MULTIVARIABLE CALCULUS VIRTUAL OFFICE HOURS IN A METAVERSE

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Vectors, Lines, and Planes in Calculus

At Old Dominion University, Calculus II is a 4-credit hour course covering a range of topics including techniques of integration, polar curves, and infinite series. The final part of the course introduces three-dimensional analytic geometry: three-dimensional Cartesian coordinate system, vector operations (addition, scalar multiplication, dot product, cross product), lines, planes, cylinders, and quadric surfaces. Many students find it difficult to deal with objects in three-dimensional space. To a large extent, these difficulties can be blamed on the inherent limitations of the (two-dimensional) media used in a typical class: blackboard, textbook, and a notebook. On the other hand, the world we live in **is** three-dimensional, therefore, one possible approach would be to present 3D concepts directly in the 3D world, bypassing the 2D media altogether. There are many instances where this can be done in a very simple manner, e.g.

- Facing the students, ask them to think of the classroom floor as the xy-plane, the wall behind you as the yz-plane, and the wall on your right as the xz-plane. (This is mentioned in some calculus books, including [11]). This can help you convey the notions of coordinates, octants, plane equations, etc.
- A number of ways exist to physically illustrate the right-hand rule for the xyz system orientation (e.g., curl the fingers of the right hand).
- The author often uses color whiteboard markers as representations of vectors in space: each marker can be thought of as a unit vector, and by snapping them together (which most of them do easily), you get longer vectors. This can help a lot with concepts like orthogonality, cross products, etc.
- As suggested by Baumslag [2], cross-sections of a three-dimensional surface (or solid) can be cleverly visualized by using a number of cardboard cutouts held in place on a pin passing through them.

As simple (or even simplistic) as some of these activities may seem, it is truly amazing how effective they can be in your classroom, when students who may otherwise be puzzled by the "abstract" three-dimensional concepts presented on the board suddenly realize these are not abstract at all!

Unfortunately, using real world to illustrate the ideas has, correspondingly, very real limitations – for instance, it would not be cost-effective to remodel the walls of your classroom to demonstrate some specific surface, or to use a hundred markers to represent a vector field. To overcome these limitations, we have been developing class demonstrations and hands-on laboratory activities using computer algebra systems Maple and Mathcad [4]. These systems allow for three-dimensional objects to be represented on the two-dimensional computer screen, with an added flexibility to dynamically alter the viewpoint (zoom in/out, rotate, etc.)

For example, in one of our lab activities, students would be asked to determine an equation of the plane passing through the given three points in R^3. The students are asked to carry out all calculations with paper and pencil, and are provided with a Mathcad document that offers them an opportunity to visually verify the validity of their answer: a three-dimensional plot in the document initially depicts the three given points and an arbitrary (incorrect) plane. By changing the equation to the correct one, they can see the resulting plane pass through the given points, regardless of the different viewpoint location. (Similar activities were designed for problems such as finding the point of intersection of two lines in space, or finding the line of intersection of two planes.)

Students have generally reacted to these activities favorably. In a student survey the author conducted in his recent Calculus II section, he asked the students to assess how helpful they found each of the lab assignments conducted throughout the course. Among the 18 students who responded (about half of the class), the "Vectors, Lines, and Planes" lab received the highest notes: 15 students (83%) said it "helped a lot", two (11%) said it "helped somewhat" and one (6%) said it "did not help". (The average responses for all remaining labs were 71%, 20%, and 8%, respectively.)

Since we have begun developing the CAS-based modules in early 1990's, a number of virtual environments became available, allowing for visualizing 3D objects that appear far more realistic than a 2D rendering on a computer screen. In this project, we seek to pilot using such technology when teaching vectors, lines, and planes in calculus. Of the many available virtual environment platforms, Second Life [9] was chosen for implementation of this project. There exist other virtual environments offering similar functionality, including the Croquet Consortium [7], Active Worlds [1], and There [12]. A number of technologies allow the user to become "immersed" in the virtual world by employing specialized (and often very expensive) hardware [3]. However, Second Life appears to offer a good combination of features and accessibility.

Second Life

Some authors (e.g., [6]) refer to Second Life as a massively multiplayer online role-playing game (MMORPG). This is somewhat misleading, since, unlike a typical game, Second Life has no clearly defined objective and no "score" is being kept. Terms such as a multi-user virtual environment (MUVE) [13] or virtual world metaverse [10] appear to describe Second Life more adequately.

One of the most striking features of Second Life (which truly differentiates it from a game) is that there is very little pre-existing content in this virtual world. There is bare virtual land surrounded by virtual bodies of water, and there is a small number of objects created by Linden Labs – a private company that developed Second Life. It is the Second Life residents (term used to refer to those that in a game would be called players) that create the bulk of the objects including buildings, furniture, vehicles, plants, and clothing for the avatars that represent them in the virtual world. A scripting language called LSL can be used to make these objects active (e.g., by writing a script that controls the building's front door, the door can be made to slide when an avatar is passing through it, or could be used to control which avatars are allowed in) – elaborate interactions between avatars and objects, or among the objects can be implemented. (Objects created by residents can then be purchased by other residents with the virtual currency called Linden dollar; this gives rise to a huge marketplace, which is another distinguishing feature of Second Life.)

Over last several years, a number of educators have experimented with using virtual worlds, and Second Life in particular, in their classes. However, very few of those are mathematics classes [5].

This is surprising, given that mathematics plays a crucial role throughout Second Life. The basic building blocks used to construct anything in SL are primitives – or "prims" – such as boxes, cylinders, prisms, tori, etc. To define properties of such objects, such as

- shape (type of solid, size),
- two-dimensional textures mapped onto the solids' faces,
- position in space (translation, rotation),
- dynamic motion (e.g., an object may contain a script that defines the manner in which it physically interacts with the user or with other objects)

requires knowledge of a number of areas of mathematics including logic, trigonometry, solid and analytic geometry, vectors, matrices. This relationship can also be reciprocated: virtual worlds can serve as a dynamic, three-dimensional canvas for learning mathematics. It would only seem natural to exploit the potential synergy there: using the context to help simultaneously learn and motivate the learning of mathematics. Why, then, does this potential remain largely untapped?

The author does not see an easy answer to that – here are just two possible explanations:

- Some of the environments (e.g., Croquet) have a great deal of potential in education (and have, in fact been used by some for that purpose [8]), but remain at the early stages of development that requires substantial commitment of time and effort of the prospective adopters.
- Given that there are many competing platforms, one often hesitates to make that kind of substantial investment betting on a single one, only to see it become overtaken by the competition.

Virtual Office Hours

The author has become a Second Life resident in 2006 and has experimented with this environment in the subsequent months. One of his ongoing Second Life projects has been the "Vectors, Lines, and Planes display" (or VLP) learning object. It allows the user to display points, line segments, lines, vectors, and planes in three-dimensional Cartesian coordinate system by issuing simple commands, e.g. "v 3 7 2" displays the vector <3,7,2>.

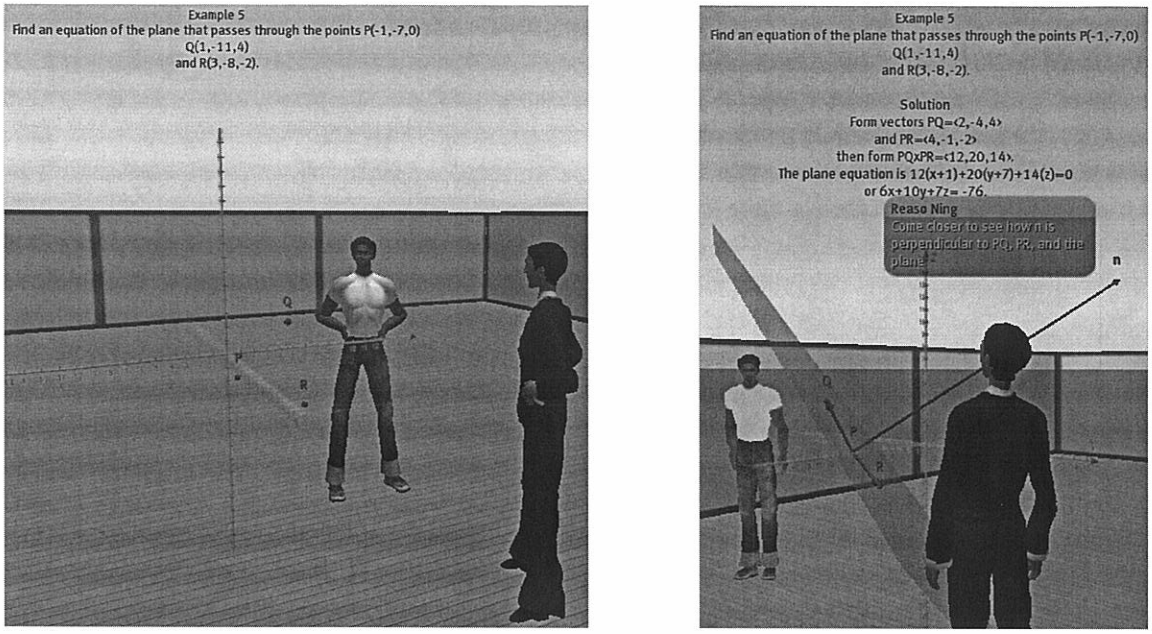

***Figure 1: Virtual Office Hour**. A student comes to the author's virtual office asking for help with determining an equation of a plane passing through given three points. The author (whose SL name is Reaso Ning) uses the VLP display to present a relevant example, and asks the student to move his avatar to fully appreciate the orthogonality of the cross product PQxPR to both PQ and PR.*

In Fall 2007, the author has opened his virtual "office" in Second Life, which hosted the VLP display and enabled him to conduct

- in-class demonstrations and
- virtual office hours.

While the demos have been quite effective, it was the virtual office hours that enabled the students (via their avatars) to become "immersed" in the three-dimensional virtual world. Given the experimental nature of this undertaking, the author chose not to require his students to participate, instead encouraging them to do so by awarding them a few extra credit test points. Several students visited the virtual office – during those visits, they would typically ask the author for some help with solving a problem such as "How do I find an equation of a plane passing through the given line and the given point?". Any shortcomings in communicating mathematical expression (the Second Life chat operates strictly in plain text mode) were more than offset by the benefits offered by the three-dimensional visualization. Each step of the solution could be progressively illustrated: e.g.: "here is a direction vector of the given line", "here is a vector connecting a point on the line to the given point", "here is a cross product of these". Using the Second Life terminology, the VLP display is set up as a phantom object – therefore, avatars can walk through the object, allowing for a more complete immersivity.

Future Plans

The author has become involved in a project that seeks to harness some of the vast potential offered by the virtual environments to enhance the learning and teaching of collegiate mathematics courses. The activities described in this article represent early stages of the project. Here are a few of the many prospective developments planned for the future:

- further develop the VLP display learning object to release it to the general public,
- produce video demonstrations involving the VLP display, and make them available on the author's website,
- investigate the prospects for using Second Life in a linear algebra course (to illustrate concepts such as spanning sets, linear transformations, eigenvalues, SVD, etc.),
- continue to examine the potential offered by virtual environments other than Second Life.

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