

MAKING ALGEBRA MEANINGFUL WITH TECHNOLOGY

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I begin by observing that this is my thirtieth year as a teacher of mathematics. Soon after I began my teaching career, personal computers also made their introduction to the classroom. I remember being responsible for the purchase of an Apple IIe computer back in 1980, intended to bring my school rapidly into the new “computer age”. It is interesting to look back over that time and, in particular, to ponder what we have learned from both classroom research and the wisdom of practice concerning the use of technology as an aid to learning.

From my perspective, as classroom teacher, researcher and academic, it is possible to make some fairly well-supported and sensible statements at this point in time concerning good teaching and learning, the teaching and learning of mathematics, and of algebra in particular. It is then possible to relate these to the appropriate and effective use of technology for the learning of algebra in a meaningful way.

1. Students learn best when they are actively engaged in constructing meaning about content that is relevant, worthwhile, integrated and connected to their world.
2. Students learn **mathematics** best when
 - ☐ They are active participants in their learning, not passive spectators;
 - ☐ They learn mathematics as integrated and meaningful, not disjoint and arbitrary;
 - ☐ They learn mathematics within the context of challenging and interesting applications.
3. Students learn **algebra** best when
 - ☐ It is not presented as meaningless symbols following arbitrary rules;
 - ☐ The understanding of algebra is based upon concrete foundations, with opportunities for manipulation and visualisation;
 - ☐ Algebra is presented as a vital tool for modeling real-world applications.

And the role of technology in the process?

Technology in mathematics and science learning plays two major roles:

- As a tool for REPRESENTATION, and
- As a tool for SCAFFOLDING of MANIPULATIONS.

Good technology supports students in building skills and concepts by offering multiple pathways for viewing and for approaching worthwhile tasks, and scaffolds them appropriately throughout the learning process.

Research over the past thirty years points to some clear steps in the process of learning algebra effectively, and the possibilities of new technologies point to some new steps with great potential to assist in bringing meaning to the learning.

1. **Begin with Number:** Just as algebra is, most purely, a generalization of the rules by which we operate with numbers, the path to algebra logically grows from students' knowledge and understanding of numbers and their operations. Number patterns, in particular, offer a perfect "jumping off" point by which students may be actively engaged in studying these rules and operations, and tables of values provide a powerful tool for exploring and conjecturing. The simple "guess my rule" games which teachers have used for many years may go well beyond just building simple patterns. They may also be used to introduce the symbolic notation of algebra in a practical and meaningful way.

From simple linear functions such as $y = 2x + 1$ students can be challenged to find the rules for variations on the same theme (what about $y = x + x + 1$? $y = 3x + 2 - x - 1$?) – Yes, that rule is correct but it is not what I have – how else could the rule be written?

Then on to factors, such as $2(2x + 1)$ – stressing the careful use of appropriate language: multiplication is always "lots of" – 3×4 is 3 "lots of" 4 and $2(2x + 1)$ is 2 lots of $2x + 1$!

The screenshot shows a software window titled "RAD AUTO REAL". It contains a table with two columns, "A x_val" and "B y_val", and a formula bar showing "=f(x_val)". The table has five rows of data. To the right of the table is a text box with the text "1. Begin with number to study patterns and to begin formal notation." and a "Define f(x)=x+x+1" button. Below the text box is a "Done" button.

A x_val	B y_val
0	1
1	3
2	5
3	7
4	9

2. The second "golden rule" from my own teaching experience and also well-grounded in classroom research concerns the appropriate use of **concrete materials** to provide a firm foundation for the symbolic forms and procedures of high school algebra. "Area models" provide a powerful and robust means for students to interact with symbolic forms in ways both tactile, meaningful and transferable.

The screenshot shows a software window titled "RAD AUTO REAL". It contains two area models. The first model shows a large rectangle divided into two parts, with dimensions labeled "x" and "one". The second model shows a large rectangle divided into four parts, with dimensions labeled "x" and "one". Below the area models is a table with three rows of data. To the right of the table is a "Define x=3" button. Below the button is a "Done" button.

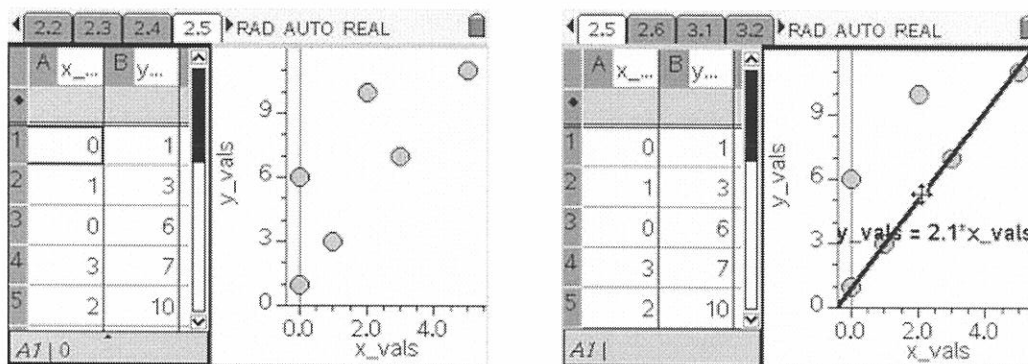
Define x=3	
$2 \cdot x + 1$	7
"What about" $\cdot 2 \cdot (2 \cdot x + 1)$	14

Two major limitations may be identified with the use of such concrete materials in this context: there is no direct link between the concrete model and the symbolic form, other than that drawn by the teacher – students working with cardboard squares and rectangles must be reminded regularly what these represent.

Of even greater concern, these concrete models promote a static rather than dynamic understanding of the variable concept. Both these limitations may be countered by the use of appropriate technology to scaffold and support the tactile forms of these models.

These basic shapes may be readily extended to model negative values (color some of the shapes differently and then these “cancel” out their counterparts) and even to quadratics, using x^2 shapes! After even a brief exposure, students will never again confuse $2x$ with x^2 since they are clearly different shapes.

3. The introduction of the **graphical representation** is too often rushed and much is assumed on the part of the students. Like the rest of algebra, the origins of graphs should lie firmly in number.



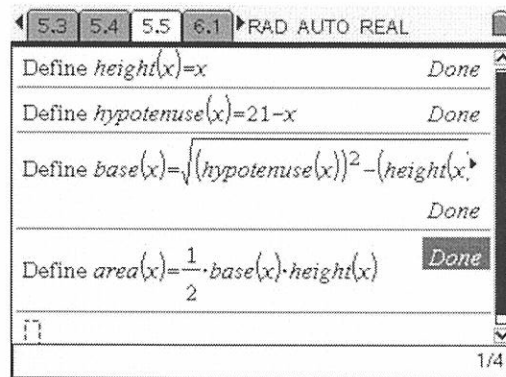
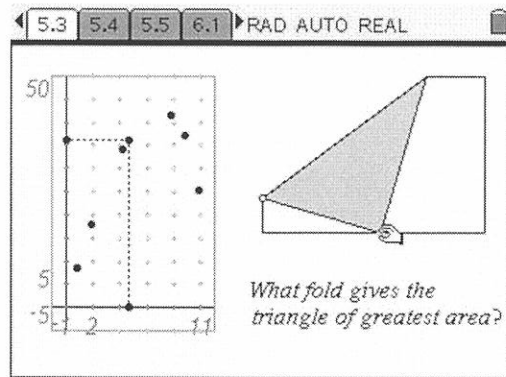
The use of scatter plots of number patterns and numerical data should precede the more usual continuous line graphs, which we use to represent functions. Such conceptual “objects” have little meaning for students, in the same way that symbolic “objects” (such as “ $2x + 1$ ”) need to be conceptually expanded to include more diverse ways of thinking.

We now have tools which make it easy for students to manipulate scatter plots and so further build understanding of the relationship between table of values and graphical representation. Only then should we encourage the use of the more formal “straight line” representation.

4. Once we have built firm numerical foundations for symbol and graph, our students are ready to begin to *use* algebra – perhaps a novel idea in current classrooms! The real power of algebra lies in its use as a tool for **modeling** the real world (and, in fact, all possible worlds!) research is clear that students in the middle years of schooling (which is when we introduce algebra) most strongly need their mathematics to be relevant and significant to their lives. Teaching algebra from a modeling perspective most clearly exemplifies that approach, and serves to bring together the symbols, numbers and graphs that they have begun to use.

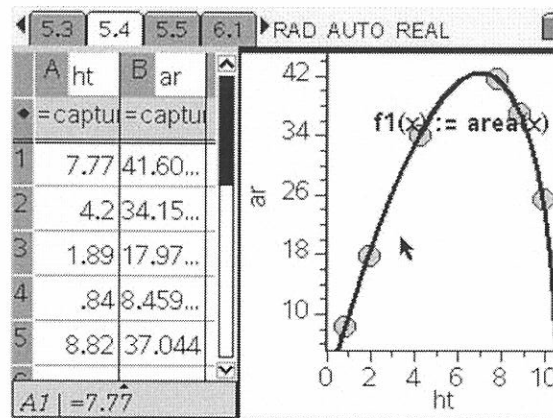
Opportunities for algebraic modeling abound, especially around such topics as Pythagoras' Theorem. The simple paper folding activity shown - in which the top left corner of a sheet of A4 paper is folded down to meet the opposite side, forming a triangle in the bottom left corner - is a great example of a task which begins with measurement, involves some data collection and leads to the building of an algebraic model. Students measure the base and height of their triangles, use these to calculate the area of the triangle, and then put their data into lists, which can then be plotted.

They may then begin to build their algebraic model, but using appropriate technology, may use real language to scaffold this process and develop a meaningful algebraic structure, as shown.



Returning to the graphical representation, students may now plot the graph of their function, $area(x)$, and see how it passes through each of their measured data points - convincing proof that their model is correct - and usually a dramatic classroom moment!

This is powerful, meaningful use of algebraic symbolism. The building of purposeful algebraic structures using real language supports students in making sense of what they are doing, and validates the algebraic expressions which they can then go on to produce. Able students should still be expected to compute the algebraic forms required and perhaps validate them using a variety of means.



5. This use of **real language** for the definition of functions and variables has previously only existed on CAS (computer algebra software) and even there only rarely been used. The new TI-Nspire is a numeric platform (non-CAS) and so allowable in all exams supporting graphic calculators, but it supports this use of real language.

Of course, it is wonderful to have CAS facilities when they are needed. Using CAS we can actually display the function in its symbolic form, and then compute derivative and exact solution, arriving at the theoretical solution to this problem. The best fold occurs when the height of the fold is 7 cm, exactly one third of the width of the page. Using non-CAS tools, this same result may be found using the numerical function maximum command, or by using numeric derivative and numeric solve commands.

6. Scaffolding is an important aspect of meaningful algebra learning, and computer algebra offers some powerful opportunities for such support. The real challenge in using CAS for teaching and learning, however, lies in finding ways to NOT let the tool do all the work!

Innovative use of CAS may include taking advantage of the algebraic capabilities of the *Lists & Spreadsheet* application, or writing programs which offer model solutions – but which stop short of giving the final result. Certainly these tools may readily provide automated solutions to extended algebraic processes, but there seems to me to be greater value in having the students do some or all of the work, and having the tool check and verify this work.

Such applications of these powerful tools remain yet to be explored.

Conclusion

Why do I like to use technology in my Mathematics teaching?

Because, like life, mathematics was never meant to be a spectator sport.

The top screenshot shows a TI-84 Plus calculator interface with the Lists & Spreadsheet application. The table has columns A, B, and C. Row 1: u(x) = 2*x. Row 2: v(x) = x^2-4. Row 3: du = 2, with a checkmark in column C. Row 4: dv = 2*x, with a checkmark in column C. Row 5: d(u*v) = u*dv+v*du. Row 6: A1, with the text 'u(x) ='. The bottom screenshot shows the same interface with the formula editor open, displaying the product rule formula:
$$d(u*v) = \frac{d}{dx}(u(x)) \cdot v(x) + \frac{d}{dx}(v(x)) \cdot u(x)$$
 Below the formula, it shows
$$u(x) = 2 \cdot x$$
$$d(u) = 2$$
$$v(x) = x^2 - 4$$
$$d(v) = 2 \cdot x$$
 The bottom right corner shows 1/1.