

EXPLORING POLAR COORDINATES WITH THE GEOMETER'S SKETCHPAD[®]

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Introduction

It's not just for geometry! In a previous paper [1] we showed how The Geometer's Sketchpad[®] can be used to create classroom demonstrations of a number of important concepts in calculus. In this paper, we give step-by-step instructions for creating several classroom demos that can help remove some of the mystery surrounding polar coordinates and the graphs of polar equations. In particular, this paper includes instructions for sketches that

- show the plotting of points in polar coordinates;
- aid the understanding of why certain polar graphs have the shapes that they do;
- show how changing a parameter changes the graph of a polar equation.

In the remainder of this paper, we assume that the reader is familiar with basic constructions in Sketchpad, and we focus on how to create the kinds of sketches described above.

Plotting Points in Polar Coordinates

- Open a new sketch, and use the Graph menu to show the (square) grid. Drag the origin to recenter the graph, and drag the point (1,0) to resize the graph.
- In the "Edit" menu, choose "Preferences," and change the angle units to radians.
- Using the straightedge tool, construct two lines, and then construct a point on each. (These lines will be used as sliders to control the values of r and θ in plotted points.) If necessary, move the constructed points so that they are between the defining points of the lines.
- Construct the segments between the left points and the constructed points on the lines. Then hide the right points and the lines themselves.
- For each segment, use the Measure menu to measure the abscissa of each point. Then choose "Calculate" from the Measure menu, and calculate the difference of the

abscissas (right minus left). Drag these measurements close to the segments, and then hide the original abscissas.

- Change the labels of these measurements, by right-clicking and then selecting “Properties” and then “Label.” Label one of the measurements “ r ” and the other “Theta1.” We could now proceed to plot the point (r, θ) . However, the θ measurement would be expressed as a real number, rather than its more convenient representation as a multiple of π . In the next step, we fix this.
- Choosing “Calculate” from the Measure menu, calculate $\text{Theta1} / \pi$, and label this as “Theta.” Move the Theta1 measurement off to the side for now, and replace it with this new value, which you should drag close to the appropriate segment. Then, using the Text tool, create a text box containing only the text “ π ” in the appropriate color and size. Drag this just to the right of the Theta measurement, so that it looks like Theta is being measured as a multiple of π .
- In the Graph menu, change to Polar grid form. Then select the “ r ” measurement and the *original* “Theta1” measurement, and select “Plot as (r, θ) ” from the Graph menu. After this, you can hide the original “Theta1” measurement. As the right endpoints of the sliders are moved, the position of the plotted point changes accordingly.
- Final touch-ups include hiding various point labels, and changing the label of the plotted point. We also recommend making the plotted point more visible to a class. To do this, draw a very small segment anywhere on the sketch. Then, selecting the plotted point and the segment, construct a circle by center and radius. While the circle is selected, construct its interior, and then change its display color to a visible color. You can then hide the segment, and add any text you wish to the sketch. Figure 1 shows the completed sketch.

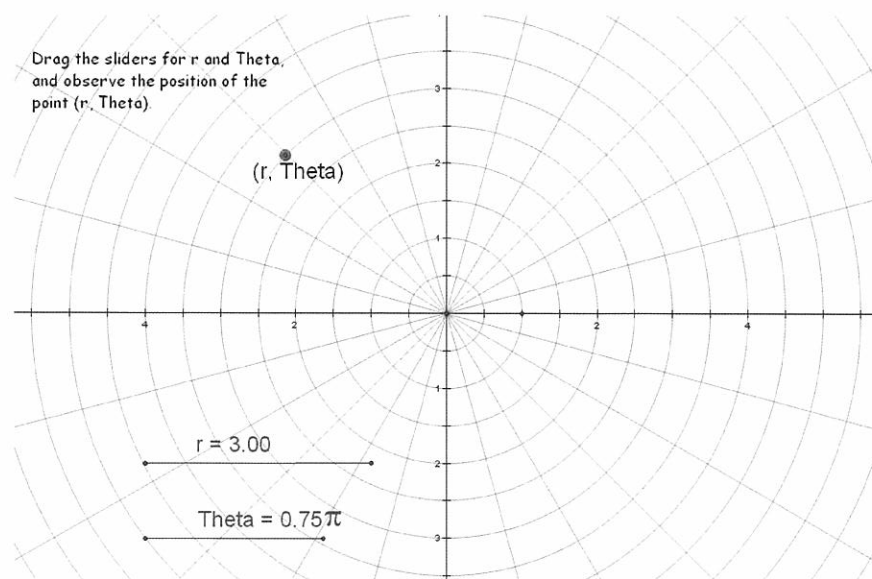


Figure 1: Plotting points in polar coordinates

How a Parameter Affects a Polar Graph

In this demo we create a graph of a polar equation that includes a parameter, whose value is controlled by a slider. As you change the parameter by dragging the slider, you can then observe the effect on the graph.

- Open a new sketch, and use the Graph menu to show a square grid. Drag the origin to recenter the graph, and drag the point (1,0) to resize the graph.
- Choose “Preferences” from the Edit menu, and change the angle units to radians.
- Using the straightedge tool, construct a horizontal line, and then construct a point on the line. If necessary, move the constructed point so that it is between the defining points of the line.
- Construct the segment between the left point and the constructed point on the line. Then hide the right point and the line itself.
- Use the Measure menu to measure the abscissa of each endpoint of the segment. Then choose “Calculate” from the Measure menu, and calculate the difference of the abscissas (right minus left). Drag this measurement close to the segment, and then hide the original abscissas. This measurement will serve as a parameter in the polar equation that you wish to explore, so change its label to “a.”
- Switch the Grid Form to Polar (in the Graph menu).
- Using the Graph menu, choose “Plot New Function.” Enter the formula for a function $f(\theta)$ in which a parameter is involved—such as $1 + a \cdot \cos(\theta)$. (To enter the parameter a , simply click on the measurement “a” of the slider.) Adjust the color and thicknesses of the graph for optimal visibility in your classroom.
- Drag the right endpoint of the slider to watch the graph morph into a new shape. Figure 2 shows one snapshot of a limaçon-cardioid-circle family of graphs.

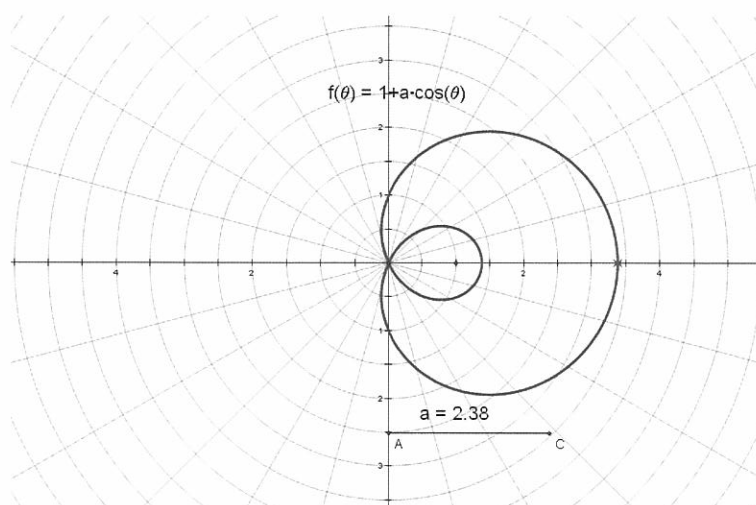


Figure 2: Snapshot of a parametric family of polar curves

Relating the Cartesian Graph of $r = f(\theta)$ to its Polar Graph

In this demonstration, we graph the function $r = f(\theta)$ in Cartesian form, with the independent variable θ on the horizontal axis and the dependent variable r on the vertical axis. We simultaneously animate the graph of the same function in polar form, and the mysteries of the shapes of various polar graphs are unlocked!

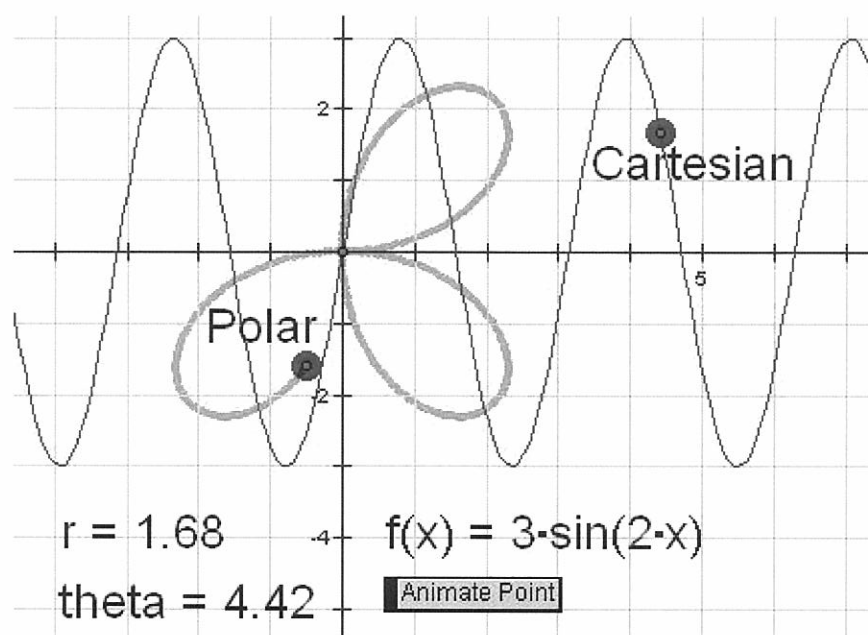


Figure 3: Plotting a function in polar and Cartesian coordinates

- Open a new sketch, and use the Graph menu to show the (square) grid. Choose “Preferences” from the Edit menu, and change the angle units to radians.
- From the Graph menu, plot a new function, $f(x)$.
- Select the graph of $f(x)$, and construct a point on the function plot. Show the label for this point, and change it to “Cartesian.”
- Measure the abscissa (x) of the Cartesian point, and change the label of this value to “ θ .”
- Create a new parameter, r , with any initial value. Then edit the parameter r to change its value to $f(\theta)$.
- In the Graph menu, change the grid to Polar form.
- Highlight r and θ , in that order, and plot as (r, θ) . Show the label for this point, and change it to “Polar.”
- Change back to the Square grid form.

- Highlight only the Polar point, and select “Trace Plotted Point” from the Display menu.
- Now highlight only the Cartesian point. From the Edit menu, create an Action Button to animate this point in the forward direction, once only, with slow speed.
- Adjust line thicknesses and label sizes so that the audience can see them. Create circles around both points to enlarge them (as described previously in this paper). It is helpful to color the circle interiors so that they contrast with the trace color. See the sample sketch in Figure 3.
- Move the Cartesian point to a good starting location, like $\theta = 0$, erase all traces, and play the animation.
- You can change the definition of $f(x)$, move the Cartesian point to a good starting location, erase all traces, and play the animation again.
- Discuss some of the relationships between the Cartesian graph and the polar graph: What’s true about the polar graph when the Cartesian graph has a maximum, or minimum? How do roots of the Cartesian function affect the polar graph? Can we tell from the Cartesian graph when the polar graph will start tracing over itself?

Conclusion

Using the Geometer’s Sketchpad[®] can help students understand the polar coordinate system. From plotting points to graphing functions, Sketchpad allows students to explore the differences between polar coordinates and Cartesian coordinates.

Reference

- [1] D’Ambrosia, Barbara K. and Carl R. Spitznagel. “Demonstrating Calculus Concepts using The Geometer’s Sketchpad[®].” *Proceedings of the 19th Annual International Conference on Technology in Collegiate Mathematics* (2008).