations. Students explored the forces acting on a bungee jumper as the weight of the jumper and elasticity of the cord changed.

The problem was to help a 160-pound, 6-foot tall bungee jumper choose a 100-foot long cord with an appropriate spring constant to avoid an unexpected water landing. They knew that the river was 174 feet below the bridge from which the jumper would step off. The students assumed that the position at the bottom of the cord is 0, the position of the jumper's feet is *x(t),* and that *x* increases in time as he/she goes down. They also had to have in mind that the air resistance increases proportionally to the jumper' speed and provides a force in the opposite direction to his/her motion of βv, where β is a damping constant and v is the velocity.

> p := piecewise (t>0 and t< 2.727, 800*exp(-t/5) – 900 + 160*t, t>2.727 and t<200, 6.626*exp(-t/10)*sin(1.67*t) + 53.72*exp(-t/10)*cos(1.67*t) + 160/14);

$$p := \begin{cases} 800\, e^{\left(-\frac{1}{5}t\right)} - 900 + 160\, t & -t < 0 \textbf{ and } t < 2.727 \\ 6.626\, e^{\left(-\frac{1}{10}t\right)} \sin(1.67\, t) + 53.72\, e^{\left(-\frac{1}{10}t\right)} \cos(1.67\, t) + \frac{80}{7} & -t < -2.727 \textbf{ and } t < 200 \end{cases}$$

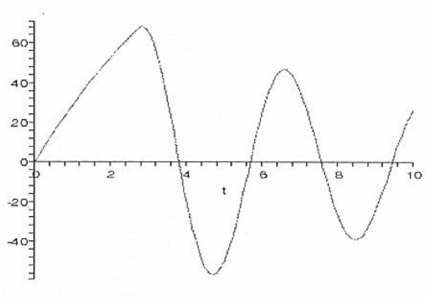

Figure 1: Model for the jump (k = 14)

First students modeled the distance the jumper falls before he/she reaches the natural length of the cord (free fall part) and then modeled the distance after the cord starts to pull. Throughout the problem they assumed that the damping constant β is 1, which for the free fall part leads to a non-homogeneous differential equation $x''(t) + \frac{1}{5}x'(t) - 32 = 0$ with the initial conditions $x(0) = -100$ and $x'(0) = 0$. The time of 2.7 seconds, at which the jumper passes the natural length of the bungee cord, was calculated from the solution of the initial value problem. This is the time at which the spring constant (stiffness of the cord), k, starts to influence the force of the cord pulling the jumper back. From the Hooks law students knew that the force of the cord pulling the jumper back is $\begin{cases} 0, & x \le 0 \\ -kx, & x > 0 \end{cases}$. They worked under an assumption that a 160 lb bungee jumper stretches the particular cord 11.4 feet, which results in a spring constant of approximately 14 lb/ft. The application of Newton's second law leads to a non-homogeneous differential equation of forced motion, $x''(t)+\frac{1}{5}x'(t)+\frac{14}{5}=32$ with the boundary conditions $x(2.7)=0$ and $x'(2.7)=67$. Students solved the boundary value problem and used Maple to define a piece-wise defined function that modeled the whole motion (Figure 1).

They concluded that the jumper is closest to the water after 3.8 seconds by finding out when the derivative is 0. At that time there are still 20 feet below the jumper to the water surface. Afterwards students explored such questions as what would happen if the jumper's 220-pound friend uses the same cord, or if the jumper uses a cord with a different spring constant.

Murder at the Mayfair

The second group of four students worked on a project involving the Laplace transform from the Chapter 7 of the textbook [4]. They used the model for Newton's law of cooling to find the time the "Mayfair Diner Murder" took place and the time when the body of an unfortunate diner's owner was moved from the kitchen of the diner to the refrigerator. This group of students mainly used Maple to check their solutions. They incorporated Maple worksheets into their final Power Point presentation. Their presentation included detailed steps of integration and calculations with partial fractions, needed for the application of inverse Laplace transform (Figure 2).

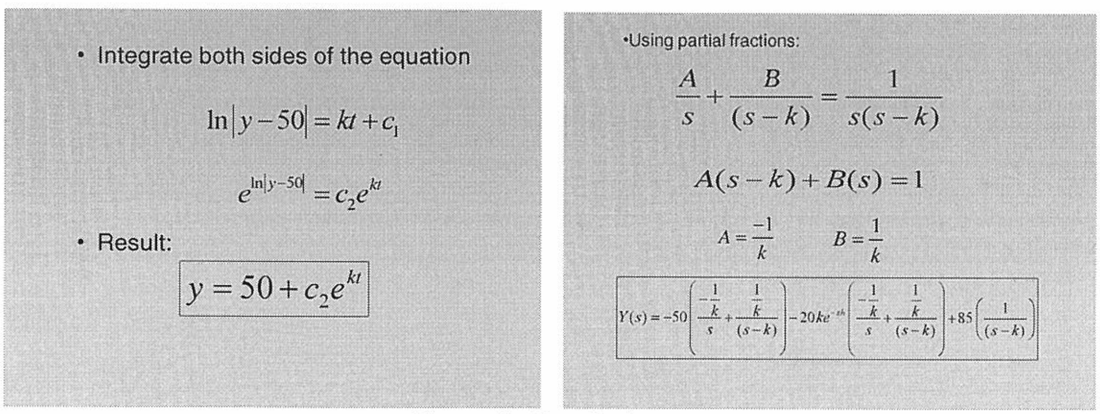

Figure 2: Detailed steps

The group presented the same project at the 2007 Spring Meeting of the Indiana Mathematical Association of America. All Power Point slides shown here are from the students' presentation [3].

In Figure 3, the group's introduction of the problem with information about the three suspects is shown. Modeling with Newton's law of cooling, working under an assumption that the body of the diner's owner was in the refrigerater since death lead to the conclusion that the approximate time of death was 12:20 am. Due to the fact that the time of death did not match the information about any of the suspects, an assumption was made that the body was possibly moved from the kitchen. In addition to the time the body has been dead, the group had to introduce another variable h to represent the time the body has been in the refrigerator. This resulted in need to introduce a unit step function into the body cooling model, to account for the difference in temperatures between the kitchen and the refrigerator. Solving such a differential equation is simplified with the use of a Laplace transform. Application of Laplace transform to the differential equation resulted in another equation. They solved the last equation for the transform of the unknown time, $Y(s)$. The method of partial fractions was used to change the form of $Y(s)$ for easier application of the inverse Laplace transform. Applying the inverse transform lead to a temperature model that included two variables for time, t and h, as well as the cooling constant of proportionality, k. The only way to find the time of death, and the time the body was moved, was to create a table of values for h, the time the body has been in the refrigerator and find the corresponding time of death in each case. The analysis of this table, for values of h from 12 to 0 lead to a conclusion that the cook, Shorty, was the only one that must be investigated further. The cook could have killed the owner in the kitchen during his unusually long break at 10:30 pm and moved the body to the refrigerator just before he left the diner at 2:00 am.

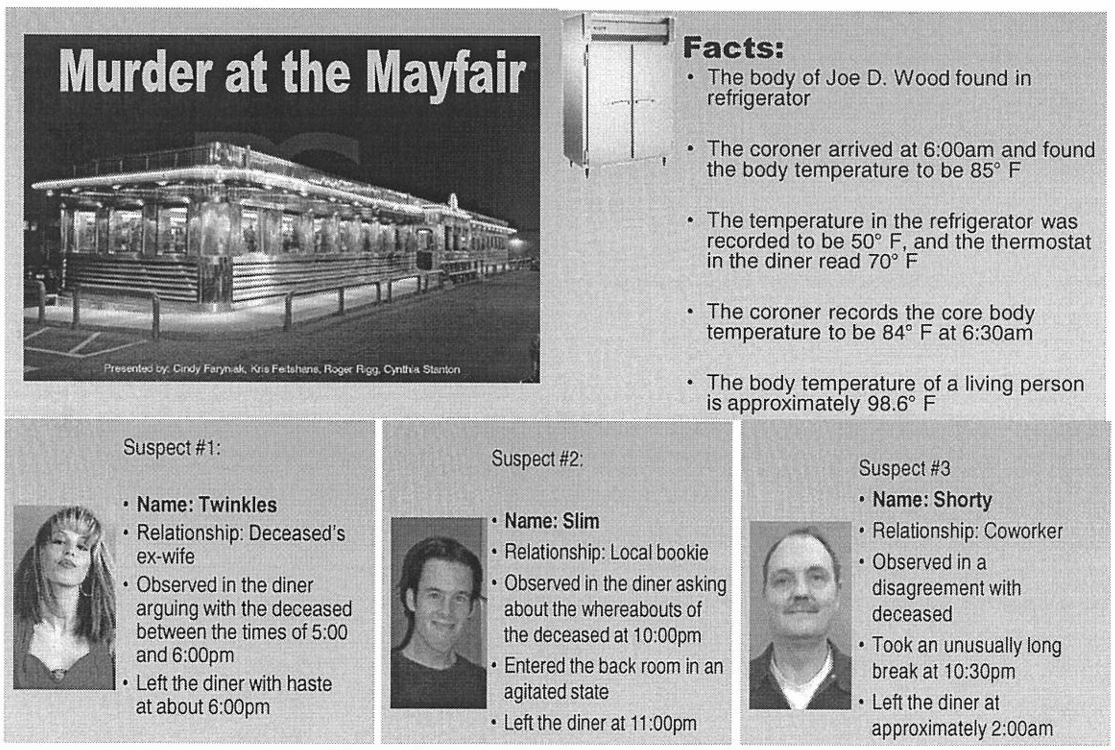

Figure 3: Introduction of the problem

Conclusion

The value of involving students in written and oral projects is enormous. It is supported by research, by national and state standards for pre-service teachers, as well as by the Mathematical Association of America. The camaraderie of working together, presentations in front of peers and at the regional professional conference, use of Maple and Oncourse will never be forgotten. These activities will help the students become better professionals and communicators. The students enjoyed researching their topics and using Maple to figure out different applications. The small class size enabled the instructor to work closely with students and give feedback at several stages of projects.

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