DYNAMIC CALCULUS TOOLS FOR VISUALIZING CALCULUS

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In this presentation, I demonstrate a series of Java applets and Excel explorations I created to help my students visualize Calculus. Over the past several years, I have developed a series of dynamic calculus applets to support both the **Anton** and **Salas** Calculus texts for John Wiley & Sons, a second series of applets to support Addison-Wesley's **Thomas *Calculus*** text, and a third series of 44 applets to support Prentice Hall's **Varberg *Calculus*** text. A fairly complete list of the applets I have created for these texts is available in a second handout (a handout from a previous presentation I have given which also includes a guided tour of my first applet, a Calculus Grapher on my webpage). See this handout online at:

http://web.monroecc.edu/manila/webfiles/pseeburger/CalcGrapherGuidedTourPublish.pdf

Some applets I will show here support these texts, but several of my best applets are found on my webpage and you can link to many others from there. Although I will not have space to describe it here, I have also created an application to explore multivariable Calculus with my students in third semester Calculus. I have received an NSF grant to convert this multivariable calculus exploration application to a web-based Java applet. This project seeks to help students develop a better geometric understanding of multivariable calculus concepts.

Calculus is the mathematics of change. The definitions of each of the basic concepts of Calculus, the limit, the derivative, and the integral are all dynamic. So, if we really want to understand the basic definitions and concepts of Calculus, we need to see them in motion… to see the **change** in a dynamic and living way. Seeing a few static diagrams can be helpful (I believe it is still very helpful to start our definitions on the chalk board in many cases), but if the diagrams come to life and we are allowed to manipulate them (and have fun with them!), our understanding of the relationships involved can improve dramatically. From what my students have told me, they find this approach to be interesting, very helpful, and many times even fun! As we explore Calculus using the explorations I am creating, I hope my students may also begin to appreciate some of the intricate and fascinating beauty of the mathematics of Calculus.

In addition to simply watching **me** interact with the concepts dynamically, I also believe my students benefit greatly when **they** interact dynamically with these concepts **themselves**. I have designed these computer explorations to be used in both of these ways, as classroom demonstrations and also as student explorations outside of class. From my experience, in order for most students to make active use of this kind of

exploration, it is very helpful to require them to complete guided exploration activities that I call "labs" outside class. I have created a number of these lab activities so far, and I will continue to create them, to help my students to get their hands "dirty" playing with the Calculus concepts visually. One of my major goals for this project is to develop my students' geometric intuition about the concepts we cover in calculus, helping them to understand these concepts more deeply and make connections between them visually in a way that has been difficult without such software.

Below are screenshots from a sampling of the Java applets I will demonstrate (as time allows). For each applet, I have included a description and how you can locate it online. My webpage is: **www.monroecc.edu/wusers/pseeburger/**

1. This applet helps students to see a clear connection between the graph of a position function and the associated motion of a race car along a straight track. The connections between the graph of the position function and the corresponding velocity and acceleration functions can also be explored with this applet.
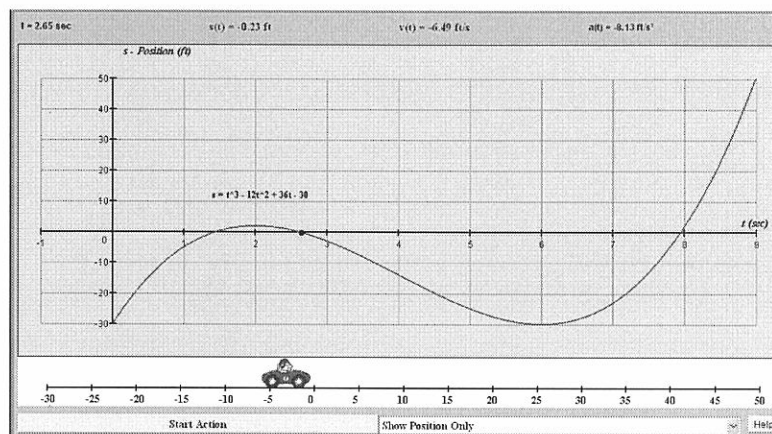

Figure 1: Horizontal Motion (Varberg, in MyMathLab)

2. Using this applet, I can quickly demonstrate the various kinds of Riemann sum approximations used in my course. I especially like the way I can show my students the Simpson Rule (Parabolic Estimate) using this applet.
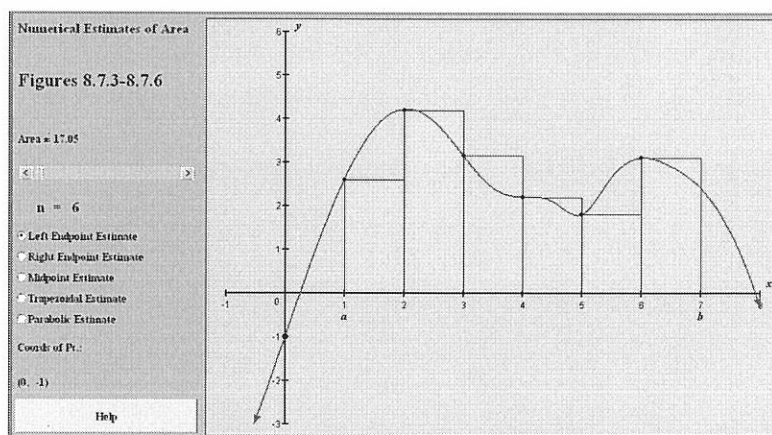
Figure 2: Riemann Sum Approximations
(**Anton** — Can link to it from my webpage)

3. As you can guess, this applet helps students to visualize the disk/washer method. It allows you to see the representative rectangle revolved about the x-axis and then you can vary the position of this representative washer. You can also show the region revolving about the x-axis. Finally you can increase the number of disks (up to 20) and even show how these revolve about the x-axis.
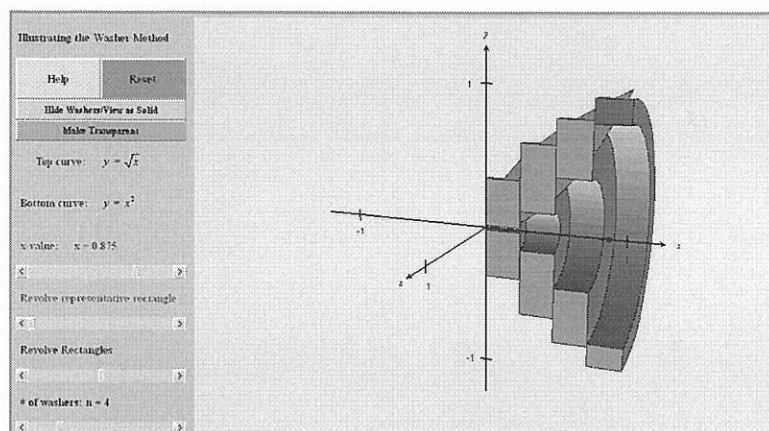
Figure 3: Volumes of Revolution–The Washer Method
(On my webpage)

4. This applet is very similar to the applet above, except that it explores the shell method.
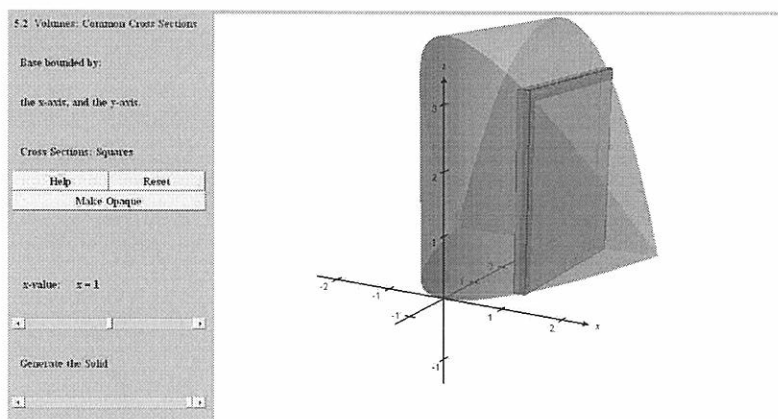
(On my webpage)
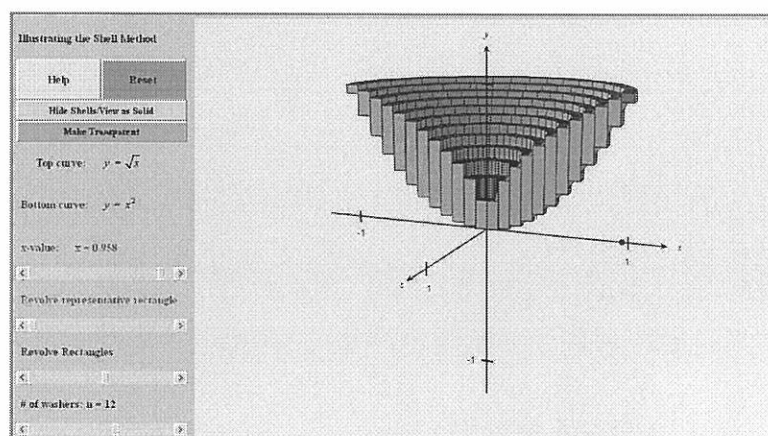
Figure 4: Volumes of Revolution – The Shell Method

5. This applet allows the user to enter the boundaries of the base using functions of x or y, and then allows the user to select the shape of the cross-sections.

(On my webpage)

Figure 5: Volumes with Common Cross-Sections

6. This Slope Field applet allows you to enter any first order differential equation that can be solved for dy/dx. It will then display a slope field for this diff. eq. and allow you to enter a possible general (or particular), solution solved for y. If you include a C with the general solution, you can then vary the value of this constant to verify that all particular solutions actually fit through the slope field correctly.
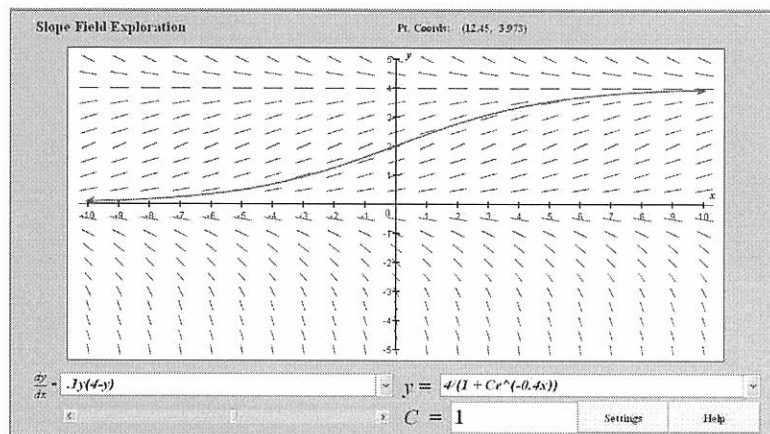

Figure 6: Slope Fields (On my webpage)

7. With this Contour Plot applet, you can show the relationship between the surface and its contours. I plan to create a general contour plotting applet sometime soon which will allow you to enter practically any function of two variables.

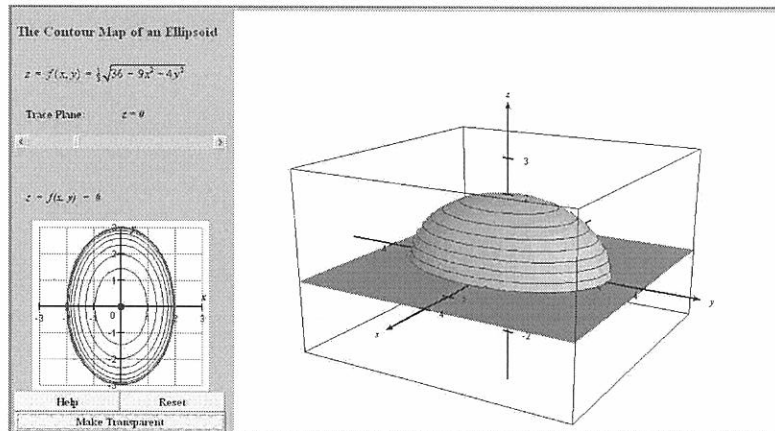(**Varberg**, but you can link to a similar applet for Anton from my webpage.)

Figure 7: Contour Plot Example

8. The concept of level surfaces is a fairly difficult topic to visualize well without software. This applet allows students to see three level surfaces for a particular function of three variables simultaneously. These can be rotated and is you use the scroll bar at the bottom, you can see the level surfaces vary by value.
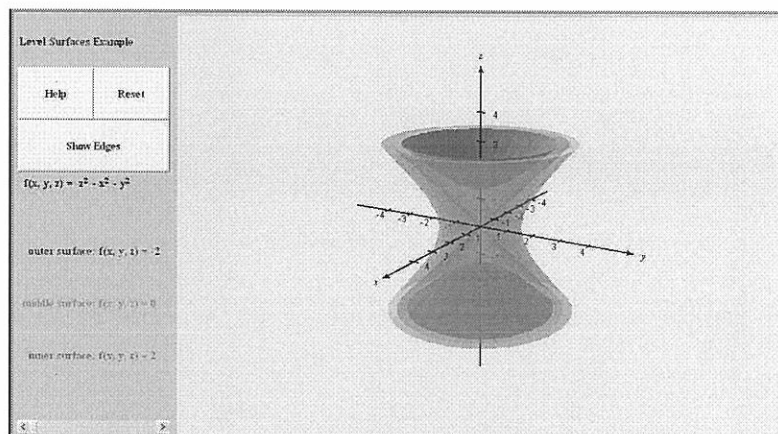

Figure 8: Level Surfaces Example
(**Anton** – link from my webpage)

9. This applet helps illustrate a classic function of two variables whose limit does not exist at the origin.
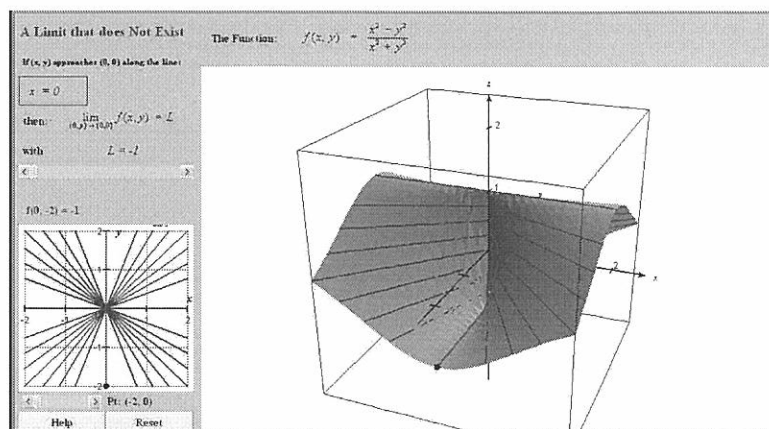
(Varberg, in MyMathLab)

Figure 9: A Limit that does not Exist

10. The applet is shown in the two screen shots to the right allows you to graph most functions of two variables and rotate them to get a better perspective. It also allows you to see a tangent plane or a trace point on the surface.

It also allows you to visualize the Taylor Polynomials of a function of two variables. Depending on the complexity of the partial derivatives involved, it seeks to display all Taylor Polynomials up to the 15th degree.

This applet forms the foundation for the NSF-funded Multivariable Exploration Environment applet that I am presently developing.
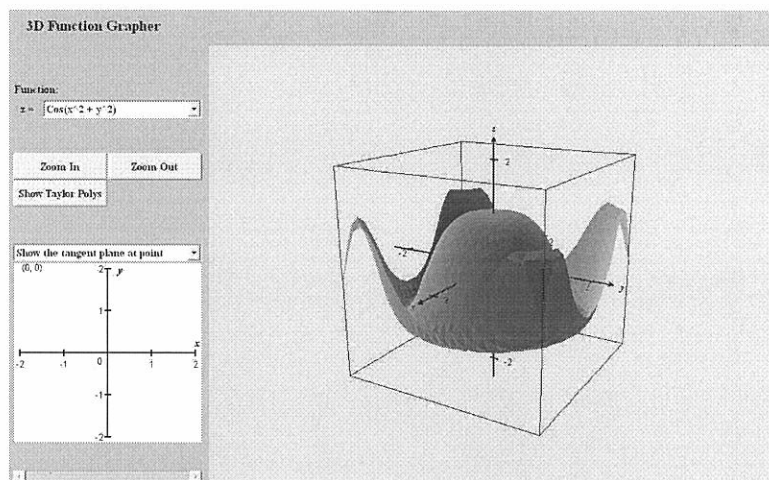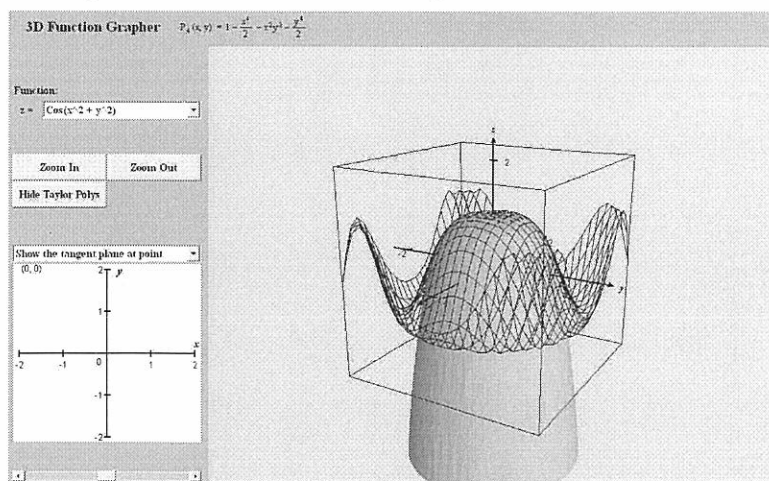
(on my webpage)

Figure 10: 3D Function Grapher

Figure 11: Taylor Polynomials in 3D

11. This applet allows you to see how the method of Lagrange Multipliers works from a graphical perspective. The relative minima and maxima occur where the constraint curve is tangent to the level curves of the surface.

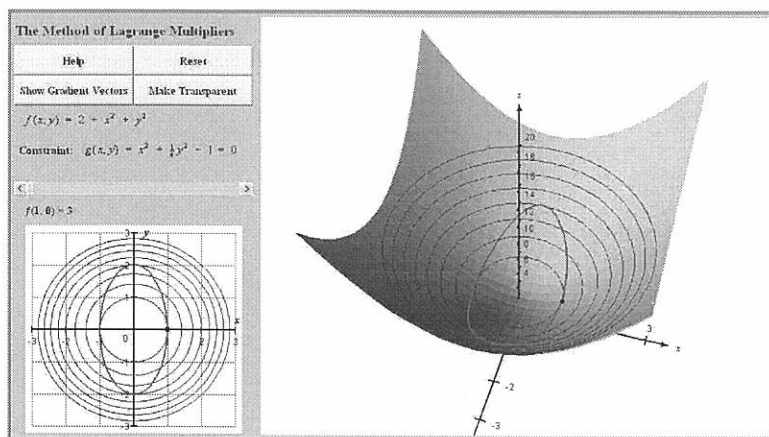It is useful to see how the constraint curve is projected up onto the surface as well.

(**Varberg**, but you can link to a similar applet for Anton from my webpage.)

Figure 12: Lagrange Multiplier Example

12. Using this applet, I can demonstrate how a double integral can be approximated using the volumes of rectangular prisms. It is fun to be able to vary the number of prisms in each direction and watch how refining the partition can cause it to fit the surface better, just as we did above for the two-dimensional case.

(**Varberg**, in MyMathLab)
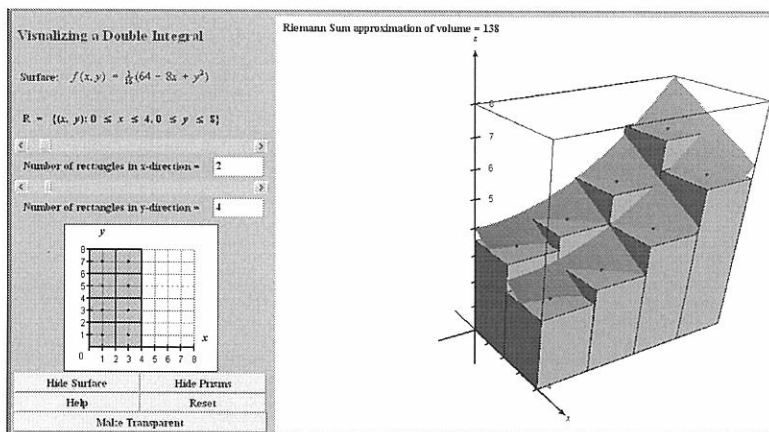
Figure 13: Visualizing a Double Integral

<u>**My Latest Invention**</u> - **A Derivative Graphing Applet** which scores students and can be used as an assignment It is located on the Dynamic Calculus part of my website: **web.monroecc.edu/pseeburger/**

Students always have some difficulty graphing a function's derivative graph by simply looking at the function's graph. For many years I have wanted to come up with a way to get students more hands-on practice with this difficult type of exercise. Finally I have been able to put the necessary pieces together to form an applet that allows students to practice drawing the graph of a function's derivative based on randomly generated functions of various types. It also provides a score that can be printed out (along with the

graph showing the student's attempted graph, the original function's graph, and the score card showing where the student had trouble, if any). Various levels are provided to allow practice and evaluation with progressively fewer tools available to assist students in getting the derivative graph nailed down. On the first level, students are able to reveal the actual derivative graph to compare with their own attempt. They can also display a tangent line to the curve and move it along the curve viewing the corresponding slope information to help locate points on the derivative graph. On level 2, they can no longer see the actual derivative graph, but they are still able to use the tangent line. On level 3, neither of these tools is available and the score also is not constantly displayed. Students can still view their score card using a menu option. There is still a lot I want to do to expand the capabilities of this applet, but it already provides a pretty fun way to practice this challenging topic.
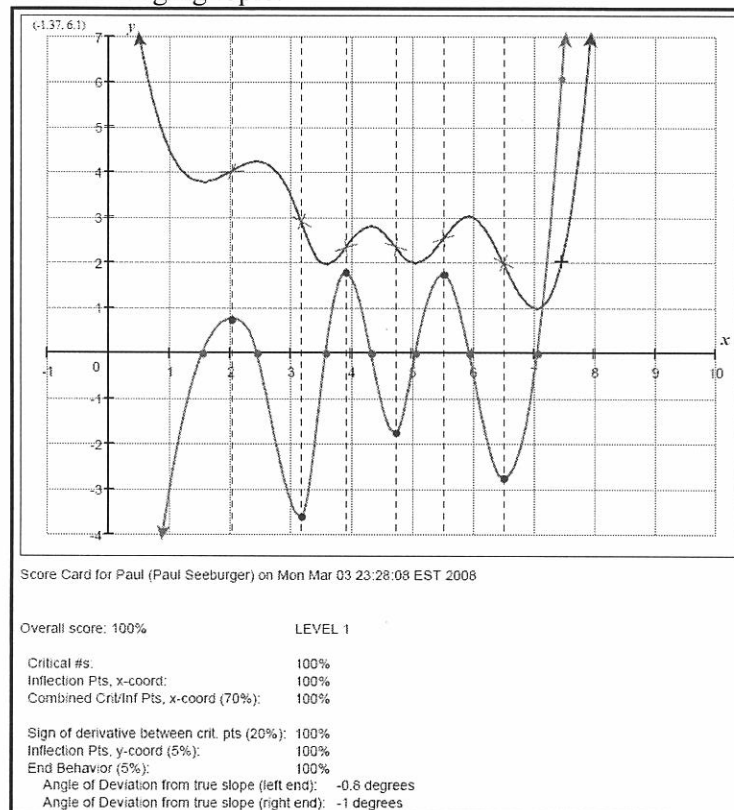
Figure 14: Sample Score Card from this applet

My next goal is to modify this applet to create a new one that allows students to use the graph of a derivative to estimate the graph of its original function, given an intial point for it to pass through. Students will locate relative extrema and inflection points, etc. In one mode, for use after the Fundamental Theorem of Calculus, students will need to be more precise, using the area under a piece-wise linear graph to give exact points (ordered pairs) at integer values of x.
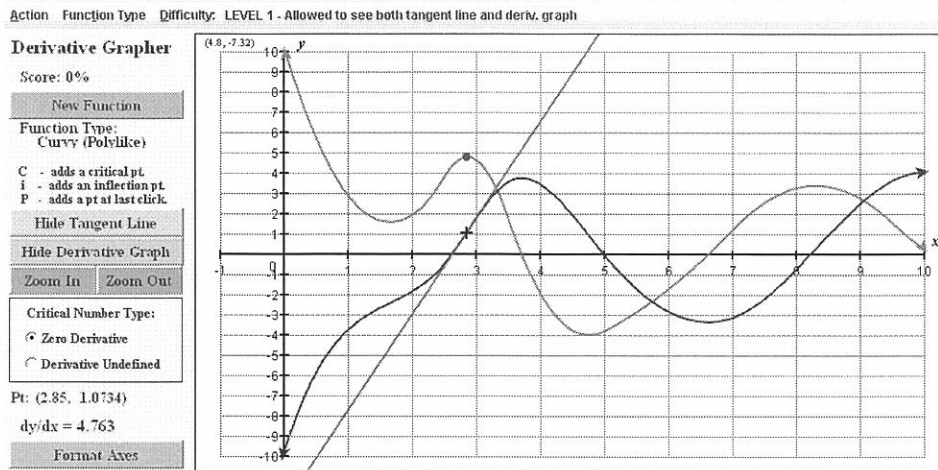
Derivative Grapher

Score: 0%

New Function

Function Type:
Curvy (Polylike)

C - adds a critical pt.
i - adds an inflection pt.
P - adds a pt at last click.

Hide Tangent Line

Hide Derivative Graph

Zoom In   Zoom Out

Critical Number Type:
⊙ Zero Derivative
◯ Derivative Undefined

Pt: (2.85, 1.0734)

dy/dx = 4.763

Format Axes

Figure 15: Derivative Grapher applet in Level 1

A Calculus Function Grapher

This is the first applet I created and I have recently done a lot of work to improve it. It allows you to demonstrate tangent lines and secant lines approaching tangent lines and the relationship between a function's graph and its derivative graph. You can also demonstrate area functions, parametric functions, polar functions, functions of the form x = f(y), and piece-wise functions. I have recently modified it to show vertical asymptotes and even jump discontinuities in the function. There are still a few places it has difficulty, but I am constantly making improvements.
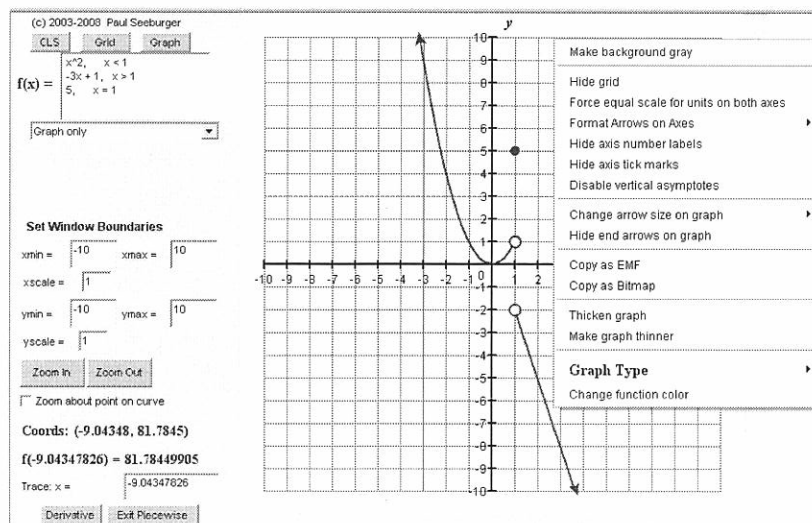
(c) 2003-2008 Paul Seeburger

CLS   Grid   Graph

$$f(x) = \begin{cases} x^2, & x < 1 \\ -3x + 1, & x > 1 \\ 5, & x = 1 \end{cases}$$

Graph only

Set Window Boundaries

xmin = -10   xmax = 10

xscale = 1

ymin = -10   ymax = 10

yscale = 1

Zoom In   Zoom Out

☐ Zoom about point on curve

Coords: (-9.04348, 81.7845)

f(-9.04347826) = 81.78449905

Trace: x = -9.04347826

Derivative   Exit Piecewise

Make background gray

Hide grid
Force equal scale for units on both axes
Format Arrows on Axes   ▶
Hide axis number labels
Hide axis tick marks
Disable vertical asymptotes

Change arrow size on graph   ▶
Hide end arrows on graph

Copy as EMF
Copy as Bitmap

Thicken graph
Make graph thinner

Graph Type   ▶
Change function color

Figure 16: Function Grapher applet – piecewise function

The other really nice thing about this grapher is that it will allow you to copy and paste graphs into WORD documents for quizzes, tests, and worksheets. You have to pick the one with a certificate to allow the copy-and-paste option though.

There is a graph formatting menu that you get when you right-click on the graph that allows many options to format the graph. To resize the graph while keeping the numbers the same size, you can resize the browser window and the graph will shrink or grow to adjust to the available space. This is helpful for optimizing the size and readability of the graphs for copying into documents.

See my other handout for a short tour of the calculus options in this applet. It doesn't say anything about the copy-and-paste option, since that is a recent addition.

Some Excel Calculus Explorations – A series of these Excel Calculus Explorations can be downloaded from my webpage for use in your classes. I have sought to update them for the new Excel 2007, but some still work best (most smoothly) on Excel 2003 or earlier.

A direct link to this part of my website is: **web.monroecc.edu/pseeburger/**

Newton's Method, Related Rates, Euler's Method, Arc Length, and Taylor Polynomials are all topics that have a geometric interpretation, and can be illustrated in a dynamic way using the computer. My goal in using Excel (and other software) to illustrate these concepts is to help my students gain a better understanding by seeing these problems come to life.

Here is a list of the Excel Explorations (with screenshots) available on my website. See my website for more detailed descriptions.

1. Tangentline/Derivative Grapher
2. Function Grapher (in Excel)
3. Related Rates - Sphere Example
4. Integration Exploration
5. Euler's Method Exploration
6. Arc Length Exploration
7. Logistic Function in Euler's Method
8. Sequences and Series Exploration
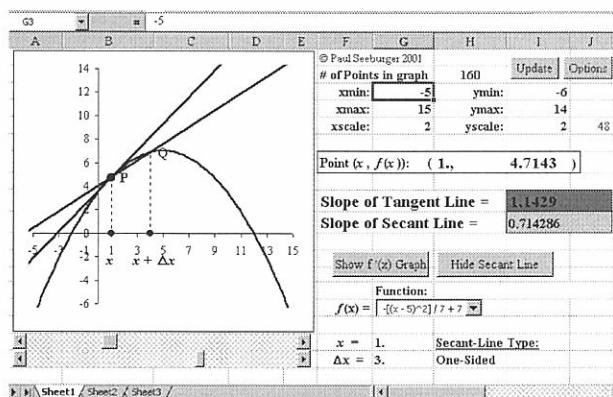9. Taylor Polynomial Exploration
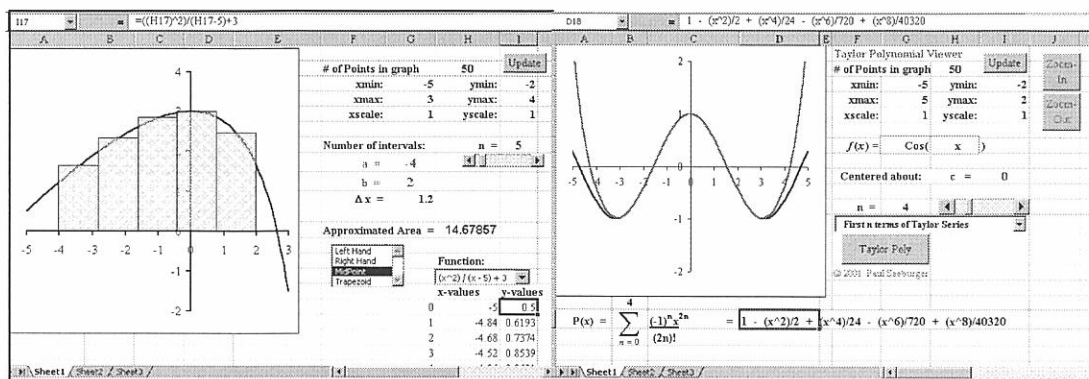

Figure 17: Tangentline Demo

Figure 18: Integration Exploration

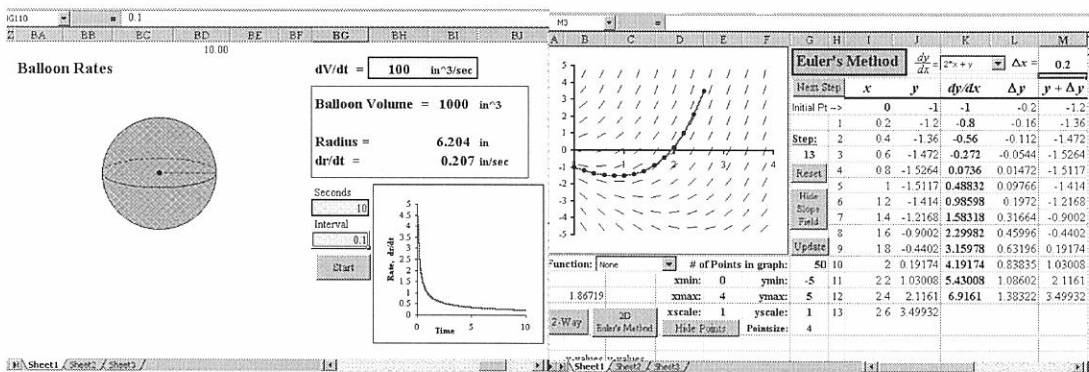**Figure 19: Taylor Polynomials**

Figure 20: Related Rates

Figure 21: Euler's Method

Some Final Reflections

The detail, action and "living color" available on the computer make it a very helpful addition to the graphing calculators many of us already use in our classes. My students have only had positive things to say about these demonstrations whenever I have used them in my classes. I usually have a classroom equipped with a computer projector which has made this a lot easier for me to do. Students often tell me that they really began to understand what was going on when they saw the concepts in motion in my computer demonstrations. And these demonstrations are not just helping them to understand. These colorful dynamic and animated calculus concepts really get their attention and make exploring the concepts of calculus more fun! And honestly, they make presenting the concepts a lot of fun as well!

I want to mention again how important I believe this kind of visual demonstration can be for many of our students. I believe it is a tool that will help the majority of our students understand the concepts of Calculus more fully (to develop their geometric intuition about the concepts), and help them to "smile and nod" with confidence in their understanding. I also hope they will come away with some sense of enjoyment in the beauty of the mathematics involved.