ON THE PREPARATION OF SECONDARY MATHEMATICS TEACHERS ON THE PROPER INTEGRATION OF TECHNOLOGY

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Abstract: Since the introduction of handheld graphing technology (HHGT), many graphical, numerical and modeling problems have been incorporated into both secondary and collegiate level mathematics textbooks; however, some important technologically-driven concepts, methods and approaches are not being included. In this paper, we examine how twelve currently popular Precalculus textbooks, published from 2000 onwards, have incorporated these concepts, and then look for connections in how these concepts are understood by a group of practicing secondary mathematics teachers.

1. Introduction:

Textbooks are a critical component in U.S. mathematics classrooms, as they help define what content is taught, as well as how it is taught (Peak, 1996). Tarr, Reys, Barker and Billstein (2006) note that since textbooks are often used for seven years, the choice of mathematics textbooks has far-reaching effects on students' mathematics development.

Recent studies have indicated some gaping holes in the content and structure of many mathematics textbooks. Data from the Third International Mathematics and Science Study indicates that the mathematics curriculum in middle schools in the United States must be reexamined. After evaluating twelve middle school mathematics textbooks for 24 criteria allied with six benchmarks, researchers conclude that most books were "inconsistent and often weak" in covering conceptual benchmarks, offering little instructional support for students and teachers, as well as "little development in sophistication" of mathematical concepts discussed (AAAS, 2000). Not all middle school textbooks are aligned with the appropriate standards and benchmarks for students using the book, and that these textbooks may not emphasize processes involving higher level thinking, such as conjecture, hypothesis-testing, and examining connections with previously learned mathematics concepts (Reys and Bay-Williams, 2003). Tarr et. al. (2006) propose that there are inconsistencies between material promoted in the main portion of textbook lessons of middle school textbooks and that presented as homework problems. In some instances, the main text promotes critical thinking, and inquiry skills, while the homework deals mainly with procedural skills.

To date, mathematics textbooks must cover a wide variety of concepts mandated by standards from different states and schools (Seeley, 2003), with the result that many textbooks are extremely large in size. Inconsistencies between the homework and main text, as well as large overall textbook size, with potentially over 100 homework problems per section, gives teachers the perplexing job of deciding what to emphasize in a text (Tarr et. al, 2006).

Similar results were found in a study of Pre-calculus college textbooks with respect to the application of technology by Quesada, Smith, and Edwards (2008). Coverage of domain and range, local and global behavior, and recursion were found to vary substantially textbook by textbook, with many not emphasizing technology-driven, graphically-based solution-finding and investigation. This study proposes to go one step farther, and examine sixteen mathematics criteria that can be conceptually aided via technology, in terms of how they are addressed in the main text versus their incorporation into homework problems.

The selection of the sixteen criteria examined in the textbooks (as summarized in Table I) reflect critical concepts allied with technology usage, addressed during our coursework with pre-service teachers, and in workshops with in-service teachers at the University of Akron (Ohio). These classes focus on conceptual changes foundational to calculus (FTC) that graphical and numerical handheld graphing technology (HHGT) facilitate on the study of functions, including the 16 criteria summarized in Table 1.

Although most of these teachers have already completed a precalculus course prior to taking the technology course/workshop, many preservice and in-service teachers have demonstrated a lack of conceptual knowledge of these critical criteria, as reflected in scores on a pretest dealing with these sixteen concepts (Table II). The textbook analysis, in part, seeks to find possible connections between weakness in teachers' grasp of these concepts, and their exposition in college precalculus textbooks.

2. Methods

Twelve commonly used college Precalculus textbooks, all published from 2000-2008, were evaluated using a Likert scale of 0-2, both in the main portion of the text, and in adjoining homework sets. Scores were later converted into average percentages, both individually, and as a whole. For a textbook to obtain a score of "1" in either the main section or the homework section, that book had to mention the concept, or present at least one detailed homework problem. To merit a "2", the textbook had to present a thorough explanation of the topic (in the main text) or have at least two detailed problems on the topic in the homework. Textbooks were separately evaluated for the criteria for each family of functions. Concepts examined included sketching polynomials, the idea of a complete graph, domain and range, finding solutions numerically via tables, graphical solutions (both $f = g$ and $f-g = 0$ approaches), a technology-balanced approach; use of transcendentals and irrationals in functions, relative growth, use of inequalities, local and global behavior, local extrema, optimization, increasing/decreasing functions, transformations, and sketching.

3. Results

The examined textbooks appear to place varying emphasis on the evaluated concepts. To illustrate, the concept of global behavior is barely mentioned (< 50%) in one textbook, while it is explored extensively

(90%) in another text. One quarter of the textbooks score poorly in almost all topics. Some books score high (> 75%) on three or four criteria, with only moderate scores (51-75%) in other categories. Scores do not necessarily increase in more recently published books.

Key ideas are normally not revisited with every family of continuous functions. All topics, with the exception of local/global behavior and extrema, were covered most thoroughly in sections on polynomials, with lesser coverage for other families of functions.

In general, weak areas, with average usage less than 50%, included such criteria as numerical solutions, graphical solutions finding zeros, a technology-balanced approach (Quesada, Smith, & Todd, 2008); the use of transcendental functions; relative growth; problems containing inequalities; optimization; sketching polynomials (turning points, effect of leading coefficient), and the concept of a complete graph. Other criteria were mentioned in almost every textbook and extensively discussed in several, including finding local and global behavior numerically (53%), local extrema (58%), and increasing and decreasing functions (60%). No single textbook addresses every topic; even the highest scoring book barely mentions complete graph and relative growth. While most books emphasize sketching (81%), transformations (67%) and domain and range (68%), they are often presented as detached concepts: few connect these ideas to the idea of a complete graph. Beyond definitions, little is done on finding the range graphically or numerically for families of continuous functions, or on the effect of transformations on domain and range. Most textbooks do not link calculator usage with numerical solutions when determining global behavior, to enable students to unify these ideas conceptually.

Table I: Analysis of 12 College Pre-calculus Textbooks published 2000-2008

TEXT AVERAGES	HOMEWORK AVERAGES	Text-HMK	Combined Average	TOPIC
0.81	----		0.81	**Sketching Polynomials (turning pts, leading coeff.)**
0.74	0.55	.19	0.65	**Complete Graph**
1.45	1.28	.17	1.36	**Domain and Range**
0.25	0.03	0.23	0.12	**Numerical Solutions**
0.83	1.00	-0.17	0.92	**Graphical Sol: f-g=0**
0.68	0.63	0.05	0.65	**Technology-balanced approach**
0.71	0.89	-0.18	0.80	**Use of Transcendentals and/or Irrational**
0.85	0.66	0.19	0.75	**Relative Growth**
0.77	0.91	-0.14	0.84	**Inequalities**
1.13	0.99	0.14	1.06	**Local and Global Behavior**
1.14	1.18	0.04	1.16	**Local Extrema**
0.62	1.05	-0.43	0.83	**Optimization**
1.26	1.15	0.11	1.21	**Interval f is increasing/decreasing**
1.52	1.15	0.37	1.34	**Transformations**
1.42	1.82	-0.40	1.62	**Sketching**

Likert scale: 0= No; 1 = mentioned; 2 = well explained with an example/several homework questions)

More books emphasize graphical solutions involving the intersection of functions (59%) rather than graphically finding zeros of the difference of these functions (46%). A few books describe both methods, and then allow students to select their preferred method when solving homework problems.

Only 17% of the textbooks emphasize the use of inequalities with all families of functions; other authors briefly work with inequalities in just one or two sections, primarily in chapters on polynomials. Solving equations rarely integrates numerical, graphical, and algebraic approaches, or contains equations involving transcendental and algebraic functions, or different transcendental functions. Most books do not include inequalities involving transcendental functions, yet inequalities make students think graphically!

Although 33% of the textbooks found numerical solutions via tables; only one examines this topic in detail. While most place an emphasis on transformations and sketching of functions, many do not relate these ideas to those of the complete graph, while over 66% do not discuss the idea of the complete graph of a function in detail. However, an awareness of when a window displays all the important features of the graph of a function is essential for approaching problems graphically.

Placement of Concept:

The presentation of most concepts was evenly balanced between main text and homework, but there were some surprising discoveries. A few concepts (particularly optimization and sketching) were presented mainly as homework, rather than in the main text; they were prominently placed at the beginning of homework sets. In contrast, relative growth problems (38%) were usually placed at the end of long homework sets, typically as extension problems, rather than being discussed in the main text; relatively few homework exercises require the knowledge of relative growth to solve problems such as finding hidden intercepts. In conclusion, these textbooks offer varying content on the concepts studied, with often inconsistent reinforcement of concepts throughout the body of the text.

Possible Connections to Student/Teacher Knowledge:

Textbook ratings were compared to pretest scores of 85 in-service teachers and 14 pre-service teachers. This 90 minute pretest consisted of 33 problems using the sixteen concepts used as criteria in the textbook evaluation. The in-service test was administered during the summer of 2007, while the preservice test was given in the spring of 2008. Each individual's score in every concept area was calculated as a percentage. For this comparison, some textbook criteria, itemized above, were combined (Table III). Clearly, there seems to be some connection between the amount of attention paid to these topics in Precalculus texts, and the conceptual knowledge of the same topics by both in-service and pre-service teachers. High textbook scores for transformations and sketching, as contrasted to pretest scores, may possibly be due to an imbalance between text and homework problems in the textbooks: students may simply not be getting all of the fundamentals from the text (with sketching and optimization) or not getting enough homework problems with which to practice (in the case of transformations.) The data suggests that further investigation, with larger groups of students, may indicate more connections.

TABLE III: Comparison of textbook, pre-service and in-service teacher scores on key concepts

Topics	Textbooks	Pre-service	In-service
Properties of functions (range, extrema, inverses…)	62.1%	47.7%	46.5%
Sketching functions (polynomials, rational, trig…)	60.8%	39.9%	58.1%
Equations from graphs/properties	52.5%	53.2%	41.6%
Global and Local behavior	53.0%	55.4%	58.7%
Transformations	67.0%	32.0%	38%
Relative growth	38%	12.8%	28%
Optimization	41.5%	25.0%	34%
Inequalities	42.0%	45.8%	60%
Average of all topics		44.5%	46.2%
			R23 = .75

4. Summary

A first glance at these textbooks reveals that we are indeed moving forward, with textbooks including many graphical ideas, and some numerical ideas and modeling problems. Our perception; however, is that there is much room for improvement. We often seem to be supplementing the traditional approach, rather than developing a cohesive new one based on research. Many relevant ideas, approaches, and tools are being ignored. Clearly, with many textbooks nearing 800 pages, the conflict continues between adding new relevant ideas to the curriculum and the need for eliminating some of the traditional material. Further research is warranted to further illuminate the effects of textbook inconsistencies on students' and teachers' understanding of these relevant concepts.

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