LINEAR PROGRAMMING VIA *EXCEL*

J Villanueva
Florida Memorial University
Miami, FL
jvillanu@fmuniv.edu

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I.   Linear programming problems and their solutions

We often encounter optimization problems in our business courses or in courses in general mathematics. In business, we may try to optimize profit given production performance from several different facilities. In engineering, we may try to optimize labor force to meet forecast demands. We may try to meet design problems to produce optimum output at minimum cost or minimum load. Optimization problems are characterized by maximizing or minimizing an output, called an *objective function,* subject to restrictive conditions, called *constraints.* The constraints are often expressed as linear inequalities, thus, the problem is called *linear programming.* When the constraints are nonlinear, then the problem becomes *dynamic programming,* and we have to resort to other methods of solution. When the problem is two-dimensional, then the natural method of solution is geometric, by graphing the *feasible region* and evaluating the objective function at the vertices, which yields the maximum or the minimum value. When the problem is of higher dimension than two, we prefer to use matrices, by the *simplex method,* developed by George Dantzig (1914-2005) during World War II. The advantage of the simplex method is that it could also solve the two-dimensional case. More recently, solutions of optimization problems are more and more being carried out by computers, especially by *Excel.*

Excel, traditionally, has been known to address routine problems in business. But it is really a very powerful tool to solve a variety of problems in business, biology, chemistry, to the very complicated problems in physics and engineering structures. Its ready availability makes it even more attractive. In linear programming, Excel has the built-in feature *Solver* that can

solve systems of linear equations, solve nonlinear equations, and do least-squares curve fitting, among others. Solver may also be used to solve nonlinear programming. It is a very capable feature, but it is not the only option. Excel also has *VBA* (Visual Basic for Applications) which may be used to solve linear programming problems and other nontraditional algorithms. This paper will provide examples on the use of Excel to solve typical linear programming problems, using Solver. The use of VBA to solve nontraditional linear programming problems we will mention in the planned future work.

II. Examples
A. Two variables
(1) Our first example is a two-dimensional product mix problem that maximizes profits, adapted from Bourg [2007, p. 360]. We use this example, because *Excel* also has graphing routines that readily display the feasible region. We have (*Figures 1-3*):
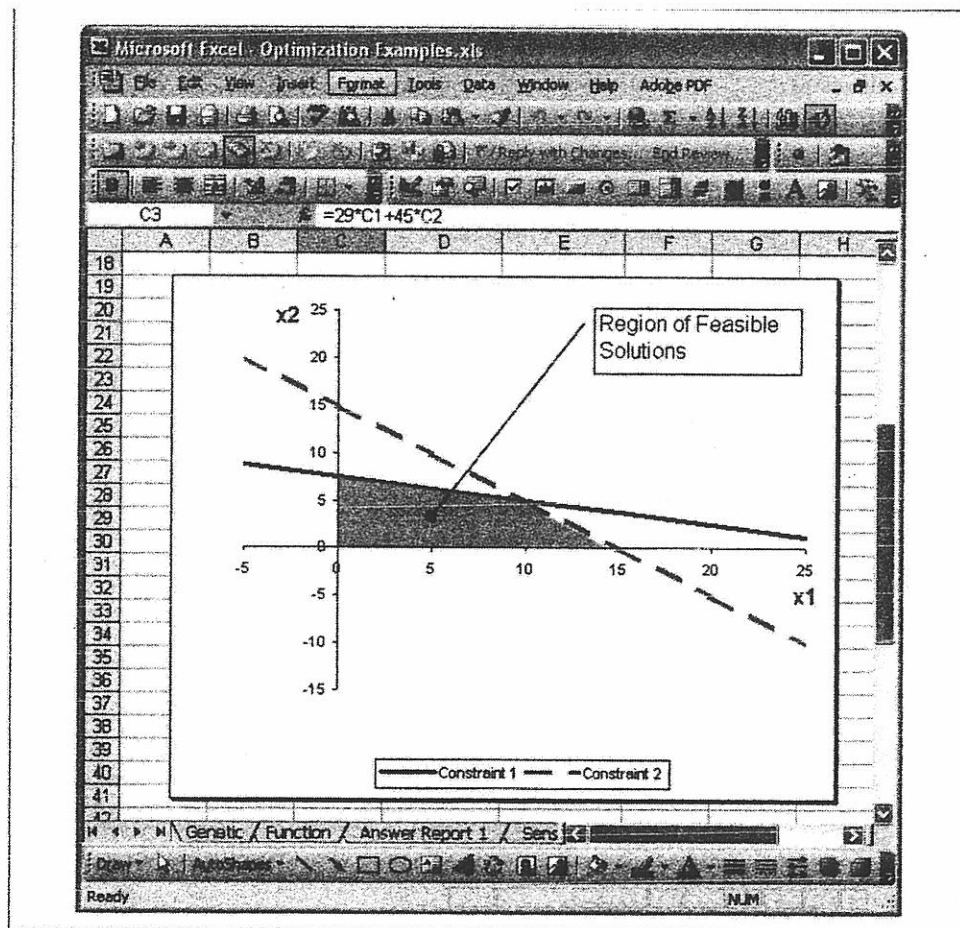

Figure 1. Graph of linear optimization example

Objective function: (maximize)	Vertex	$f(x_1, x_2)$
$f(x_1, x_2) = 29x_1 + 45x_2$	(0,0)	0
	(15,0)	435

Constraints:

$$x_1 \geq 0, \quad x_2 \geq 0$$

$$2x_1 + 8x_2 \leq 60$$

$$4x_1 + 4x_2 \leq 60.$$

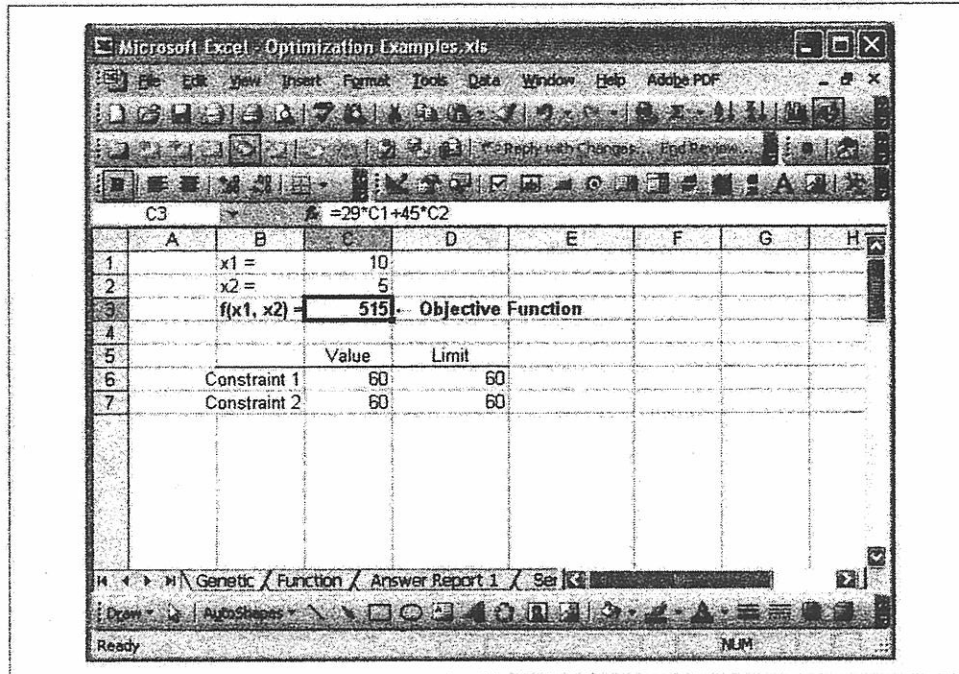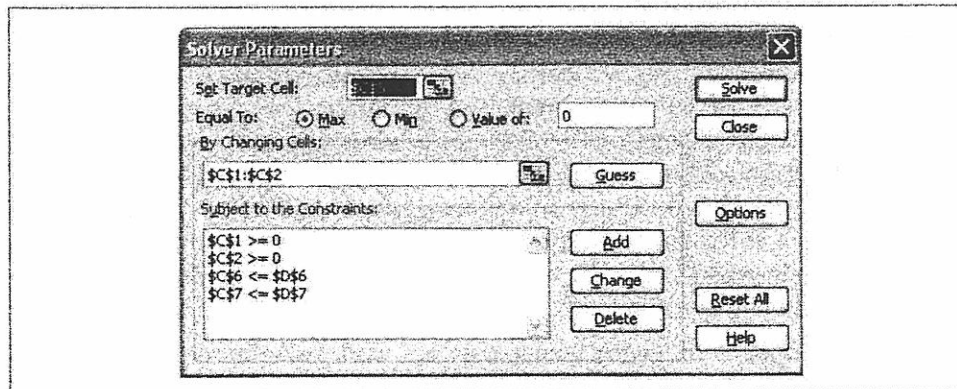|	
(10,5)	515*
(0,10)	450

Figure 2. Linear optimization example

Figure 3. Solver model for linear optimization

(2) The next example is a nutrient mix problem, requiring certain amounts of nutrients, at minimum cost for the food sources, adapted from Lial & Hungerford [1999, p343]. Laboratory animals must have at least 30 grams of protein and 20 grams of fat per feeding. These nutrients come from foods A and B. Each unit of A costs 18¢ and supplies 2 grams of protein and 4 of fat; each unit of B costs 12¢ and supplies 6 grams of protein and 2 of fat.

Under a long-term contract, at least 2 grams of B must be used per feeding. How many units of A and B must be used per feeding so that cost will be minimized while satisfying the nutrient requirements of the lab animals? (*Figure 4*)

Food	Number of Units	Grams of Protein	Grams of Fat	Cost
A	x	2	4	18¢
B	y	6	2	12¢
Minimum required		30	20	

FIGURE 4
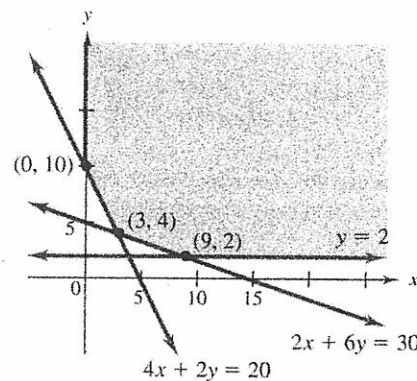
Objective function: (minimize)

$$f(x_1, x_2) = 0.18x_1 + 0.12x_2$$

Constraints:

$$x_1 \geq 0, \quad x_2 \geq 2$$
$$2x_1 + 6x_2 \geq 30$$
$$4x_1 + 2x_2 \geq 20.$$

Vertex	$f(x_1, x_2)$
(0,10)	1.20
(3,4)	1.02*
(9,2)	1.86

Thus, 3 units of A and 4 units of B will produce a minimum cost of $1.02 per serving while satisfying the dietary requirements of the lab animals.

B. More than two variables

(3) Transportation problem, adapted from Goldstein, Schneider, & Siegel [1998, p. 128]. A Maryland TV dealer has stores in Annapolis and Rockville with warehouses in College Park and Baltimore. Annapolis orders 25 TV sets, Rockville orders 30. Shipping costs are: $6 per set from College Park to Annapolis, $3 from College Park to Rockville, $9 from Baltimore to Annapolis, $5 from Baltimore to Rockville. College Park keeps a stock of 45 sets, Baltimore 40. What is the most economical way to supply the requested TV sets? (*Figure 5*)

We picked this problem because we can reduce it to two dimensions, and thus, we can apply the geometric solution again.

(a) The 4 variables in this problem:

x_1 = number of sets from CP to Rockville

x_2 = sets from CP to Annapolis

x_3 = sets from Baltimore to Rockville

x_4 = sets from Baltimore to Annapolis

can be reduced to 2:

$$x_3 = 30 - x_1$$
$$x_4 = 25 - x_2$$

since Rockville ordered 30, and Annapolis 25.

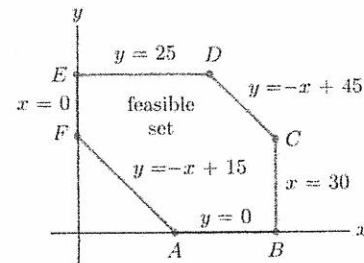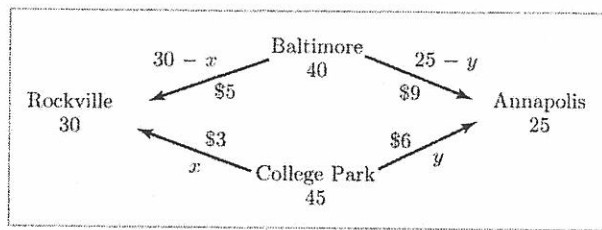x_4 = sets from Baltimore to Annapolis   and Annapolis 25.

FIGURE 5

Objective function: (minimize)

	Vertex	$f(x_1, x_2)$
$f(x_1, x_2) = 3x_1 + 6x_2 + 5(30 - x_1) + 9(25 - x_2)$	(0,25)	300
$\qquad = 375 - 2x_1 - 3x_2$	(0,15)	330
Constraints:	(15,0)	345
$\qquad x_1 \geq 0, \ x_2 \geq 0$	(30,0)	315
$\qquad x_1 \leq 30, \ x_2 \leq 25$	(30,15)	270
$\qquad x_1 + x_2 \leq 45$	(20,25)	260*
$\qquad (30 - x_1) + (25 - x_2) \leq 40 \ $ or $ \ x_1 + x_2 \geq 15.$		

(b) Using *Excel,* we can employ all 4 variables at once.

Objective function: (minimize)
$$f(x_1, x_2, x_3, x_4) = 3x_1 + 6x_2 + 5x_3 + 9x_4$$

Constraints:
$$x_1 \geq 0, \ x_2 \geq 0, x_3 \geq 0, \ x_4 \geq 0$$
$$x_1 + x_2 \leq 45$$
$$x_3 + x_4 \leq 20$$
$$x_1 + x_2 = 30$$
$$x_3 + x_4 = 25.$$

Solutions are: $x_1 = 20, \ x_2 = 25, \ x_3 = 10, \ x_4 = 0; \ f = \$260.$

(4)   Resource allocation., adapted from Barnett, Ziegler, and Byleen [1999, p.321]. A company manufactures 3-speed, 5-speed, and 10-speed bicycles that require hours of fabrication, painting & plating, and final assembly, as shown in the table. How many bicycles of each type should the company manufacture per day to maximize its profit?

Objective function: (maximize)

$$f(x_1, x_2, x_3) = 80x_1 + 70x_2 + 100x_3$$

Constraints:

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0$$
$$3x_1 + 4x_2 + 5x_3 \leq 120$$
$$5x_1 + 3x_2 + 5x_3 \leq 130$$
$$4x_1 + 3x_2 + 5x_3 \leq 120.$$

Solutions are: $x_1 = 10, \quad x_2 = 10, \quad x_3 = 10; \quad f = \$2,500.$

(5) Resource allocation., adapted from Barnett, Ziegler, and Byleen [1999, p.339]. A school district has two high schools that are overcrowded and two that are underenrolled. To balance the enrollment, the school board decided to bus the students from the overcrowded to the underenrolled schools. North High School has 300 more students than it should have, and South High has 500 more students. Central High can accommodate 400 more students, and Delta High can accommodate 500 additional students. The weekly cost of busing a student from North to Central is $5, from North to Delta is $2, from South to Central is $3, and from South to Delta is $4. Determine the number of students that must be bused from each of the overcrowded schools to each of the underenrolled schools to balance the enrollment and minimize the cost of busing?

Objective function: (minimize)

$$f(x_1, x_2, x_3, x_4) = 5x_1 + 2x_2 + 3x_3 + 4x_4$$

Constraints:

$$x_1 \geq 0, \quad x_2 \geq 0, x_3 \geq 0, \quad x_4 \geq 0$$

$$x_1 + x_2 \geq 300$$
$$x_3 + x_4 \geq 500$$
$$x_1 + x_3 \leq 400$$
$$x_2 + x_4 \leq 500..$$

where: x_1 = number of students bused from North to Central

x_2 = bused from North to Delta

x_3 = bused from South to Central

x_4 = bused from South to Delta.

Solutions are: $x_1 = 0, \quad x_2 = 300, \quad x_3 = 400, \quad x_4 = 100; \quad f = \$2,200.$

(6) We will just mention an example of a problem in 6 variables [Bourg, p. 363]. This is significant in that it seems that *Excel* can solve a linear programming problem to any number of independent variables. The answers are output instantaneously. However, there is another significant feature to this example. If the answers required are in integer format, not decimal numbers, as in the case here for the problem asks for the number of regular and part-time employees that must be hired or fired, *Solver* can take several hours to find a solution.

III. Suggested future work

We have demonstrated that *Excel,* through its tool *Solver,* is a very convenient and precise method of solving linear programming problems for any number of dimensions. When the constraints, or the objective function, is nonlinear, or not readily convergent, then it would be preferable to use another approach, *Visual Basic for Applications (VBA).* VBA allows the solution of dynamic programming problems by successive generations of solutions, called genetic algorithms. This is made possible through the ease in writing subroutines in VBA. We plan to continue the solution of nonlinear optimizations in the near future.

IV. Conclusion

Optimization problems in business and the sciences can be readily solved by *Excel* using its tool *Solver.* This works best when the problem is linear. For the case of nonlinear programming, or the solution is not readily convergent, *Excel* has another tool, *VBA,* which promises to be adaptable to such problems.

References:

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