"FUNNY FACE CONTEST": A FORMATIVE ASSESSMENT ON TRANSFORMATION OF FUNCTIONS

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Many high school students throughout America initially take the Algebra I, Geometry, and Algebra II sequence. Some mathematically-apt students take the honors equivalents, and many curricula are designed to permit the mathematically talented students to take these courses in the middle school. Students with "average" or "below average" mathematical ability have a dilemma in choosing a mathematics course after Algebra II. Nevertheless, college admissions counselors strongly encourage these students to take more mathematics in preparation for college entrance and postsecondary study. To succeed in higher mathematics, students' understanding of the concept of functions is paramount. Hence, mathematics departments at the secondary level have found creative ways to provide more treatment of functions in courses like Advanced Algebra, Mathematics IV, and FST (Functions, Statistics, and Trigonometry).

H School, an independent upper school located in the northeast region of Ohio, usually offers two sections of FST course for juniors and seniors. (The actual course title came from the University of Chicago School of Mathematics Project's *Functions, Statistics, and Trigonometry* textbook (1998).) The foremost content difference between FST and a traditional Precalculus course is FST incorporates the concepts of introductory statistics and expounds functions and trigonometry portions less indepthly.

Two weeks into the 2001-2002 academic year, I observed FST students' inability, despite having had two years of prior course work, to neither recall algebra facts nor demonstrate procedural understanding. Moreover, they displayed lower-than-expected motivation for learning mathematics. Mathematics teachers can attest to this cyclical observation: Students' poor comprehension resulting in lower motivation and lower motivation then leading to less effort and less learning. To help overcome these shortcomings, I designed "Funny Face Contest," a formative assessment on transforming functions.

The spirit of the "Funny Face Contest" is in agreement with the first recommendation of the President's Committee of Advisors on Science and Technology, Panel on Educational Technology (1997): "Focus on learning *with* technology" (p. 7). Jiang and McClintock (2000) further advocate that "we, as mathematics educators, should . . . encourage and help our students to apply multiple approaches to mathematical problem solving and engage them in creative thinking" (p.19). To attain this goal, curriculum developers, Schwarz and Hershkowitz (1999), promote the "lever" of graphing calculators to learn mathematics through investigations, and they emphasize the importance in developing curriculum that takes advantage of this technology. Echoing this sentiment, the *Principles and Standards of School Mathematics* (NCTM 2000) states "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning" (p. 24).

To make informed pedagogical decisions, mathematics teachers must become familiar with the advantages as well as disadvantages in incorporating graphing calculators (Garofalo et al. 2000; Hong et al. 2000). For example, Boers and Jones (1994) and Ruthven (1990) found utilizing graphing calculators only in supplemental exercises created disconnect in learning since students had difficulty integrating this information with the more traditional approaches found in the textbook. Furthermore, it is necessary for teachers to model appropriate use of graphing calculator. According to Porzio (1999), students, quite often, "behave" as how they were taught. Hence, teacher training should support the technology innovation as a tool that makes teaching more efficient and not as a disconnected layer in the curriculum (Swan and Dixon 2006).

Having studied linear, parabolic, square root, absolute value, and semi-circle functions, each FST student was encouraged to create a unique "funny face" using at least one from each parent function. With TI-83 Plus technology, students were restricted to graphing at most ten functions and were also permitted to use STAT PLOT option of the calculator to plot points.

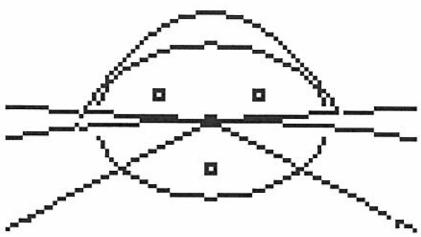
Most of the students come to my class knowing how to input functions into the graphing calculator. Students are also proficient in setting the WINDOW to view particular portion of the function. I, nonetheless, reviewed some important calculator features that should have aided in their effort:

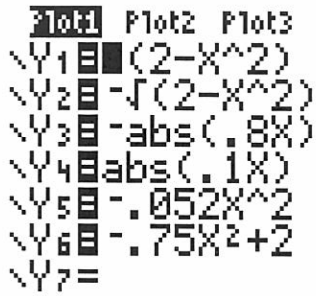
- Choose ZStandard from the Zoom option. This will provide [-10, 10] by [-10, 10] window. Then, modify the window to provide the optimal view of your funny face.

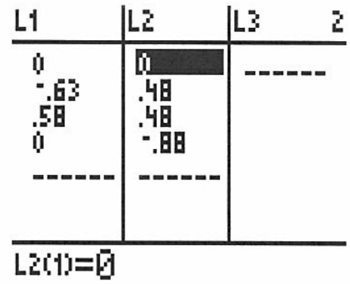
- When formatting the screen, turn on the axes. This setting should help you when transforming functions. When you complete your funny face, make sure to turn off the axes.

- If needed, do not forget to restrict the domains of the functions. This may enhance the quality of your funny face.

To verify that students fulfilled the "at least one from each parent function" criterion, I captured, in addition to their funny faces, the screenshots of Y = (equation editor) and L1-L2 (list editor). For example, Brandon created "Von Steubing." The screenshots below represent his funny face, six functions, and four points. (The radical symbol in Y1 equation is not apparent due to the cursor. When capturing a screen, the cursor should not be on the relevant content. One way to remedy this is to place the cursor over = .)

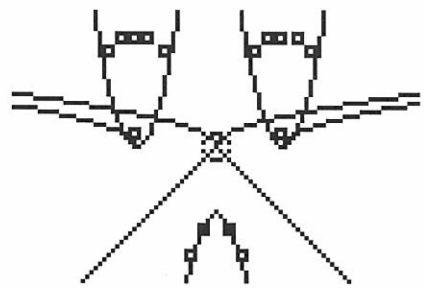






Here is a sample of the funny faces created by Brandon's classmates:

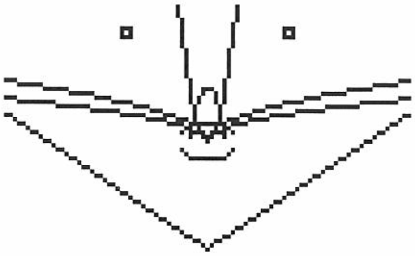
Alex' Allen Billoops



Grayson's Jimmy

Michael's Dan K.

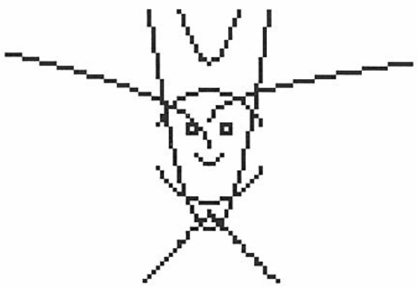
Julie's Yogi



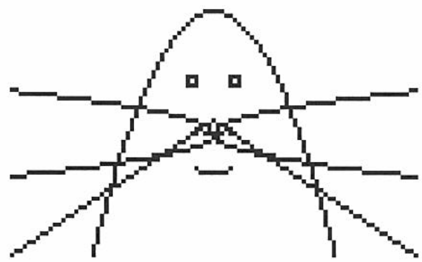
Rachael's Big Kitty

Doug's Rat Boy

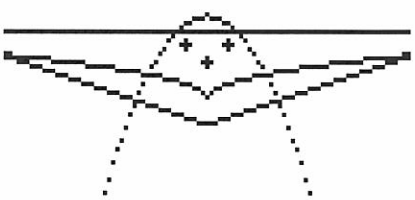
Students in the other class were equally creative. Here is their sample:



Allie's Samurai Molar Man
All Dressed For Work



Jillian's Randall

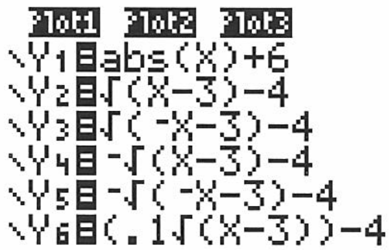


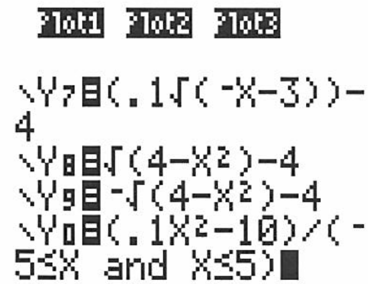
Tom's Wolfort

Rachel's Unhappy Clown-Man

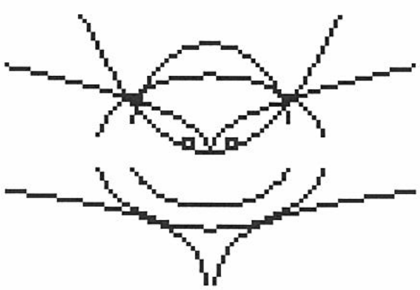
Since this was a contest, I had each student in one class choose the two best funny faces of the other class and visa versa. Students did not have prior knowledge that I would use this form of peer judging.

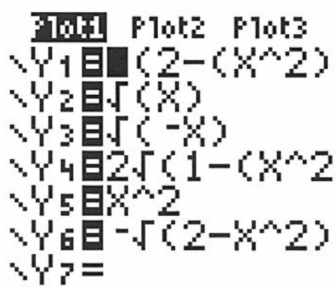
Seventy percent (the most frequent vote) of students in one class chose Brandon's "Von Steubing" as the funniest face. In the other class, while "Rufus" had a strong showing, "Wack–Teenage Mutant Turtle" received 100% endorsement. (The screenshots of these funny faces, along with the equation editors, are shown below.) Hence, the creators of "Von Steubing" and "Wack–Teenage Mutant Turtle" were declared the winners, and each received a school T-shirt as the prize.

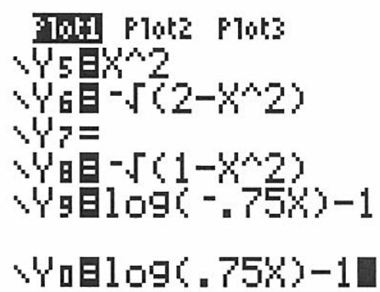




Jason's Rufus







Sam's Wack–Teenage Mutant Turtle

To complete this assignment, students brain-stormed possibilities during one 45-minute class period. Some collaborated while others worked independently. Throughout the day, students sought me for assistance. Their inquiries ranged from "How do I restrict the domain of this function?" to "Is this what you want?" This one-on-one tutorial provided an enormous opportunity to probe for student understanding much deeper—and correct any misunderstanding.

On the following day, students turned in their completed funny faces at the beginning of the class. While I captured the screenshots, I asked them to ponder how they created their faces. One student wrote:

> *After making two eyes and a nose with STAT PLOT, I incorporated the other equations around the nose which was the center. I learned how to correctly place a certain shape from the equations by guessing and checking and I gained a better understanding of the equations.*

Other students reflected:

> *The functions I used for selected parts were decided by the shapes each function made. Like a semi-circle for a mouth and a V shape for the body.*
>
> *I began by experimenting with different formulas. I wanted to create a face that resembled something important to me. I chose to add whiskers so my face would resemble my kitty. The nose wound up being a circle with a parabola after seriously contemplating where I should put those two functions.*
>
> *To make my funny face, I graphed a circle face. Then I played with the window in order to get the face to the left side of the screen. Once that was there, I played around with the different functions to make the body.*
>
> *I listed all types of functions I needed to use and checked them off as I used them. I experimented with different equations until I had them into an arrangement that looked like a face.*
>
> *I knew I could have hair coming off the side of the head by using* $y=\sqrt{x}$ *graphs and moving them up to the top of the head. To make a mouth, I used the semi-circle equation and lowered it down.*

Once students had handed in the above written reflections, I asked them to share with the classmates their approaches, insights, and intuitions. Several students mentioned that at the onset they sketched their funny faces on paper. Others described how they transformed certain functions. At times, I asked them to elaborate—in other words, this discussion was helpful in guiding further instruction (Ashline 2005). This "debriefing" session and the peer-judging afforded opportunities for students to learn from one another and to praise not only the creative aspects of the final products, but also their effort.

Grading formative assessments can be difficult. The above student's inquiry—"Is this what you want?"—indicates I was not clear how I would grade the assignment. Initially, I had conceived grading this 20-point assignment, representing about 2% to 3% of the quarter grade, would not pose much difficulty. I was sorely mistaken. ALL students expected to earn 20 points for their funny faces, yet some did not. Students' and my expectations did not necessarily concur. Simply put, the verbal direction, "using at least one from each parent function," was not enough. I should have specified the following criteria and provide these as a handout:

1. Did the student use all five parent functions?

2. Did the student provide the screenshots of the funny face, the equation editor, and the list editor?

3. Is the window configuration at a desired setting? Provide a screenshot of this as well.

Teachers, implementing this activity, can modify these requirements.

From the above reflections, readers get a sense that this activity provided an opportunity for students to examine the effects of the various dynamic transformations that the graphing calculator allowed (Kastberg and Leatham 2005). Contrary to students solving routine textbook problems, creating funny faces involved "engaging in a task for which the solution method is not known in advance" (NCTM 2000, p. 52). In fact, students were challenged to create "solutions" that were not unique—an approach quite foreign to many mathematics students.

Mathematics teachers should consider utilizing observations, projects, and student reflections in assessing students' mathematical understanding, and "such a collection of both informal and formal assessments can provide teachers . . . with a more complete picture of student performance" (NCTM 2006, p. 4). Valuing problem solving approach to mathematical learning, I firmly believe this formative assessment activity provided students an opportunity not only to solve an interesting, engaging problem, but to reveal clarity to their understanding of functions.

Students were able to explore and observe transformations due to the use of graphing calculator technology. Their reflections suggest, in some instances, students' use of mathematical knowledge while others employing a "trial and error" strategy. Recognizing students' use of a "trial and error" strategy was beneficial. Students and I were able to address the conceptual shortcomings by explicitly describing the mathematics behind the transformations.

Despite the assignment being called a "Contest," student cooperation permeated in and outside the classroom. My students had fun, and they displayed much pride in their creations. As their teacher, I was particularly pleased to witness motivation sprout from within.

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