MULTIVARIABLE CALCULUS PROJECTS

Shari Prevost
Department of Mathematical Sciences
Clemson University
Clemson, SC 29634-0975
sprevos@clemson.edu

The third semester of calculus is a wonderful course to teach. I have spoken to many instructors who jump at the chance to teach a class in multivariable calculus. The consensus among these instructors is that this material is a lot of fun to teach.

Unfortunately, many of our students do not share their instructors' appreciation of multivariable calculus. I think this stems from the students' inability to visualize many of the concepts they encounter in the course. To help my students gain a better geometric understanding of the calculations they come across, I created a collection of Maple-based multivariable calculus projects designed to (1) enhance their understanding of the material, (2) encourage students to use Maple to verify their calculations visually and analytically, (3) require students to write [about] mathematics, and (4) give experience using a computer algebra system.

The typical student in a third semester calculus course at Clemson University is a sophomore engineering student. This student usually carries a heavy course load, has a laptop, and has never used Maple. The third semester calculus course currently meets four times a week for 50 minutes a day and has a syllabus that is packed. As a result, there is little time to spend in class on "project" activities. Furthermore, any outside assignment cannot be so onerous that the engineering students feel that the time taken for the project is adversely affecting their ability to complete work in their engineering classes.

The creation of Maple projects that would enhance the understanding of topics that typically give students trouble, but at the same time be assignments that novice Maple users could complete without an overload of frustration with the CAS, was difficult. After several revisions, I found each project needed a complementary Maple document that gave examples of how to use relevant Maple commands to check "paper and pencil" calculations and construct graphics. In addition, a (voluntary) Maple help session at the beginning of the semester is held to "jump start" student Maple usage.

I typically assign four Maple projects during the semester. Topics include graphing planes and lines in 3-space, finding lines of intersection of two planes, finding parallel planes containing skew lines, computing the equation of an osculating plane and the parametric equations of an osculating circle for a given space curve, and then graphing the osculating plane, the osculating circle and the space curve together, geometry of the directional derivative and the gradient, geometry of the Lagrange multiplier method, and parameterization of surfaces. Each project will have a set of questions to answer and graphics to create that support the analytical work used to answer the questions. Students are required to submit a report for each project: the report consists of the Maple worksheet used to answer the questions and create the required graphics. Any calculations are to be word processed within the worksheet. A portion of the grade for each report is based on the student's presentation of work.

Below I will describe four of the projects I typically use. The descriptions are rather brief. If you would like to obtain actual copies of the projects, please feel free to contact me at sprevos@clemson.edu.

Using Maple to Graph Lines and Planes: The objectives of this project are to (1) get students familiar with Maple and its 3-dimensional graphing capabilities; (2) encourage students to create graphics that "visually" verify their paper and pencil calculations; (3) strengthen students' understanding of how to find equations for lines in 3-space and planes; (4) get students writing about mathematics. The project consists of two problems. Problem 1 requires students to find parametric equations for the line of intersection of two planes and then use Maple to graph the line and the two planes together. The second problem gives two lines. Students are then asked to give an analytical argument that shows the two lines to be skew. Next students must find equations for two parallel planes, each containing one of the skew lines. Finally, they must use Maple to sketch the two planes and two lines together.

Using Maple to Graph Space Curves and Circles of Curvature: The objectives are to (1) compute parametric equations for the circle of curvature to a smooth space curve at a point; (2) compute equations of the osculating plane at a point along a smooth space curve; (3) use Maple to create a graphic that "verifies" the equations obtained earlier are those of the circle of curvature and the osculating plane at a point along a smooth space curve; (4) get students writing about mathematics. In this project, students are given a space curve (twisted cubic) and are required to compute $\kappa(t)$, $\mathbf{T}(t)$, $\mathbf{T}'(t)$, $\mathbf{N}(a)$, $\mathbf{B}(a)$, an equation of the osculating plane at $t=a$, and parametric equations of the circle of curvature at $t=a$. After the computations are complete, students must produce a graphic that sketches the curve, circle of curvature at $t=a$, and the osculating plane at $t=a$ together on the same set of axes.

Visualizing the Directional Derivative and the Gradient with Maple: The objectives are to (1) give students a deeper understanding of the directional derivative by having students create graphics that depict tangent lines to curves of intersection; (2) reinforce skills needed to find equations of (a) lines in 3-space, (b) planes, and (c) curves of intersections of two surfaces; (3) use Maple to create graphics that verify the plane obtained using the gradient is indeed the tangent plane to a given level surface and that the gradient is normal to the surface; (4) get students writing about mathematics. Problem 1 requires students to find the directions that produce a directional derivative of 0 at a given point in the domain of a function of two variables. Then students must graph the surface near the given point, the vertical plane determined by the directions, the curve of intersection of the vertical plane and the surface, and the tangent line to the curve of intersection at the given point. Problem 2 asks the students to find the directional derivative to a function at a given point. Then students are asked to provide a graph of the surface near the given point, the vertical plane determined by the direction vector, the curve of intersection of the vertical plane and the surface, and the tangent line to the curve of intersection at the given point. Finally, Problem 3 requires students to find an equation of the tangent plane to a level surface at a given point. After the plane has been determined, the student must create a graph that depicts the sketches of the level surface, the tangent plane and the gradient vector.

The Geometry of Lagrange Multipliers: The objectives of this project are to (1) provide a visualization of the Method of Lagrange Multipliers; (2) use Maple to graphically verify the points selected using the Method of Lagrange Multipliers are valid; (3) get students writing about mathematics. Problem 1 requires students to analytically find the absolute extrema of a function of two variables along an ellipse using the Lagrange Multiplier Method. Then students are asked to compute the gradients to both the (appropriate) level curves for the function and the ellipse at the points identified using Lagrange multipliers. Next the students are required to create a static graphic that depicts the (appropriate) level curves for the function, the ellipse, and the gradients computed earlier. Lastly students are to create an animation that shows how the different level curves of f intersect with the ellipse. Problem 2 requires students to analytically find the point on a plane that is closest to a given point using Lagrange multipliers. Students then are asked to create a graphic that sketches both level surfaces and the corresponding gradients at the point(s) identified by the Lagrange multiplier method. Lastly students must make an animation that shows how the level surfaces that measure distance from the point intersect the plane.