Incorporating GeoGebra into Teaching Mathematics at the College Level

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Abstract

The open source software GeoGebra provides an easy way to create interactive and dynamic web pages. In this article we explain how this is done and present first experiences of the use of this new tool for teaching mathematics at the college level.

Introduction

GeoGebra is dynamic mathematics software for teaching and learning mathematics from middle school through college level (see Hohenwarter & Preiner, 2007). It combines the ease-of-use of dynamic geometry software with some basic features of computer algebra systems. Although primarily focused on secondary school curriculums, GeoGebra is also an interesting tool for college level courses as it can help to bridge concepts from geometry, algebra, and calculus. The software is open source under GNU General Public License and freely available for multiple platforms (Windows, Mac OS X, Linux) from www.geogebra.org.

> Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances student's learning. (NCTM, 2000)

GeoGebra was created to help students gain a better understanding of mathematics. You can use it for active and problem-oriented teaching and to foster mathematical experiments and discoveries both in classroom and at home. The software may be used both as a learning and as a teaching tool. On the one hand, students can create constructions from scratch on their own. By doing so, they have the opportunity to solve problems by creating mathematical models and investigating mathematical relations dynamically. On the other hand, the software makes it very easy to create interactive and dynamic online materials for demonstrations or *dynamic worksheets*.

Creating Teaching Materials

A *dynamic GeoGebra worksheet* is an interactive web page (html file) that consists of a dynamic figure (interactive applet) with corresponding explanations, questions, and tasks for students. The dynamic figure is constructed in GeoGebra and can easily be exported to an html file. Students can use this dynamic worksheet both on local computers or via the Internet to work on the given tasks by modifying the dynamic figure.

Example: The worksheet Shape of a Slope Function allows students to find the shape of the slope function of a cubic polynomial by interactively modifying a dynamic figure. We will now show you how to create this worksheet step by step.

Creating the Dynamic Figure

First, you need to construct the dynamic figure including the cubic polynomial *f*, point *P* and the tangent line *t* as well as a special point *M* whose *y*-coordinate equals the slope of the tangent line *t*. To do this, you can use the interactive applet below that represents the user interface of GeoGebra but includes a customized tool bar to simplify the construction process. It enables you to do the construction and export it as a web page. Since this setting should only provide an environment to gain first experiences with the use of GeoGebra while reading this article, we recommend to install GeoGebra and to have a look at the GeoGebra Quickstart document for additional information.

1. Type `a = -0.5` into the input field at the bottom of the GeoGebra window and hit the Enter-key. Right click (Mac OS: ctrl-click) number *a* in the algebra window and select "Show object" to create a slider for this parameter. Repeat this for `b = 2`, `c = 0` and `d = -1`. Tip: You can also create the sliders directly by using the "Slider" tool from the toolbar.
2. Type `f(x) = a x^3 + b x^2 + c x + d` into the input field and hit the Enter-key.
3. Select the tool "New point" in the tool bar. Move the mouse over the graph of function *f* and click on it when it is highlighted to get a point on the graph. Right click on this new point (Mac OS: ctrl-click) and choose "Rename". Change the name of the point to *P* and click "Apply".
   Tip: Check if point *P* really lies on the graph of function *f* by selecting the tool "Move" and dragging point *P* along the graph.

4. Select the tool "Tangents" and successively click on point *P* and the graph of function *f*. Rename the tangent to *t*.
5. Type the command `m = Slope[t]` into the input field and hit the Enter-key.
6. Create point *M* by typing `M = (x(P), m)` into the input line.

Tip: *x(P)* gives you the *x*-coordinate of point *P*.

Exporting the Dynamic Worksheet

After creating the dynamic figure you are now ready to export it as an html-page. When you export a construction from GeoGebra both the graphics window and the algebra window will be shown on the dynamic worksheet. However, you can hide the algebra window using the "View" menu if you don't need it. To save space for text on screen you should decrease the size of GeoGebra's application window by dragging its lower right corner before you proceed. If you are using the applet above, you don't need to worry about this.

1. In the menu bar select "File" - "Export" - "Dynamic Worksheet as Web page (html)...".
2. Fill in the text fields of the appearing export dialog (Title; Author; Date).
3. Add some text that should be displayed above the construction in the corresponding text field, for example: "Below you can see the graph of a cubic polynomial *f(x)*."
4. In the last text field you can provide tasks for your students.
5. Click the "Export"-button and save your dynamic worksheet.
   Tip: We recommend to export your worksheet into a new folder as GeoGebra will create several files during the export. Do not use any space or special characters in your filename to prevent problems.
6. To access your dynamic worksheet open the html-file in a web browser.
   Note: Internet Explorer may restrict interactive html pages opened from your local computer. In this case you need to click on a yellow bar at the top of your browser window and approve a security question to use the dynamic worksheet. However, this problem does not occur when you upload your dynamic worksheet to the Internet.

Deploying your Dynamic Worksheet

GeoGebra creates several files when you export a dynamic worksheet:

- The files with the extension ".jar" contain the interactive GeoGebra applet.
- The file with the extension ".html" represents the actual dynamic worksheet and can be opened with any web browser.
- The file with the extension "_worksheet.ggb" contains the dynamic figure which is displayed in the interactive GeoGebra applet.

To give your dynamic worksheet to your students you need to keep all of these files together in one folder (e.g. copy them to a CD, USB drive or web server). If you don't have web space, you can use the GeoGebra Upload Manager to publish your dynamic worksheets on the Internet. If you do so, we would also like to invite you to provide a link to your worksheet in the GeoGebraWiki. In order to use dynamic worksheets your students need a web browser and Java 1.4.2 or later (free download from www.java.com) on their computers. Note, that they don't need to have GeoGebra installed.

For more information on creating dynamic worksheets with GeoGebra, please have a look at the "Help" section on www.geogebra.org.

First Experiences

Markus Hohenwarter and Judith Preiner, FAU

During the past year, GeoGebra has become an integral part of the NSF math and science partnership project Standards Mapped Graduate Education and Mentoring of Florida Atlantic University and the Broward County School District. In this project, in-service middle school teachers can earn a master's degree in science teaching by attending a series of evening classes at the university. One part of their assignments is to create dynamic worksheets for their own teaching. Some of these worksheets are available on GeoGebraWiki, a collaborative pool of teaching materials.

Our experience with the teachers participating in our project shows that the creation of dynamic worksheets helps them to increase their computer literacy and confidence concerning the use of technology in their classrooms. While creating their own instructional materials they reflect upon their usual teaching style which allows them to get a new pointof view on the contents that they are teaching. By using their own materials in their classroom they experience that they are not necessarily restricted to methods and materials provided by the textbooks. Many of the teachers are now starting to use the method of guided discovery learning in their dynamic worksheets which encourage active learning and give their students the chance to discover some mathematical concepts on their own. Additionally the kids like to work with online materials their teacher created specifically for them. This can increase their motivation to solve the given tasks and to deal with mathematics.

Taeil Yi, University of Texas at Brownsville

One of the features in GeoGebra is 'Navigation bar for construction steps' which is an important tool to export constructions as interactive web pages. It can be used for a teaching tool for class teaching. In a typical traditional classroom setting teachers deliver their lectures by hand writings and drawings on the board. There are two main difficulties for this. First, these writings and drawings on the board are not consistent among teachers, and usually are not accurate. Second, they cannot be changed unless the teacher draws another one, and sometimes it is difficult for the student to see the connection between them.

Using 'Navigation bar' teachers can give better and accurate drawings and explanations. Even after the drawing and/or construction the teacher is able to change the shape of the drawing by changing parameters or moving points. This gives a much better insight and understanding of the subject to the students. Let's consider the use of 'Navigation bar' in some courses.

In Calculus 1 course we use GeoGebra for the following lectures:

- Definition of the graph of a function,
- Drawing graphs of exponential, logarithmic, and trigonometric functions,
- Graph transformations & shifting,
- Inverse functions,
- Definition of the limit of a function at a point,
- Idea of derivative,
- Graphical relation between derivative and anti-derivative functions, and
- Riemann sum.

In Geometry 1 course we use GeoGebra for

- Graphical examples before checking the rigorous proof of theorems, and
- Step by step construction of geometric figures.

The following examples show the usage of 'Navigation bar' for an exponential function in Calculus 1 and the construction of the perpendicular bisector in Geometry.

- Exponential function
- Perpendicular Bisector

Ben Ford, Sonoma State University

The following e-mails show a discussion on the NExT mailinglist where Ben Ford, Sonoma State University, uses GeoGebra to answer the following calculus related question.

Question:

> I am trying to come up with a good Polar Coordinate demonstration for my Calculus class, and I am trying to use Maple to get it done.
> We are starting to look at the different shapes that can arise from some of the basic polar coordinate equations. For example,
> r = cos(3 * theta), 0 <= theta <= 2*Pi
> I am trying to come up with an animation in maple that will trace out the equation, as opposed to just popping the entire thing up at once. I have tried a web search, as well as perused the books in my office, but I have found no help.
> Does anyone know of a way to do this in Maple? If so, I would appreciate any insights.

Answer by Ben Ford:

> Don't know how to do it in Maple, but I just put up an applet that does this.
> On Monday, I saw a presentation at the University of Nebraska-Lincoln by the creator of GeoGebra (www.geogebra.org), and it took me about 20 minutes to figure out how to do polar coords in this program and create the page.
> http://www.sonoma.edu/users/f/fordb/polarcoordstrace.html
>
> The program is open source and available for Mac, Windows, and Linux.
> The geogebra file I used to create the applet is at
> http://www.sonoma.edu/users/f/fordb/polarcoordstrace.ggb

Tom Hull, Merrimack College

The following e-mail from April 2007 describes the experiences of Thomas Hull, Merrimack College, when using GeoGebra in his geometry course.

> I've been teaching our upper-level geometry course at Merrimack College for a number of years now, and usually I'd use Geometer's Sketchpad. That worked out OK, but this past Fall I switched to GeoGebra and I was much happier with it. One of the big factors was that it's free, so there's nothing stopping students from putting it on their own machines and for our IT people from installing it on any campus machine I want. But even better, there is a java-based version of GeoGebra which can be launched from the GeoGebra web page, http://www.geogebra.org (provided that your computer has a recent version of Java installed). So you don't even need to install GeoGebra.
>
> But my students loved GeoGebra because of its dual algebra-geometry nature. They really liked being able to make lines, circles, and points and then be able to see the coordinates of those points and the equations of the circles and lines. They liked being able to type in the formula for a function and see it graphed, like a supped-up graphing calculator, in addition to manipulating geometric objects with the mouse. It made advanced geometry seem very "grounded" in what they had learned before in courses like calculus. It allowed me to flexibly discuss different approaches one can take to geometry (axiomatic, coordinate/vector-based, transformational, and projective) in hands-on (or perhaps "mouse-on"?) ways.