Let's visualization the differentiation of the trigonometry though the mathematical activity using CBL

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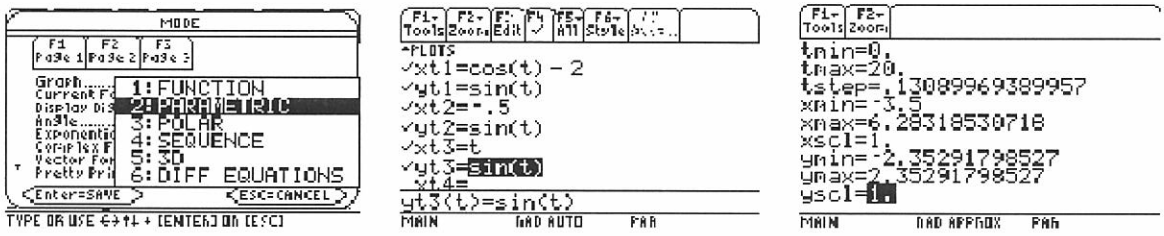
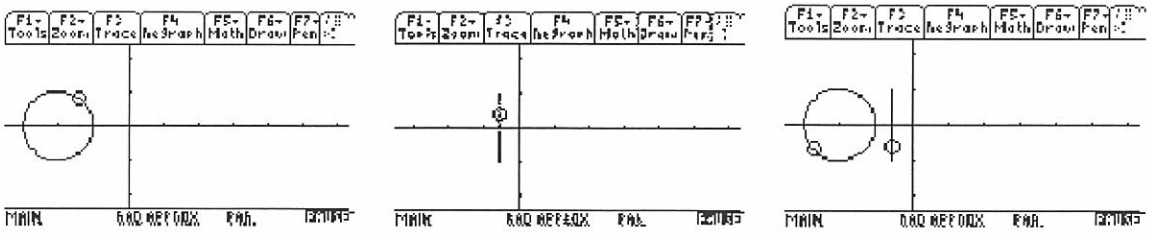
It is difficult to understand the abstract mathematics. We have helping tools for students. With the graphic calculator, the student is learning mathematics more easily. They can visualize the definition and the differentiation of the trigonometry function with the concrete method. Next we can understand them using the body with CBL.

1.Definition of the trigonometry function on the graphic calculator

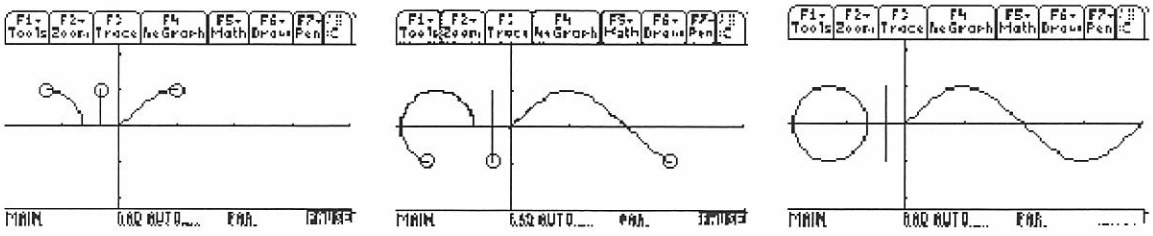
In the lesson of the trigonometry, we can see the definition of the trigonometry function using TI graphic calculator. This screen is the circle having a radius of one at the left side and the right is the straight line. We have two valuables, one is the angle other the distance. We can see the definition of the trigonometry with the animation. We use the TI-89 with the parametric function.



using graphs TI-89



angle valuable distance valuable two valuables

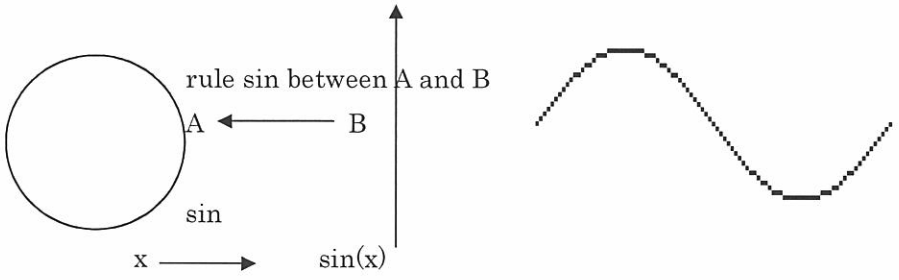


moving with animation drawing sine curve

It is interesting to learn mathematics with TI-89. But this experiment is only seeing by eye. So next we want to use the body not only seeing by the eye.

2. The learn by experience of the definition of the trigonometry with CBL

We use the hiker program with CBL with the distance sensor. In this time, two valuables are the time and the distance. We cannot see the time variable. We want to see all variables and drawing the graph on the screen. We get the moving thing on the straight line with CBL system. One student A is the part of the angle valuable on the circle; other student B is the part of the distance valuable. First time, the student A is moving on the circle with the unit clockwise. And other student B is moving on the striate line, seeing A. This movement of the student B is drowning the sine curve that is the trigonometry function of y=sin(x). That is the direction is plus on the mathematics. Then we get the graph of y=sin(x).

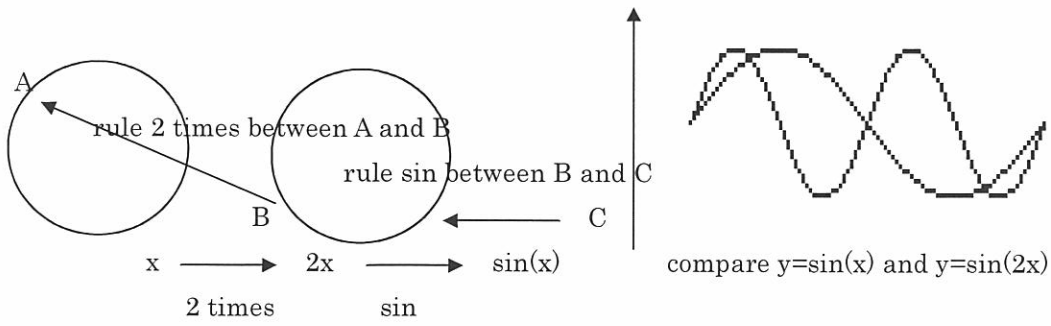


A moves on the circle　　B moves on the line　　drawing the graph of B

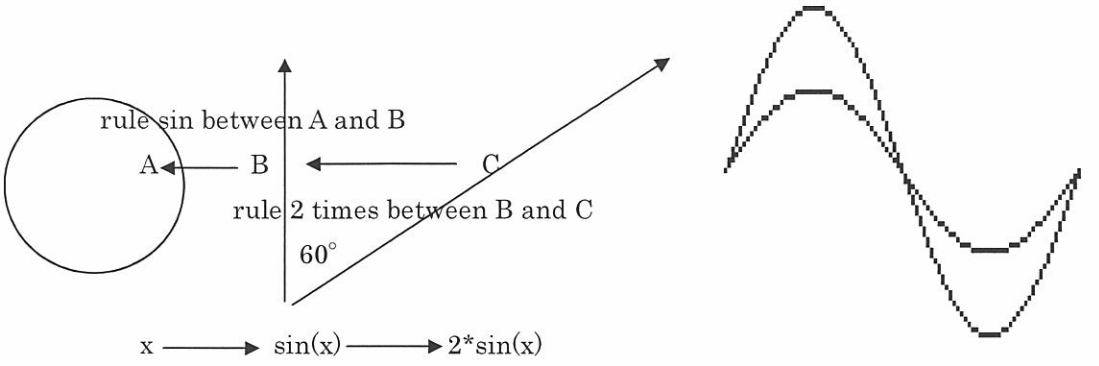
A is angle variable x　　B is y=sin(x)　　this graph is same y=sin(x)

The move of the student B is changing at every time. So this graph is not strait, we get the graph of drawing the sine curve. Students can understand the sine curve with their bodies. They are interesting for the trigonometry when they understand it very well.

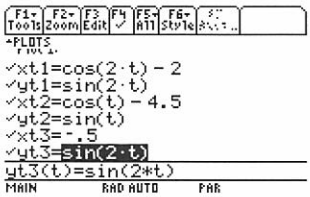
Next we want to study the composite function, y=sin(2x) and y=2*sin(x). These functions are difficult. The composite function y=sin(2x) is making with the moving three students, A, B and C. The student A is walking the same speed on the circle, then B is moving on the other circle. In this time there is the rule of 2 times between A and B. And C is seeing B and walking on the line. We get the graph of C. This graph is the composite function y=sin(2x). The independent variable is the angle by the student A. One student A moves on the circle, this moving is the independent variable of the angle. And other B is moves on the striate line; this moving is the dependent toward the angle valuable. We get the data of the dependent with CBL moving on the line that is the student B. Then we can get the curve of the sine curve. We can see the definition of the trigonometry. In this experiment, we can understand two valuables, the angle and the distance. It is interesting for students on the experiment. They have the good lessen for the mathematics.

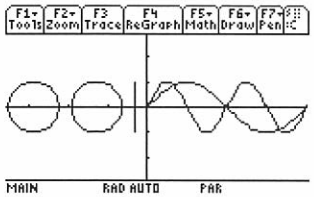


The composite function has two rules, one is 2 times and other is “sin”. It is important that the rule has order, x ⟶2x ⟶sin(x.). The student A is moving on a round of the circle, and then C is two waves. We change the order of the rule; first time using “sin” and next 2 times. We get other functions. x ⟶ sin(x) ⟶2*sin(x.).

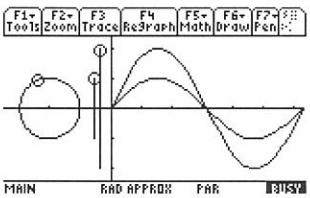


We can see these graphs on the graphic calculator.





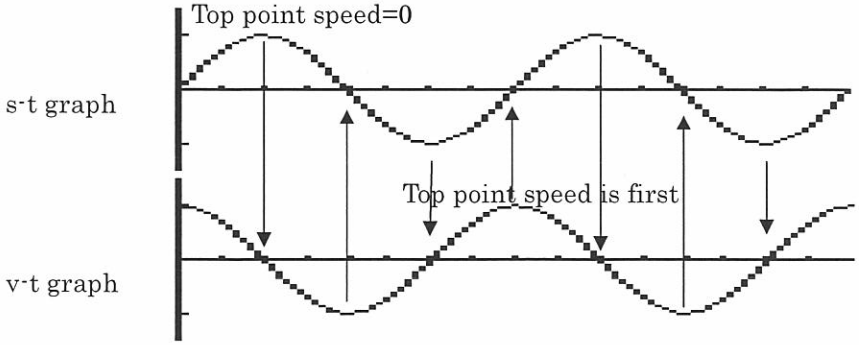
y= sin(2x)



y=2*sin(x)

3. The learn by experience of the definition of the differentiation

We can see the velocity of the dependent moving of the student on the line. The most fast on the x-axis is the speed +1 or -1. We can draw the next graph form the sine curve to the velocity graph that is the cosine curve. So we make the next graph at four times, then we can understand that the derivative of y=sin(x) is y=cos(x). If we show the fourth derivative of y=sin(x) is the same function. We learn the differentiation without the precise calculation. It is most important to use the precise calculation for learning mathematics. The students move on the line with the rule of “sin”. They have the experience that is changing everywhere. They can draw the sine curve graph, and can read the sense of this graph.



The experience of the making the sine curve graph with their bodies is interesting to understand on the mathematics. They can estimate the differentiation of y=sin(x) that is y=cos(x).

So we check this differentiation of y=sin(x).

Two points on the y=sin(x), P(x, sin(x)) and Q(x+ Δ x,sin(x+ Δ x)).

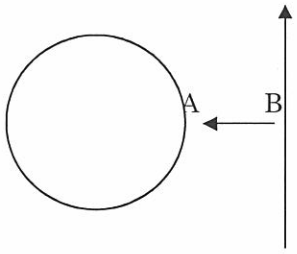
The slop of the line PQ = (sin(x+ Δ x)- sin(x))/(Δ x)

=2*cos(x+ Δ x/2)*sin(Δ x/2)/ Δ x

Δ x/2→0 then we estimate that the differentiation of y=sin(x) is y'=cos(x),

So we want this formula that is 2*sin(Δ x/2)/ Δ x →1 (Δ x/2→0).

We show it using the walking.



The A is moving on the circle and the B on the line.

At one point the speed of A and B are same.

Speed of A x/t = sin(x)/t speed of B.

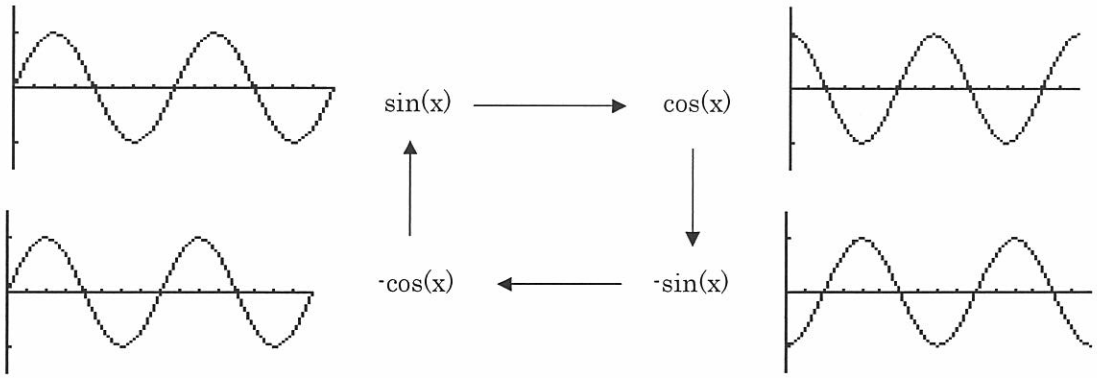
x →0 then (sin(x)/t)/(x/t) →1

Then we get the formula: the differentiation oh sin(x) is cos(x).

PQ = (sin(x+ Δ x)- sin(x))/(Δ x)

= 2*cos(x+ Δ x/2)*sin(Δ x/2)/ Δ →cos(x) (Δ x/2→0).

In this time it is important to get the estimating of the differentiation oh sin(x) is cos(x). When we have the estimating the result of calculating, we can understand how to show them. In my class, teachers give all the knowledge about the mathematics to students. We have the concrete experience for the abstract mathematics. We can get the formula of the differentiation of the trigonometry function differentiation for the walking with CBL.



4. Reference

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