RADIOACTIVE DECAY: DATA COLLECTION AND ANALYSIS OF A REAL POISSON PROCESS

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Introduction: The clicking of a Geiger counter measuring radioactive decay (on a time scale short compared to the half life of the radioactive source) is the standard example of a Poisson process. Good (albeit more extensive than usual) examples of a typical treatment can be seen in the texts [1,2]. radioactive decay is also commonly alluded to as a source of genuine random numbers: the website HotBits [3] uses radioactive decay to provide "Genuine random numbers, generated by radioactive decay." However, students rarely hear or see (much less collect) a real data set from a Poisson Process. This paper demonstrates a simple experiment to record, extract, and analyze such data using an inexpensive ($15-$30 on Ebay) dosimeter, household radioactive sources, and standard PC software.

The mathematical definition of a Poisson Process is as follows. Write $X(t)$ for the number of events that have occurred up to time t and $p_{i,j}(s, t)$ for the probability that $X(t) = j$ given that $X(s) = i$. Note that $p_{i,j}(s, t)$ is the probability that the number of events (or in our case clicks) is $i - j$ in $(s, t]$. The two defining conditions for a Poisson Process from [1] are.

1. Events occurring in non-overlapping intervals of time are independent of each other.
2. For a sufficiently small Δt, there is a constant λ such that the probabilities of occurrence of events in the interval $(t, t + \Delta t]$ are given as follows
   a. $p_{i,i}(t, t + \Delta t) = 1 - \lambda \Delta t + o(\Delta t)$
   b. $p_{i,i+1}(t, t + \Delta t) = \lambda \Delta t + o(\Delta t)$
   c. $\sum_{j=i+2}^{\infty} p_{i,j}(t, t + \Delta t) = o(\Delta t)$
   d. $p_{i,j}(t, t + \Delta t) = 0$ for $j < i$

The general physical meaning of these conditions are:

1. Events in non-overlapping intervals are independent if on the large scale each individual event is insignificant and does not trigger (or suppress) other events.
2. The constant rate λ satisfying the conditions exists if events do not cluster or avoid each other and the rate at which individual events occurs is constant. In which case, for small Δt the following interpretations hold:
   a. The probability no event occurs in $(t, t + \Delta t] \approx 1 - \lambda \Delta t$.
   b. The probability a single event occurs in $(t, t + \Delta t] \approx \lambda \Delta t$.
   c. The probability of multiple events in short time intervals is negligible.
   d. The count can not decrease.

The Geiger counter is the most common introductory example because it is familiar (from movies and Physics courses) and the mathematical characteristics of a Poisson Process can be relatively easily and convincingly interpreted in the physical setting of slow radioactive decay. Natural radioisotope decay satisfies the defining conditions for a Poisson Process (to extremely high accuracy) because there are a very, very large number of identical unstable atoms in even a very small radioactive source. The decay of any individual atom is independent of the fate of any other atom and moreover natural radio-isotopes have very long half-lives. For example, one microgram of ThoriumOxide from an old gas mantle has over 10^{16} identical unstable Thorium atoms each with a half life of 1.4×10^{10} years.

The experiment described is accessible, inexpensive, and safe. A Russian military surplus geiger counter (dosimeter) can usually be purchased on Ebay for between $15 and $30. There are a variety of radioactive sources available . Background levels will typically produce a rather sparse recording but older gas lantern mantles (which contain radioactive Thorium), vintage Vaseline Glass (which contains Uranium), or granite boulders are readily available and will produce a significantly elevated reading. An audio recording is made of the clicks using a standard microphone and a PC (which brings up the interesting question "What does random sound like?"), the events (or click times) are easily extracted using a freeware audio package Audacity [4] to give a data file of event times, this extensive (thousands of clicks can be easily obtained) data file is imported into Mathematica and analyzed.

The obvious question is how good are our random numbers from a $15 detector? The properties of the collected time series of clicks is analyzed to see how well it conforms to the assumptions of a Poisson Process. This analysis and the discussion serves to illustrate the meaning (physical and mathematical) of the defining characteristics of a Poisson Process. As expected the data collected is clearly a good fit to a Poisson Process with the only obvious discrepancy being a deficit of extremely short inter-arrival times. Finally, the physical reason (a comparatively long dead time for the software and inexpensive detector) for this anomaly is explained.

Data Collection: Data collection is simple. Place the detector and microphone near the source as shown in Figure 1.

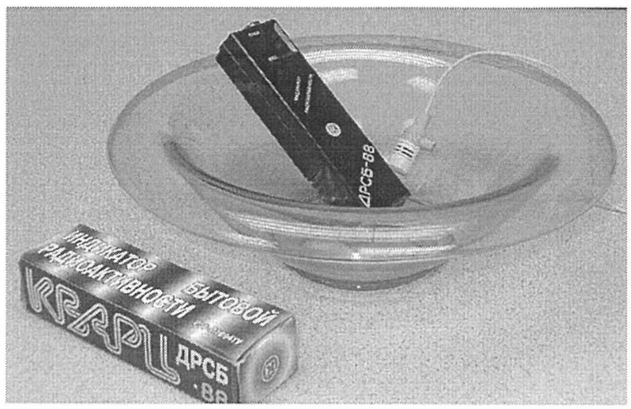

Figure 1: Sound Recording

Record the clicks (a screen shot of the Audacity [4] window with an open recording of some clicks is shown in Figure 2) the clicks (with minimal background noise) for a few minutes, and save the file in any standard (for instance .mpg or .wav) audio format. The result should be a clear recording of the Geiger counter clicks.

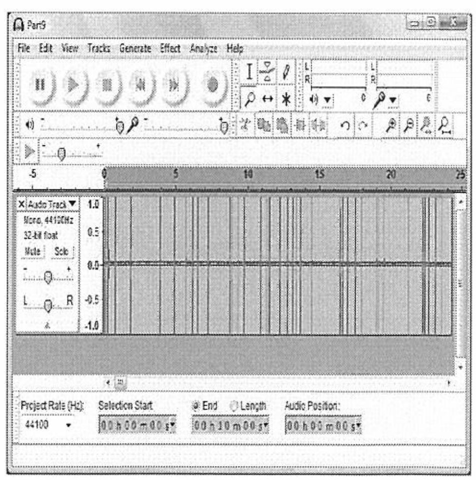

Figure 2: Audacity Recording

Event Time Extraction: Extracting the event times is simple with Audacity's built-in silence detector. Open the recording and select (under edit there is a select all option) the entire track then select silence finder from the analyze menu. If you zoom in your window should be similar to the screenshot in Figure 3. Set the place label slider to zero seconds, set the minimum silence duration down to 0.01seconds (you may need to type in the box rather than use the slider to set it this low), and experiment with the silence level setting (as shown in Figure 3 I used 17db but it depends on the recording) to capture all the visible clicks.

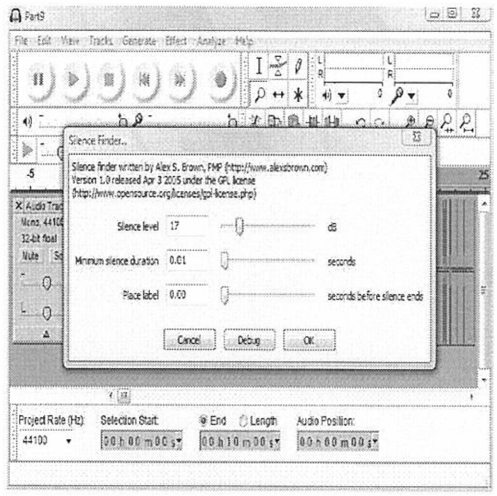

Figure 3: Audacity Silence Finder

After scanning through the entire recording to verify the accuracy of your silence flags save the labels using the export labels command under the file menu. Your window should look like Figure 4 with a visibly accurate labels track directly underneath the visual representation of the audio track. The saved label file is simply a text file of the times marked by the flags. This file can be readily imported into any data analysis program.

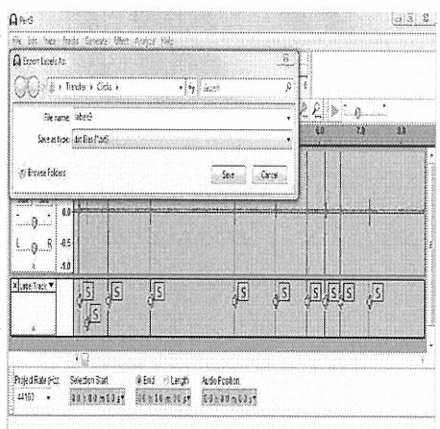

Figure 4: Saving Labels from Audacity

Data Analysis: Arrival Rate Conditions: The data file of times was imported into Mathematica and analyzed. A plot (shown in Figure 5) of the first 10 minutes of the data shows roughly 1100 clicks and (since it is visibly straight) that the rate of clicks is approximately constant with a linear fit giving the estimate $\lambda = 0.520757 \pm 0.0007$ clicks per second as a 95% confidence interval for the arrival rate. The extremely small error estimate reflects accuracy (the R^2 value is 0.999454) of the linear fit to the data.

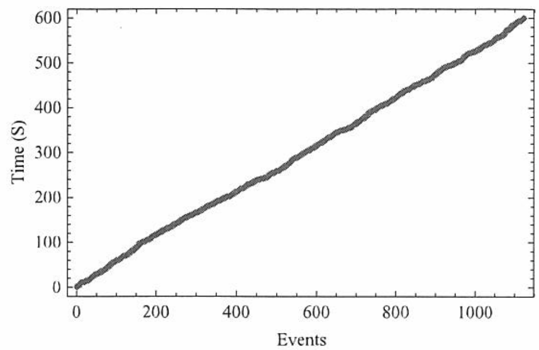

Figure 5: Event Times vs Count for 1st 10 minutes

The estimate $\lambda = 0.520757$ is used to generate Figure 7 of comparing the observed frequencies (in samples of 500 intervals of various lengths $0.01 \leq \Delta t \leq 1$) to the probability prediction $1 - \lambda \Delta t$ specified in conditions 2a which is the black line on the graph. It is clear that for small Δt the sampled zero counts (the bottom category in the stacked bar chart) matches well with the predicted line. It is also clear that the observed single counts (the second category in the barchart) fill in the gap: this means that the single counts match well with the predicted $\lambda \Delta t$ while the multiple counts (the higher categories in the barchart) disappear as Δt gets small. We can see that the measured data matches well with the assumptions 2a, 2b, 2c, and 2d.

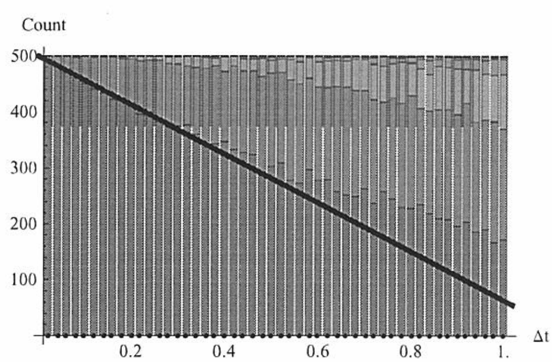

Figure 6:Observed Number of Events in a sample of 500 time intervals

Data Analysis: Interarrival Time Distribution: The independence condition and the conditions on the arrival rate imply that the interarrival times $t_{i+1} - t_i$ (where t_i is the time of the ith click) must have an exponential distribution with mean $1/\lambda$. Figure 7 is a histogram of the recorded interarrival times counted into bins of length 0.1 seconds with a superimposed exponential distribution $\frac{1}{\lambda} e^{-\Delta t/\lambda}$ with a count in the first bin of 3267 except for the significant deficit (indicated by the light rectangle) of short inter-arrival times in that first bin. This deficit can be easily explained by the data extraction technique used with a minimum silence length of 0.01 seconds: as a result many short gaps are missed. In fact, all Geiger detectors have a dead time during which they are recharging a capacitor and can not measure another event.

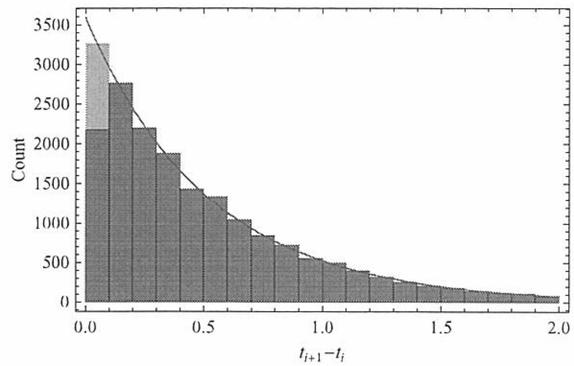

Figure 7:Histogram of recorded Interarrival times

Conclusions: The primary conclusion is that the radioactive decay process recorded is indeed well approximated by a Poisson Process. The only discrepancy is a deficit of short inter-arrival times which is easily explained physically. The benefits of using easily accessible real data are numerous: it motivates students; provides an endless supply of custom assignments; prompts students to consider extensions; connects seemingly disparate courses; and convinces students of the applicability of the mathematics they are learning.

References

1. U. N. Bhatt and G. K. Miller, Elements of Applied Stochastic Processes (3rd edition), Wiley, New Jersey, 2002.
2. R. M. Gray and L. D. Davisson, Random Processes: A Mathematical Approach for Engineers, Prentice-Hall, New Jersey, 1986.
3. HotBits, "http://www.fourmilab.ch/hotbits/", retrieved September 5th 2007.
4. Audacity, "http://audacity.sourceforge.net/", retrieved September 19th 2007.