DISCUSSION ACTIVITIES IN AN ONLINE ABSTRACT ALGEBRA COURSE

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Abstract

In this paper we present several discussion activities designed for an online abstract algebra course. The activities are designed to engage student in material beyond the textbook and to build a sense of community among the students.

Introduction

Discussion is key to the success of any class, even those that are taught in the online environment. However, in the online environment discussions do not happen as naturally as they do in the face-to-face classroom; the instructor must work to design activities that foster interaction among the students [1]. This paper explores several discussion activities that the author and her colleague, Rick Klima, have designed for an online abstract algebra course. The topics are specific to algebra, but the structure could be adapted to any upper level online course.

In order to understand the discussion that we have used in our course, the reader must first know a little about the structure of the course itself. We designed the course using Joseph Gallian's textbook *Contemporary Abstract Algebra* [4]. The course contains twelve content modules, each approximately one week in length. Each module contains approximately three lessons. Every lesson contains a small traditional homework assignment and one of the lessons also contains a discussion activity. In addition to the twelve content modules the course contains three review modules. Like the content modules, the review modules are approximately one week in length and contain a discussion activity. We designed the discussion activities to encourage interaction among the students in hopes that once the students become comfortable with each other in the online environment they will begin to discuss other aspects of the course, for example their traditional homework assignments, and thus build a true learning community.

The content modules contain a variety of discussion activities. Some activities ask students to look at extensions and applications of traditional abstract algebra concepts, others require students to read and comment on non-technical journal articles about topics in abstract algebra, and still others ask students to complete activities that are designed to explore new topics in abstract algebra. The final discussion activity encourages students to explore some of the history of abstract algebra. In addition, each review module contains a true/false discussion activity. The remainder of this paper describes one activity in each of the previously mentioned categories.

Extension and Application Activities

The majority of the discussion activities in the course fall into the category of extension and application activities. As an example, we will consider the discussion activity presented during the second content module in the course, which pertains to the dihedral groups. The discussion activity for this module involves extending the textbook (*Contemporary Abstract Algebra* by Joseph Gallian [4]) discussion of finite symmetry groups. The first step of this activity asks students to search the web and find an example of a corporate logo that has dihedral symmetry and an example of a corporate logo that has cyclic symmetry. Each student creates a post to a specified discussion board that includes a link to his or her chosen logos and a description of the logos themselves. The description should not include the type of symmetry present. The second step of the activity asks the students to reply to someone's previous posting letting the author know which of his or her graphics has cyclic symmetry and which has dihedral symmetry. Finally, the author of the original post should reply to the student who responded in step two and let him or her know if the author agrees or disagrees with his or her opinion. By the end of the discussion thread the author and the responding student should agree on the types of symmetry present in the corporate logos.

Other extension and application activities in the course occur during the module discussing permutation groups, the module discussing normal and factor groups, the module reviewing modular arithmetic, the module discussing cyclic groups, and the module introducing the concept of rings. During the permutation group module, the students are asked to explore the idea of the perfect card shuffle. As part of the normal subgroups and factor groups module the students are asked to explore connections between group theory and music theory. In the modular arithmetic module students are asked to look at the UPC codes on items around their homes and explore the idea of check digits. The discussion activities in the cyclic groups and rings modules explore shift and affine ciphers, respectively.

Reading Activities

The first reading discussion activity in the course occurs in the module concerning the introduction to the concept of a group. We ask the students to read Arie Bialostocki's article "An Application of Elementary Group Theory to Central Solitaire" that can be found in *The College Mathematics Journal* [2]. This article discusses the use of the Klein Four group to determine the possible final pin positions in a game of central solitaire. The first step of this activity asks students to read the article and create a summary and critique of the article. We severely limit the length of the summary in order to force the students to choose what they consider to be the most important aspects of the article to summarize. In addition to a purposely brief summary, we ask the students to answer the following questions concerning the article. Did you learn anything? What parts of the article could be written or presented differently to make the article better? In the second step of the activity students are to respond to someone's posting that differed in some way from their own. The response should politely point out the differences and discuss why the responding student felt differently about the article. In the third step the original author should respond and continue discussing the differing opinions.

The course contains one additional reading activity which occurs during the module concerning homomorphisms and isomorphisms, and asks the students to discuss Michael Bernnan and Des MacHale's article "Variations on a Theme: A_4 Definitely Has No Subgroups of Order Six!" which can be found in *Mathematics Magazine* [3]. In this article the author presents eleven proofs that A_4 has no subgroup of order six, using many of the topics that the students have studied in this and previous modules. We ask the students to read the proofs and discuss which ones they like, which they do not, and why. This article as well as the one mentioned in the previous paragraph are both listed in "Suggested Readings" sections of Joseph Gallian's *Contemporary Algebra* [4] textbook. The reader can find many other interesting articles written at a level that is accessible to beginning algebra students in Gallian's textbook.

Discovery Activities

Students first encounter a discovery activity during the module concerning subgroups of groups. This module introduces the idea of a cyclic subgroup, and the activity is designed to help students explore the cyclic subgroups (and thus all of the subgroups) of $\mathbb{Z}_n$ for various n. The discovery activities allow students to experience the joy of uncovering mathematical relationships for themselves, rather than simply being told about these relationships. During the first step of the activity students learn that they should let $n = b \mod 5 + 8$ where b is the day of the month that they were born. Thus, most of the integers n between eight and twelve will be represented. The students then find the cyclic subgroup of $\mathbb{Z}_n$ generated by a for each a in $\mathbb{Z}_n$ (recall each student has already chosen a specific n) and post their solutions to the discussion board. The second step of the activity asks the students to review all of the postings and check that all students with the same value for n have posted the same cyclic subgroups. During the third step of the activity students are asked to make conjectures and send these conjectures to the instructor via private message. Each student is to use the data provided by all of the students in order to answer the following questions. What type of relationships do you observe between the order of a cyclic subgroup of $\mathbb{Z}_n$ and the order of $\mathbb{Z}_n$? Suppose a is a generator of $\mathbb{Z}_n$, what relationship have you observed between a and n? In the next module, we use the students' conjectures as a starting point for a discussion of the fundamental theorem of cyclic groups.

The course contains one additional discovery activity. During the module concerning cossets and Lagrange's theorem, the students explore operations on the left cosets of a group. We ask the students to look at some Cayley tables and conjecture when the left cosets along with the given operation form a group and when they do not. We then use this discussion to begin the module concerning factor groups.

History Activity

The final content module of our course is the longest of all the modules, lasting ten days. The discussion activity for this module is more involved than the previous discussion activities. The first step in this activity asks students to choose an algebraist whose work interests them and post the algebraist's name to the discussion board. Joseph Gallian's testbook [4] contains many short biographies of algebraists and this list should help the students easily choose an interesting algebraist whose work the student would like to explore further. In the second step of the activity each student posts a 600-word report about his or her algebraist and the work of that algebraist. For the third step, each student must send the instructor (via private message) five multiple choice questions whose answers can be found in the reports of his or her classmates. No more than two of these questions can be about a single algebraist. For the final step of the activity each student takes a quiz that we compile from the multiple choice questions received in step four.

True/False Activities

After each set of four content modules, the students participate in a review module that requires them to reflect upon the content of the previous four modules. Every review module has the same type of discussion activity, a true/false activity. The activities begin with a list of fifteen to twenty true/false questions that pertain to the previous four modules. We encourage the students to use the discussion boards to converse concerning the correct answers to these questions. After the discussion we ask the students to answer the true/false questions via private message and remind them that they should all have the same answers because they should have previously discussed the questions until the class as a whole had reached a consensus.

Conclusions

Discussion activities are important to an online classroom because they help to build a sense of community. It is our hope that the discussion activities that we have presented here will do just that. However, we have not designed these activities with only this goal in mind. Our activities add variety to our course and in this way help to hold the students interest. Most importantly, we have designed the activities to challenge the students to use the knowledge that they collect in the modules in a non-traditional sense, helping the students to develop a deeper understand of the topics covered in our abstract algebra course.

References

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