

BIOCALCULUS COMPUTER LABORATORY PROJECTS: EXCEL AND MAPLE

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Introduction

Reports including Bio 2010: Transforming Undergraduate Education for Future Research Biologists [2] and Math and Bio 2010 [4] emphasize that aspects of biological research are becoming more quantitative and that there is a need to introduce future life science researchers to a greater array of mathematical and computational techniques and more sophisticated mathematical reasoning. Moreover, one of the themes discussed at the Biology CRAFTY Curriculum Foundations Project was that “Creating and analyzing computer simulations of biological systems provides a link between biological understanding and mathematical theory,” [1], and the Bio 2010 report asserts the importance for biologists to be able to use computers as tool: “Computer use is a fact of life of all modern life scientists. Exposure during the early years of their undergraduate careers will help life science students use current computer methods and learn how to exploit emerging computer technologies as they arise,” [2].

Creative solutions can be employed to achieve the desired integration of mathematical, computational, and biological content without radically changing major requirements or requiring additional credit hours of course work. Additionally, presenting quantitative approaches to biological problems to all biology majors, not just those who intend to pursue research careers, in their introductory college mathematics courses provides these students with a wider range of tools and can better motivate the mathematics. This paper focuses on one example an activity that is used in the Calculus with Analytics I Laboratory at Benedictine University. The students enrolled in this course are concurrently enrolled in Calculus with Analytics I or Biocalculus I. Other sample activities are available on the author’s web site:

<http://www.ben.edu/faculty/tcomar/>

Curve Fitting, Nonlinear Scales, Allometry

First semester calculus courses often begin with a brief review of functions. Valuable connections can be made between mathematics and biology right from the beginning. Allometric relationships, which are scaling relationships between different features of an organism, provide simple examples of power functions. Fitting a curve represented

by a power function provides a contextual way to review the precalculus topics of power functions, exponential functions, and logarithmic functions to introduce the concept of nonlinear scales. One such example we study in a lab activity is the allometric relationship between the height H and diameter D of a tree:

$$H = aD^b, \quad (1)$$

where a is the scaling coefficient, and b is the scaling exponent. Students are given a data set of measurements of tree heights and diameters that were collected as part of an activity in a botany course. The students are then asked to find the best-fitting power equation given by Equation (1) for the data set. To accomplish this, the students must transform the power Equation (1) into the linear Equation relating $\ln(H)$ and $\ln(D)$:

$$\ln(H) = \ln(a) + b \ln(D) \quad (2)$$

so that they can use linear regression to find the fit.

We now demonstrate how this allometric fit can be performed in Maple with the data stored in an Excel file.

1. We will first assume that the data set has been stored in an Excel file called TreeData.xls. The a sample data set is given in Table 1. This data set was collected by Dr. Larry Kamin of Benedictine University and his Biology 204 Advanced Botany students [3]. In the Excel data table, include the column headings in Row 1 and place the diameter measurements in Column A and the height measurements in Column B. Highlight the entire table, including the column headings and name the region "TreeData".
2. Open up a new Maple worksheet in Worksheet Mode. Load the Statistics package using the command:

$$\text{with(Statistics):}$$
3. We now import the data into the Maple worksheet as follows:

- (a) Click on Tools > Assistants > ImportData... .
- (b) Locate the file TreeHD.xls and select the Excel format. Click Next.
- (c) Click on the Named Range pulldown menu and select TreeData. Click Done.
- (d) Move the cursor to the beginning of the *ExcelTools:Import...* command line and enter:

$TD:=$

to the left of *ExcelTools:Import...* Now TD is an array.

Table 1: Tree Diameters and Heights [3]

Diameter (in.)	Height (ft.)	Diameter (in.)	Height (ft.)
3.18310	19.0800	5.72958	16.5262
35.01409	77.5800	3.50141	17.0920
14.32395	40.5500	2.38732	14.6883
20.05352	49.3000	25.78310	80.1883
9.84771	26.9000	10.66338	30.1883
3.18310	26.4147	21.64507	61.1883
18.93944	61.7480	2.87116	22.1883
27.53381	73.9143	13.05071	22.4042
11.85704	56.9143	25.14330	77.7505
35.33240	46.0813	10.50423	35.5892
13.05071	32.6351	14.00564	32.3651
7.63944	10.4588	23.23344	68.3731
10.50423	18.9630		

- (e) By looking in the Excel file, you will notice that the first column of the array provides the diameter measurements, and the second column provides the height measurements.
4. We now proceed to find a log-log regression line for the data set. That is we must find modify the data set by computing the natural logarithms of both the inputs (diameters) and the outputs (heights), and then fit a line to the modified data set. The line you find is a linear relationship between $\ln(H)$ and $\ln(D)$. We transform the data with the following commands:

$$\text{logd} := [\text{seq}(\log(TD[i, 1])), i = 1 \dots 25]; \text{logh} := [\text{seq}(\log(TD[i, 2])), i = 1 \dots 25];$$

The first of these commands transforms the list of tree diameters into a list, logd , of the logarithms of the tree diameters. The second command creates the list, logh , of the logarithms of the corresponding tree heights. Now create a list of points which we can plot in the $\ln(H)$ $\ln(D)$ -plane:

$$\text{logTD} := [\text{seq}([\text{logd}[i], \text{logh}[i]], i = 1 \dots 25)];$$

5. The line that you find is a linear relationship between $\ln(H)$ and $\ln(D)$:

$$\ln(H) = \ln(a) + b \ln(D).$$

The linear fit is obtained via the command:

```
lineq := Fit(b*x+c, logd, logh, x);
```

Maple returns:

```
lineq := .598379842657030924 * x + 2.09372547898997530
```

6. We plot the transformed data and the regression line with the command:

```
plot([logTD, lineq], x = 0 .. 4, style = [point, line], symbol = circle, labels = [log(diameter), log(height)]);
```

This plot is shown in Figure 1.

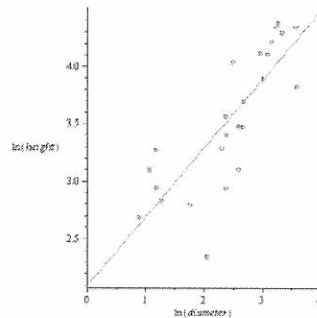


Figure 1: The log-log plot of the data set and the regression line for log-log data

7. We then exponentiate to find the power curve that fits the original data set. We accomplish this with the following command:

```
allom := x -> x^.598379842657030924 * exp(2.09372547898997530)
```

8. We can then plot the this power curve and the original data set using the following command:

```
plot([TDplot, allom], 0 .. 40, style = [point, line], symbol = circle, labels = ["height (ft.)", "diameter (in.)"]);
```

This plot is given in Figure 2.

Students also perform a similar exercise fitting an exponentially growing data set. From these activities, students are exposed to observing data presented with nonlinear scales: if a data set comparing two quantities appears to be linear on a log-log plot, the relationship between the quantities is a power relationship; if a data set comparing two quantities appears to be linear on a semi-log plot, the relationship is

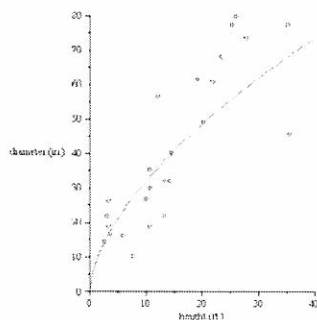


Figure 2: The original data set and power curve fit

exponential. An additional benefit for including this activity is that the concept of nonlinear scales (as well as allometry) is new to most students in a first semester calculus, even to those who had a previous calculus course in high school. This activity would also be appropriate in a precalculus course when exponential and logarithmic functions are introduced.

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