A JAVA SIMULATOR FOR VOTING METHODS

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In recent years, election theory has become part of the mathematics curriculum at many colleges and universities, from small parts of introductory liberal arts courses to complete advanced courses for both mathematics and non-mathematics majors. One source of frustration for many students in these courses is the tedious arithmetic that often accompanies the voting methods, which can prevent some students from appreciating the breadth of real-life applications of the methods. In this paper, I will describe a Java simulator for voting methods that automates the calculations required for most of the methods that are usually presented in introductory liberal arts courses, including the plurality, Borda count, single and instant runoff, Coombs', sequential pairwise, and pairwise comparison methods. In addition to automating the calculations required for these methods, this simulator also identifies majority winners and Condorcet winners and losers, whenever they exist. The simulator was written by one of the author's students, Mr. Brian Onstot, as part of his senior honors thesis in mathematics, and has been used by the author several times in his own introductory liberal arts mathematics courses.

A Brief Introduction to Election Theory

Although the topic itself may sound simple, there is a lot more to voting theory than appears on the surface. This is especially true for elections in which there are more than two candidates and we allow for the possibility that voters can rank all of the candidates according to their own personal preference. For example, consider the voter preferences in the following table from the 1998 gubernatorial election in the state of Minnesota. The candidates in this election were St. Paul Mayor Norm Coleman, Minnesota Attorney General Skip Humphrey, and former professional wrestler and radio shock-jock Jesse "The Body" Ventura.

Percentage of Voters	**35%**	**28%**	**20%**	**17%**
1st choice	Coleman	Humphrey	Ventura	Ventura
2nd choice	Humphrey	Coleman	Coleman	Humphrey
3rd choice	Ventura	Ventura	Humphrey	Coleman

Table 1: Voter preferences in the 1998 gubernatorial election in Minnesota

Of course, the data in Table 1 is based on exit polling. Voters in Minnesota were not actually asked to rank all three candidates as indicated in the table, but rather just to vote for the candidate that they wanted to win (although among political analysts, the data in

the table is generally regarded as very accurately representing the true opinions of the voters). The winner of this election was determined using the **plurality method**, which *declares the winner of an election to be the candidate who receives the largest number of first-place votes*. Thus, Ventura claimed a (stunning) victory with 37% of the vote, followed by Coleman with 35% of the vote, and Humphrey with 28% of the vote. With the additional information revealed in the voter preferences in Table 1, we can see just how remarkable this outcome really was. While it is true that Ventura was the first-place choice of the largest number of voters, albeit just barely, it is also true that Ventura was the last-place choice of the largest number of voters as well, and for this the "victory" was a landslide! This simple little example truly demonstrates just how accurate it is that there is a lot more to voting theory than appears on the surface.

Voting Methods Automated By the Simulator

The easiest and most common method for determining the winner of an election is the plurality method, which we described in the previous section. While this method does have some very serious problems (like the problem that we described in the previous section), it is used in some notable types of elections. For instance, the governor of each of the 50 states is determined using the plurality method, as is the presidential candidate who receives all of the electoral votes in 48 of the 50 states. The reason that the plurality method, despite its shortcomings, is used in such important types of elections is because it is the only method that really makes sense if the voters are just going to be asked to vote for the candidate that they want to win, and it is also the easiest method for quickly determining the winner of an election. However, speed is frequently not a factor in determining the winner of an election. For example, the producers of the FOX television program *American Idol* want to drag their election (of the winner of *American Idol*) out for many weeks. As such, they use a method called the **instant runoff method**, in which *a series of runoff elections are held, with the candidate who receives the smallest number of first-place votes eliminated each time*. Rather than actually holding a series of runoff elections, the instant runoff method can also be used with the voters voting just once, provided that the voters each submit a ranking of all of the candidates in the election from first through last, like the voter preferences shown in Table 1 for the 1998 gubernatorial election in Minnesota. If the instant runoff method had been used in this election, then Humphrey would have been eliminated first since he had the smallest percentage of the first-place votes. We could then remove Humphrey from the table of voter preferences, and move the remaining candidates up into the vacated positions in the table. This would give Coleman an extra 28% of the first-place votes in the second runoff election, and cause Ventura to be eliminated second. Thus, Coleman would have been the winner of this election if the instant runoff method had been used to determine the winner.

Several variations of the instant runoff method exist. One such variation is **Coombs' method**, which is identical to the instant runoff method except that *the candidate who receives the largest number of last-place votes is eliminated in each runoff election.* Coombs' method is used on the CBS television program *Survivor*, with the added variation that the voters are also the candidates. Also, it is worth noting that if Coombs'

method had been used in the 1998 gubernatorial election in Minnesota, it would have been Ventura who was eliminated first instead of Humphrey. Coleman would still have been the overall winner of the election under Coombs' method though, just as he would have been under the instant runoff method.

Another variation of the instant runoff method is the **single runoff method**, in which *the two candidates who receive the largest and second-largest number of first-place votes compete in a single runoff election.* The single runoff method is used in the election of the President of France, with the voters going back to the polls a few weeks after the initial election to vote for their preferred candidate between the two who received the largest and second-largest number of votes in the initial election.

Yet another variation of the instant runoff method is the **sequential pairwise method**, in which *a series of pairwise (i.e., head-to-head) runoff elections are held, with the candidate who is ranked higher on fewer ballots than the other eliminated each time, and new candidates introduced into the sequence of pairwise runoff elections according to some pre-determined list (called the* agenda*) of all of the candidates in the election.* In the sequential pairwise method, it is clearly beneficial to a candidate to be introduced into the sequence of pairwise runoff elections as late as possible, for then to win the overall election (i.e., to win the final pairwise runoff election) the candidate would have to win a smaller number of pairwise runoff elections than another candidate who was introduced into the sequence of pairwise runoff elections earlier. However, the idea of treating some candidates this way (unfairly in their favor) is perfectly legitimate in some situations. The whole basis of seeded tournaments like the NFL playoffs and NCAA men's and women's basketball tournaments is to reward teams for exceptional regular seasons by giving them an easier path to the championship than teams who did not perform as well during the regular season.

A variation of the sequential pairwise method that removes the inherent unfairness in the method is the **pairwise comparison method**, in which *every possible pair of candidates compete in a pairwise runoff election, with the winner being the candidate who is ranked higher on more ballots than the other, and the overall winner of the election being the candidate with the largest number of pairwise runoff election wins.* The pairwise comparison method is used to determine winners and final rankings of the competitors in many figure skating competitions, including Olympic and World Championships, under the name *OBO* method, which stands for One-By-One.

The final voting method that is automated by the simulator is the **Borda count method**, in which *points are assigned to each candidate for each ballot cast: 1 point for the lowest ranked candidate, 2 points for the next lowest ranked candidate, 3 points for the third lowest ranked candidate, etc., and the winner of the election being the candidate with the largest overall point total.* The Borda count method is used in many sports rankings, including the Associated Press and ESPN/*USA Today* college football and basketball polls, as well as to determine the winners of a variety of awards in sports, including the Heisman Memorial Trophy and the Most Valuable Player in each of Major League

Baseball's American and National Leagues. If the Borda count method had been used in the 1998 gubernatorial election in Minnesota, then treating the percentages of voters in Table 1 as actual numbers of voters, Coleman would have won the election with $(35 \times 3) + (48 \times 2) + (17 \times 1) = 218$ points, compared to $(28 \times 3) + (52 \times 2) + (20 \times 1) = 208$ points for Humphrey and $(37 \times 3) + (63 \times 1) = 174$ points for Ventura.

Fairness Criteria Checked By the Simulator

There is a rich assortment of criteria that election theorists use to judge the fairness of particular voting methods, including such criteria as monotonicity, citizen sovereignty, and independence of irrelevant alternatives. We will restrict our attention to three criteria that are checked by the simulator.

The most basic fairness criterion in election theory is the **majority criterion**, which states that *if a candidate in an election is a majority winner (i.e., if the candidate receives more than half of the first-place votes), then that candidate should be the overall winner of the election*. This criterion seems reasonable; if more than half of the voters in an election favor a particular candidate over each of his or her opponents, then it would only seem proper for that candidate to be the winner of the election. However, of the voting methods described in the previous section, Coombs' and the Borda count methods can both violate the majority criterion. As an example of the Borda count method violating the majority criterion, consider the following table, which shows the 1971 Preseason Associated Press Top 20 college football poll, in which there were a total of 50 voters.

Team	Borda Points	First-Place Votes
1. Notre Dame	885	15
2. Nebraska	870	26
3. Texas	662	5
4. Michigan	593	1
5. Southern Cal	525	1
6. Auburn	434	1
⋮	⋮	*(all 0)*
20. Northwestern	58	1

Table 2: 1971 Preseason Associated Press Top 20 College Football Poll

The poll shown in Table 2 illustrates a violation of the majority criterion because Nebraska, despite receiving more than half of the first-place votes (26 out of 50), does not win (i.e., finish first in) the poll. Another election that illustrates a violation of the majority criterion is the U.S. presidential election of 1876, in which Rutherford B. Hayes received more than half of the popular vote, but lost the election to Samuel Tilden.

Another basic fairness criterion in election theory is the **Condorcet winner criterion**, which states that *if a candidate in an election is a Condorcet winner (i.e., if the candidate*

would win a pairwise runoff election against each of his or her opponents), then that candidate should be the overall winner of the election. As an example of the plurality method violating the Condorcet winner criterion, consider the voter preferences shown in Table 1 for the 1998 gubernatorial election in Minnesota. In this election, Coleman would have won a pairwise runoff election against both Humphrey (55% to 45%) and Ventura (63% to 37%), but Coleman was not the winner of the election under plurality. Another election that is generally accepted among election theorists as illustrating a violation of the Condorcet winner criterion is the U.S. presidential election of 2000, in which Al Gore would have almost certainly won a pairwise runoff election against each of his opponents, but lost the election to George W. Bush.

The 1998 gubernatorial election in Minnesota also illustrates a violation of yet another basic fairness criterion in election theory, the **Condorcet loser criterion**, which states that *if a candidate in an election is a Condorcet loser (i.e., if the candidate would lose a pairwise runoff election against each of his or her opponents), then that candidate should not be the overall winner of the election.* In the 1998 gubernatorial election in Minnesota, Ventura would have lost a pairwise runoff election against both Coleman and Humphrey (both 63% to 37%), but Ventura was the winner of the election under plurality.

The Simulator

The Java simulator for voting methods that was demonstrated by the author at the 2008 ICTCM was written by one of the author's students, Mr. Brian Onstot, a 2007 graduate of Appalachian State University, as part of his senior honors thesis in mathematics. For the sake of space, screen shots of this simulator cannot be included in this paper. The simulator begins by giving the user the option of entering a list of candidates and complete voter preferences for an election, or loading a list of candidates and complete voter preferences that had been entered and saved by the user previously. Then by utilizing dropdown menus, the user can cause the simulator to determine and display the winner and final ranking of the candidates in the election using each of the voting methods that are described in this paper (plurality, Borda count, single and instant runoff, Coombs', sequential pairwise, and pairwise comparison). In addition to automating the calculations required for each of these methods, the simulator also identifies and displays majority winners and Condorcet winners and losers, whenever they exist. This simulator is platform-independent, and is available along with all source code and instructions upon email request of the author. Also available upon email request of the author is a copy of the senior honors thesis written by Mr. Onstot. Much of the material in this thesis as well as the examples that are presented in this paper are discussed in more detail in a book that was co-written by the author and published by the AMS in 2005 ([1]).

Reference

[1] Hodge, J. K. and Klima, R. E. (2005). *The Mathematics of Voting and Elections: A Hands-On Approach*. Providence, RI: American Mathematical Society.