TEACHING TECHNICAL MATHEMATICS WITH CALCULUS USING CAS CALCULATOR SUPPORT
or
HOW YOU CAN GAIN MORE TIME FOR MATHEMATICAL TOPICS AND ENGAGE YOUR STUDENTS IN THEIR LEARNING

Ben Rushing, Jr.
Northwestern State University of Louisiana
435 D Kyser Hall
Natchitoches, La., 71497
brushing@nsula.edu

This paper outlines two methods that have been effective in increasing student interaction with course content. Without student interaction with content, there is no learning. Despite best teaching practices, you cannot push a rope. Effective teachers see the educational process as a journey of experimenting, adapting, and learning on the part of the instructor and the student. They are willing to try new ideas in the hope of increasing student learning. CAS calculators can increase student learning.

Student learning increases when educational activities are relevant to the students' need and require students' active participation in their learning. In a technological savvy learning culture, appropriate use of technology provides additional time for task and time on task. These two aspects of time management can result in increased student learning.

The processes outlined in this paper are used for a two semester mathematics sequence. In the first six-semester hour course, technical mathematics, students begin with basic algebraic operations and end with vectors. Enrollment in this first course requires a math ACT score of 18 or higher. The second six-hour course, calculus for electronics covers the calculus topics of derivatives, integrals, MacLaurin, Taylor, and Fourier series. Many of these students are first generation, non-traditional students.

Currently, each student is required to purchase one of the TI ® CAS calculators (TI-89® , TI-92®, Voyage 200®) for use in these courses. Non-calculator based material and processes are presented during the first week of class but thereafter, most of the content is heavily CAS technology dependent. This choice is appropriate for the culture and demographics of this student population.

Vectors

Vectors are useful to model situations where quantities are described as having magnitude and direction. Most examples in the technical mathematics course use 2-dimensional vectors. Vectors are represented in either rectangular form or polar form. In the rectangular form, horizontal and vertical components are given explicitly as [*horizontal component, vertical component*].
This form is useful when adding or subtracting vectors. The polar form gives the magnitude and direction angle in the form, [*magnitude, direction angle*].

Example 1:

Figure 1 shows the vector, [*3, 4*]. The magnitude, 5, is computed using the Pythagorean theorem. The direction angle, θ, is $\theta = \tan^{-1}\left(\frac{4}{3}\right) \approx 53°$. The polar form for this vector is [*5, ∠53°*].

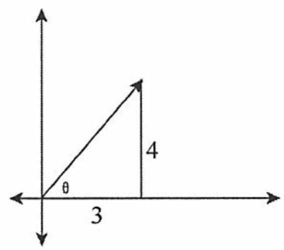

Figure 1 Vector [*3, 4*]
in rectangular form

In general, when a vector, [*a, b*] is in rectangular form, the polar form for the vector is [*R, ∠θ*] where $R = \sqrt{a^2 + b^2}$ and reference angle, $\theta_{ref} = \tan^{-1}\left(\frac{b}{a}\right)$. In working with polar form vectors, attention must be given to the angle measurement unit.

Example 2: Convert each vector to polar form using decimal degrees for angle measurement.

1. [2, -6] 2. [0, 10] 3. [-1.3, 2.4]

Solutions: Because the desired angle measure is degrees, the calculator is set to degree mode (note the DEG at the bottom of the calculator screen. Figure 2 illustrates the calculator result of converting each of the three rectangular form vectors to polar form.

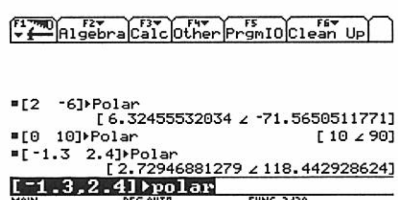

Figure 2 Converting
rectangular to polar form

A vector, V = [*R, ∠θ*] in polar form with magnitude, *R* and direction *∠θ* has rectangular form, [*R cos θ, R sin θ*]. Using non-calculator processes for adding or subtracting vectors, one must determine the horizontal and vertical components for each vector, then combine the horizontal and vertical components as indicated and convert the resultant rectangular form vector to polar form to obtain the magnitude and direction. The calculator streamlines the process, leaving yields substantially more time for richer examples.

Example 3: Suppose a boat travels at 12 knots toward a compass direction of 63° in a river that flows at 3 knots toward a compass direction of 175° with wind blowing at 5 knots from a compass direction of 137°. Find the actual velocity and direction of the boat.

Solution: Vectors are used to represent the motions of the boat, the river, and the The boat vector is [12, ∠63°]. The river vector is [3, ∠175°]. Because the wind direction is given as a direction from, the wind vector [5, ∠223°]. Figure 3 illustrates the calculator result in both rectangular and polar forms. The resultant direction angle is in compass degrees.

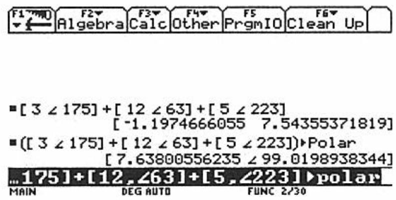

Figure 3 Resultant

The velocity of the boat is about 7.6 knots toward a compass direction of 99°.

Fourier Series

Fourier series are used to model periodic behavior of sound waves, heat distribution, vibration analysis, optics, and others. They give efficient models for piecewise defined periodic functions.

The Fourier series for a 2π periodic function, *f(x)*, is

$$f(x) = a_0 + a_1 \cos x + a_2 \cos 2x + \ldots + a_n \cos nx + \ldots + b_1 \sin x + b_2 \sin 2x + \ldots + b_n \sin nx + \ldots$$

The coefficients are defined by

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \cos(nx)dx \quad (\text{n} = 1, 2, 3, \ldots)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cdot \sin(nx)dx \quad (\text{n} = 1, 2, 3, \ldots)$$

The initial coefficient, a_0 is obtained using the calculator. The process for finding the coefficients a_n $(n = 1, 2, 3, \ldots)$ can be done using "clever" calculator processes. The calculators are programmed with a "when" process that allows for a single input of a complex formula and minor changes to obtain results for multiple values. The calculator syntax for determining the coefficient, a_1 is $a_1 = 1 \div \pi\left(\int_{-\pi}^{\pi} f(x) \cdot \cos(n \cdot x)\right) | n = 1$. The $| n = 1$ part of the syntax after the integral is the "when" process. The b_n coefficients $(n = 1, 2, 3, \ldots)$ are found by replacing "cos(" with "sin(" in the expression and adjusting the indexing value, n.

Example 4: A half-wave rectifier allows an electric current to pass through in only one direction. Give four terms of the Fourier series for a rectified wave where the (2π) periodic function is defined by

$$f(t) = \begin{cases} \sin t & (0 \le t < \pi) \\ 0 & (\pi \le t < 2\pi) \end{cases}$$

The coefficients for this Fourier series require computing the following integrals. A typical first year student may find the computation of these integrals intimidating, if not impossible.

$$a_o = \frac{1}{2\pi}\left(\int_0^{\pi} \sin t \cdot dt + \int_{\pi}^{2\pi} 0 \cdot dx\right) = \frac{1}{\pi} \quad \text{(not too difficult by hand)}$$

$$a_1 = \frac{1}{\pi}\left(\int_{-\pi}^{0} \sin t \cdot \cos t \cdot dt + \int_0^{\pi} 0 \cdot \cos t \cdot dt\right) \quad \text{(the second integral is zero } \forall \text{ n)}$$

$$a_2 = \frac{1}{\pi}\left(\int_{-\pi}^{0} \sin t \cdot \cos 2t \cdot dt\right) \qquad a_3 = \frac{1}{\pi}\left(\int_{-\pi}^{0} \sin t \cdot \cos 3t \cdot dt\right)$$

$$b_1 = \frac{1}{\pi}\left(\int_{-\pi}^{0} \sin t \cdot \sin t \cdot dt + \int_0^{\pi} 0 \cdot \sin t \cdot dt\right) \quad \text{(the second integral is zero } \forall \text{ n)}$$

$$b_2 = \frac{1}{\pi}\left(\int_{-\pi}^{0} \sin t \cdot \sin 2t \cdot dt\right) \qquad b_3 = \frac{1}{\pi}\left(\int_{-\pi}^{0} \sin t \cdot \sin 3t \cdot dt\right)$$

Figure 4 shows the calculator syntax and result of computing the a_0 coefficient.

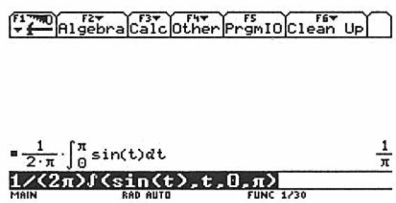

Figure 4 Computing a_0

Figure 5 illustrates the calculator syntax required for computing the a_1, and a_2 coefficients. Figure 6 illustrates the result of the computation of these coefficients.

$$a_1 = 1 \div \pi \int(\sin(t)\cos(n \cdot t), t, 0, \pi) \mid n = 1$$

$$a_2 = 1 \div \pi \int(\sin(t)\cos(n \cdot t), t, 0, \pi) \mid n = 2$$

$$a_3 = 1 \div \pi \int(\sin(t)\cos(n \cdot t), t, 0, \pi) \mid n = 3$$

$$a_4 = 1 \div \pi \int(\sin(t)\cos(n \cdot t), t, 0, \pi) \mid n = 4$$

Figure 5 Computing a_n

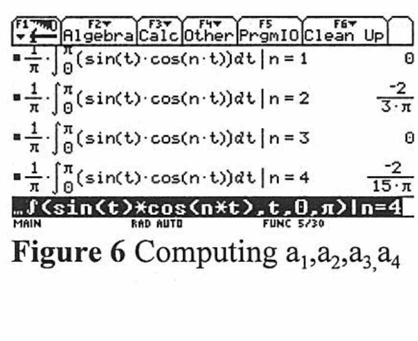

Figure 6 Computing a_1, a_2, a_3, a_4

Figure 7 shows the result of finding b_1, and subsequent b_n for $n>1$. The value, $sin(n\cdot\pi)$ is zero for integer values, $n>1$. It follows that all b_n coefficients for $n>1$ are zero in this Fourier expansion.

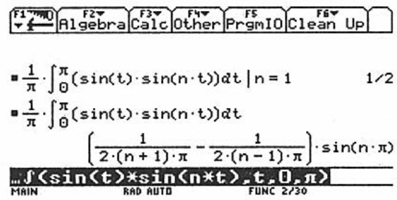

Figure 7 Computing b_n

The rectified half-wave function is approximated with the four terms of the Fourier series.

$$f(x)=\frac{1}{\pi}-\frac{2}{3\pi}\cos(2t)-\frac{2}{15\pi}\cos(4t)+\frac{1}{2}\sin(t)$$

The graph of this estimating function for the rectified half-wave is shown in Figure 11.

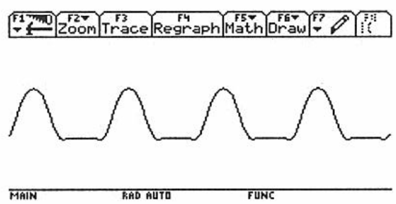

Figure 11 Fourier estimation

This CAS process for computing Fourier coefficients facilitates a level of understanding that is not possible when computing these integrals by hand. The time and possible complexity required to compute Fourier coefficients by hand can mean that the pattern of subsequent coefficients is missed. Because the coefficients can be computed quickly using the CAS calculator process, recognizing the pattern of subsequent coefficients is easier.

Teachers wanting to employ CAS calculator support face some manageable disadvantages. IN particular, evaluations and tests must change to account for the technology. Teachers must adapt to and learn the technology and seek useful methods for integrating technology support into their classroom experiences. Students must be taught some of the "button pushing" processes of the technology. The advantages of implementation include the fact that CAS calculators increase student participation in their learning. Problems can be "reality based" rather that "computationally based," and there is more time for richer problems. In a technology driven curriculum, learning appropriate use of technology is a desirable result. It is better to have students learn to do something rather than to continue to fail to do anything. Properly used, CAS calculators provide a powerful tool for teaching challenging mathematical topics.