The Topics of Taylor Series with TI-89

—Math Education is Interesting—

Shin Watanabe
Tokai University
watanabe@scc.u-tokai.ac.jp

Today's problem using Taylor Series

We find out the functions which are the solution of the differential equation $y^{(n)} = y$.
There are functions which are number of n.

Theorem　There are n functions when the function is same of the n times differential

Before we use the basic theorem of Algebra
n-dim algebraic equation has n solutions

In this time we use the n-vector space has n vectors with the basis
the basis is the function

1. The one time differential function is same

$f(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + \cdots$

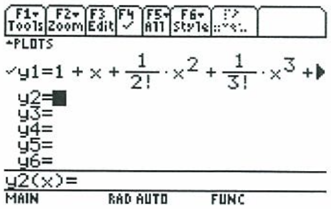
↓　differential (we use only one differencal formula

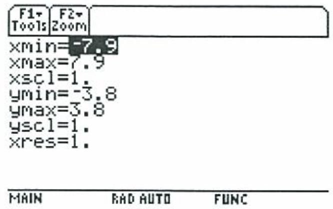
$f(x) = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + 5a_5x^4 + \cdots$
$= a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + \cdots$

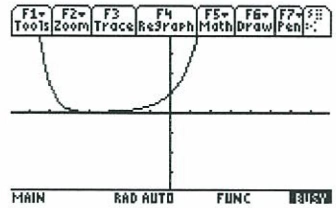
so $a_0 = a_1$
$a_1 = 2a_2$
$a_2 = 3a_3$
$a_3 = 4a_4$
$a_4 = 5a_5$
………

if $a_0=1$, then $a_1 =1$
$a_2 = 1/2a_1 = 1/2$
$a_3 = 1/3a_2 = 1/3*1/2a_1 = 1/3!$
$a_4 = 1/4a_3 = 1/4*1/3a_2 = 1/4*1/3*1/2a_1 = 1/4!$
$a_5 = 1/5a = 1/5*1/4a_3 = 1/5*1/4*1/3a_2 = 1/5!$
…………

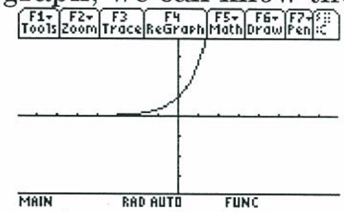
We get the function f(x)= $1+1x+1/2x^2+1/3!x^3+1/4!x^4+1/5!x^5+1/6!x^6+\cdots$,we draw this graph with TI-89

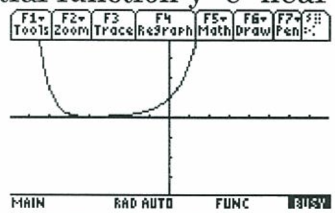






We see this graph, we can know the exponential function $y=e^x$ near x=0.





the graph of $y=e^x$ f(x)= $1+x+1/2x^2+1/3!x^3+1/4!x^4+1/5!x^5+1/6!x^6$

If we use the differential equation $y^{(1)}=y$, then we get the solution $y=e^x$.

Using the method of the linear, D= $y^{(1)}$

so Dy=y

then D=1 (here is the linear equation, this solution is only one)

we get $y=e^{1x}$

2. The two times differential function

We know this functions are $y=e^x$ and $y=e^{-x}$.

Now we use the function f(x) = $1+1x+1/2x^2+1/3!x^3+1/4!x^4+1/5!x^5+1/6!x^6+\cdots$

We break two parts with modulo 2

$Y1=1+1/2x^2+1/4!x^4+1/6!x^6+\cdots$

$Y2=x+1/3!x^3+1/5!x^5+1/7!x^7+\cdots$

If we difference Y1 two times, then we get same Y1.

$Y1=1+1/2x^2+1/4!x^4+1/6!x^6+\cdots$

↓ differential (we use only one differencal formula

$=1x+1/3!x^3+1/5!x^5+1/7!x^7+\cdots$

↓ differential (we use only one differencal formula

$=1+1/2x^2+1/4!x^4+1/6!x^6+\cdots$

=Y1

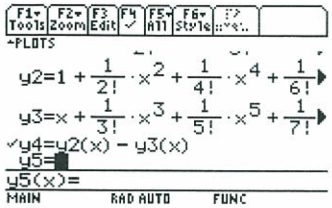
Y2 is same

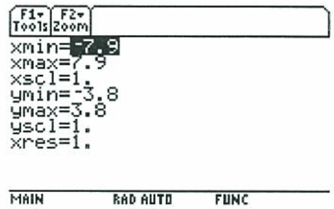
These two functions, Y1 and Y2 are 2 basis.
We make the functions from Y1 and Y2.

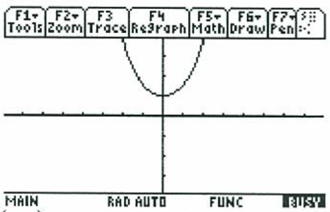
$f(x) = Y1+Y2 = e^{x}$

$g(x) = Y1-Y2 = e^{-x}$

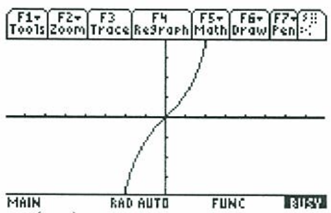
We get functions, we draw this graph with TI-89



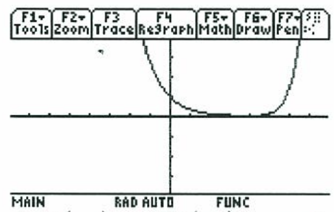




Y!(x)=1 + 1/2! $x^2 \cdots$

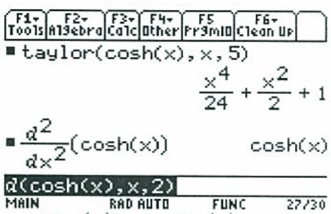


Y2(x)=x+ 1/3! $x^3 \cdots\cdots$ composite

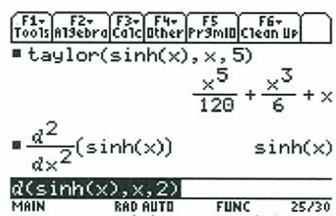


Y1(x)−Y2(x)= e^{-x}

note : We know that Y1 = cosh x $=(e^{x}+e^{-x})/2$ and Y2= sinh x $=(e^{x}-e^{-x})/2$
y=cosh(x) and y=sinh(x) are famous functions.



Y1(x)=cosh(x)



Y2(x)=sinh(x)

If we use the differential equation $y^{(2)}=y$, then we get the solution $y=e^{x}$ and $y=e^{-x}$.

Using the method of the linear, D= $y^{(1)}$
so $D^2y=y$
then $D^2=1$ (here is the 2-dim equation, solutions are two values)
then D=1 or -1
we get $y=e^{1x}$ and $y=e^{-1x}$

3. The three times differential function
We do not know this function, so we use modulo 3.
Now we use the function $f(x) = 1 +1x + 1/2x^2 +1/3!x^3 + 1/4!x^4+ 1/5!x^5+ 1/6!x^6+\cdots$
We break three parts with modulo 3

$Y1= 1 \quad + 1/3!x^3 \quad + 1/6!x^6 \quad +1/9!x^9 +\cdots\cdots$

$Y2= \quad x \quad + 1/4!x^4 \quad + 1/7!x^7 \quad + 1/10!x^{10}+ \cdots$

$Y3= \quad 1/2!x^2 \quad + 1/5!x^5 \quad + 1/8!x^8 \quad + 1/11!x^{11} + \cdots\cdots$

If we difference Y1 three times, then we get same Y1.

$Y1= \quad 1+ \quad 1/3!x^3 + 1/6!x^6+ 1/9!x^9 + 1/12!x^{12} +\cdots\cdots$

↓ differential (we use only one differencal formula

$= 1/2!x^2 + 1/5!x^5+ 1/8!x^8 + 1/11!x^{11} +\cdots\cdots$

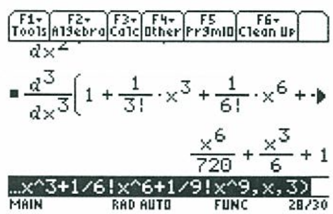
↓ differential (we use only one differencal formula

$=1x \ + 1/4!x^4+ 1/7!x^7+\cdots$

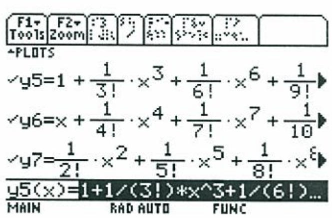
↓ differential (we use only one differencal formula

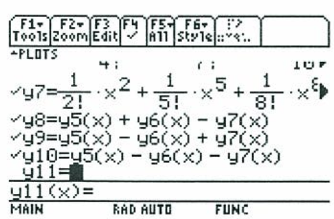
$=1+1/3!x^3 \ + 1/6!x^6+ \ 1/9!x^9+\cdots$

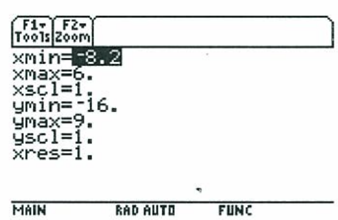
=Y1

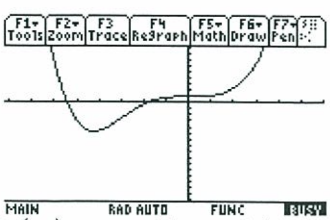


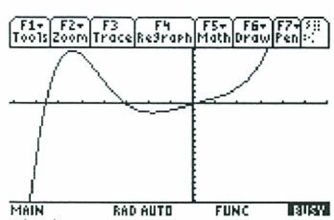
Y1 difference three times with TI-89

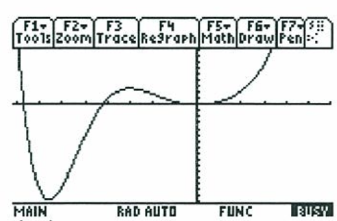




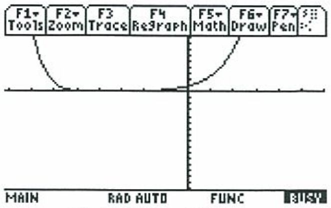



$Y1(x)=1 +1/3!x^3+1/6!x^6+\cdots$

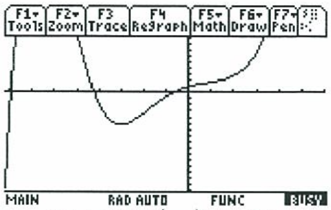

$Y2(x)=x +1/4!x^4+1/7!x^7+\cdots$


$Y3(x)=1/2!x^2+1/5!x^5+1/8!x^8\cdots$

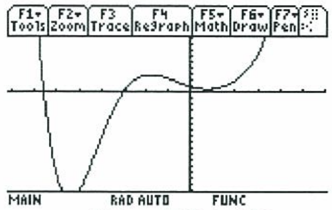
We composite with Y1, Y2 and Y3.



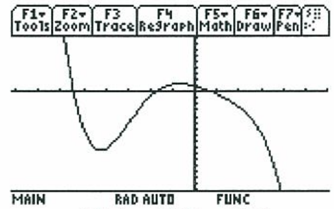
$Y1+Y2+Y3=e^x$



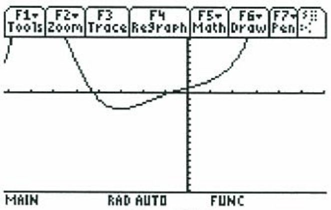
$Y1+Y2(x)-Y3$



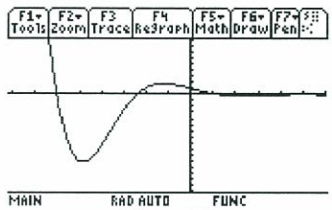
$Y1-Y2+Y3$



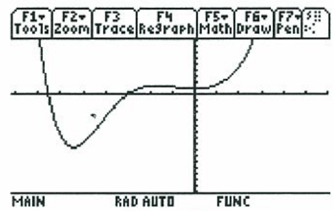
$Y1-Y2-Y3$



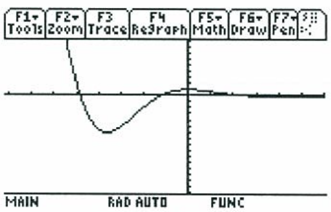
$Y1+Y2$



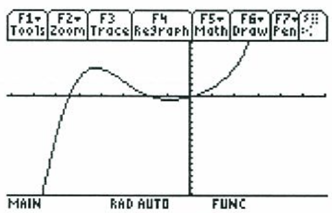
$Y1-Y2$



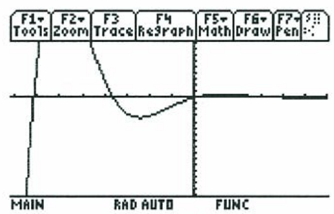
$Y1+Y3$



$Y1-Y3$

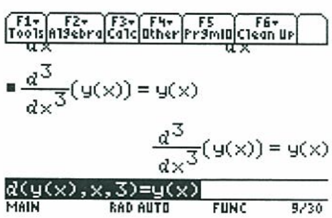


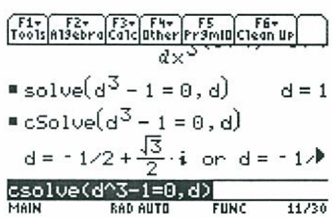
$Y2+Y3$

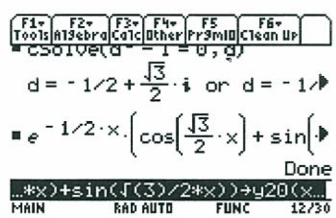


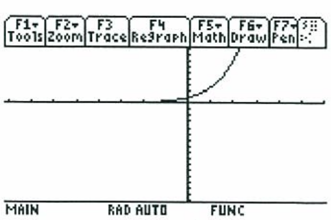
$Y2-Y3$

We use the differential equation $y^{(3)}=y$. Using TI-89 CAS.



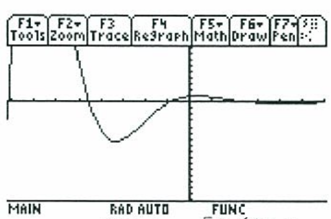






$y=e^{1x}$

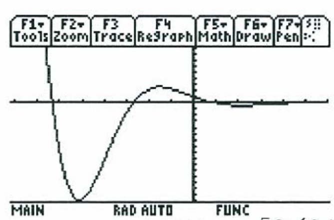
$=Y1+Y2+Y3$



$y=e^{-1/2x} * e^{\sqrt{3}/2\, i\, x}$

$=e^{-1/2x} * (\cos\sqrt{3}/2\, x+\sin\sqrt{3}/2\, x)$

$=(Y1-1/2Y2-1/2Y3)+\sqrt{3}/2(Y2-Y3)$



$y=e^{-1/2x} * e^{-\sqrt{3}/2\, i\, x}$

$=e^{-1/2x} * (\cos\sqrt{3}/2\, x-\sin\sqrt{3}/2\, x)$