CAN MATH SOFTWARE IMPROVE HOMEWORK EFFECTIVENESS?

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Abstract

Most commercial math practice software suffers both from inadequate feedback due to use of short answer problems and from inadequate problem variety. Three programs published by individual authors over the Internet are more responsive and intelligent and provide more problem variety than most.

What Do Our Students Need?

Our students need whatever legitimate help best prepares them to earn high grades in the courses in which we enroll them. Our examinations rarely require higher order problem-solving skills or critical thinking skills. We don't ask for new proofs or for solutions to original problems. Instead, we mostly require students to solve problems very similar to those covered in class and especially similar to those assigned as homework. The only way to acquire the needed manipulative skills is: practice, practice, practice. Passively watching an instructor solve problems is insufficient. Students must actively solve such problems for themselves over and over again to obtain the skills that we demand and that we reward.

No instructor can realistically be present for even a small fraction of the time students must spend solving repetitive practice problems. Our students must rely on their text materials, their fellow students, tutoring centers and such other tools as faculty can provide or recommend for use when working alone. Of course, it has occurred to many faculty that personal computers could and should be a good source of the needed help. Computers surely have the potential to provide intelligent assistance to students in solving large numbers of practice problems. The ideal is that computer software should offer a reasonable emulation of a human instructor or tutor.

Limitation 1. Prevalence of Short Answer Problems

Many instructors have noted one obvious limitation of most current practice problem software. The problems are usually of the "short answer" variety: true-false problems, multiple choice problems and problems requiring only the entry of a completely simplified final answer. The same objection that prevents most instructors from using short answer problems on examinations applies even more to homework. For any non-trivial problem, a short answer contains insufficient information to evaluate a student's skills and provide useful feedback. Yet the entire point of solving practice problems is for students to obtain such feedback, in order to improve their skills.

After an incorrect short final answer is selected or entered, typical software displays a stored solution, unaffected by the student's specific errors. From this stored solution the student must infer the nature and location of any and all errors. If the student made multiple errors, this is clearly unlikely. Moreover, non-trivial math problems can always be solved in more than one way. For example, the equations in Figure 1a. below can initially be converted to polynomial equations by at least three distinct general strategies:

1. multiply all terms on both sides by the L.C.M. of the denominators, or any integral multiple thereof, and reduce.
2. transpose the fraction to isolate the variable, add the fractions and cross multiply.
3. add or subtract the two fractions on the left and cross multiply.

Even if the student and the stored solution use the same general method, differences in the details may still make an incorrect step difficult to find. Inferring the reason for the error may be even more difficult. The stored solution model emulates a problem workbook. Such workbooks were widely used before personal computers. But if a computer merely emulates a workbook, why not simply recommend a workbook?

To help a student in person, no instructor would use short answer problems. An instructor would monitor the student's actual solution steps and explain how and why to correct the specific step or steps whenever errors actually occur.

Limitation 2. Lack of Problem Variety

Most practice software problems are "algorithmically generated," a marketing phrase meaning "generated by varying the coefficients." A more accurate phrase would be "template generated." Consider the following lists of problems:

$$\frac{1}{2}x-\frac{4}{3}=\frac{7}{9}$$
$$\frac{3}{2}x-\frac{8}{3}=-\frac{1}{2}$$
$$\frac{1}{2}x+\frac{2}{5}=\frac{3}{2}$$
$$\frac{1}{3}x-\frac{5}{6}=\frac{2}{3}$$
$$\frac{4}{3}x+\frac{1}{2}=\frac{4}{3}$$

Figure 1a. Algorithmically Generated Fractional Linear Equations

$$\frac{1}{2}x=\frac{7}{9}$$
$$\frac{2y}{3}=-\frac{3}{8}$$
$$\frac{3x}{2}-\frac{2}{5}=\frac{5}{2}$$
$$\frac{3}{2}=8+\frac{2}{5}y$$
$$\frac{2}{5}z-\frac{3}{2}=\frac{1}{2}z$$

Figure 1b. Manually Generated Fractional Linear Equations.

A currently popular program that describes its problems as "algorithmically generated" posed the list of equations in Figure 1a and appeared likely to continue with this type indefinitely, at least until the student entered correct short answers to six of them. No equations reflecting the variety of those in Figure 1b were forthcoming. Solving this second list would evidently require a much greater range of skills, and would thus be much more instructive for the student. No conscientious instructor would assign only equations of the type shown in Figure 1a in preference to a list of equations similar to those in Figure 1b.

The Common Villain: Stored Solutions

Both limitations – prevalence of short answer problems and lack of problem variety – result from typical programs' reliance on stored solutions to their problems.

If the only help available is to be a stored solution, there is nothing to be gained by monitoring individual steps in a student's solution. Stored solution programs might just as well settle for short answers from students. Of course, this falls far short of the goal of emulating a human instructor.

If each problem requires a stored solution, programmers tend to use templates to generate all-but-identical solutions to many all-but-identical problems. This appeared to be the case with the program used in Figure 1a above. The resulting lack of variety is unfortunately difficult to detect. Whereas a textbook evaluator can quickly look through a problem set and assess its problem variety, a software evaluator must completely solve many problems correctly, while taking careful notes, to encounter and document the full range of problems available on even a single topic.

The Common Remedy: Computed Solutions

Computer software runs on computers. The remedy for both limitations is for practice problem software to **compute** solutions rather than **storing** them. This allows intelligent step-by-step responses to solutions entered on a step-by-step basis.

Two algorithms are needed.[1] One algorithm should determine if a student step is **equivalent** to the preceding step in the appropriate sense. The other should generate, for any given step, an **appropriate next step** and a **description of the procedure** used to obtain it. Equipped with these two algorithms, a computer can reasonably emulate a human instructor. It can accept solutions entered step-by-step and confirm each step as correct or incorrect using the first algorithm. At any point it can either describe or actually provide an appropriate next step using the second algorithm. A complete solution to any problem, or a completion of any correct partial solution, can also be generated by applying the second algorithm repeatedly.

Availability of a solution generating algorithm also encourages programmers to offer more problem variety, as the programmer need not be concerned with providing solutions for each type of problem generated.

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[1] Miller, John C., "A Beginner's Guide to Offering Intelligent Help at Every Step" at www.xyalgebra.org/Downloading/A_Beginners_Guide_11.doc has more details on such algorithms.

Three Intelligent Programs Using Computed Solutions

MathXpert (www.HelpWithMath.com).

In Figure 2a below MathXpert suggests and demonstrates combining the fractions on the left. In Figure 2b, the student has instead asked MathXpert to multiply both sides of the equation by the product of the denominators, and MathXpert has complied. It then changes its suggested strategy accordingly and hints that now the distributive law should be applied. When asked, MathXpert demonstrates this procedure in Figure 2c.

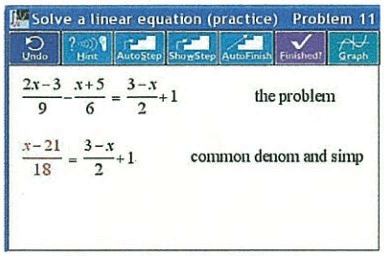

Figure 2a. MathXpert

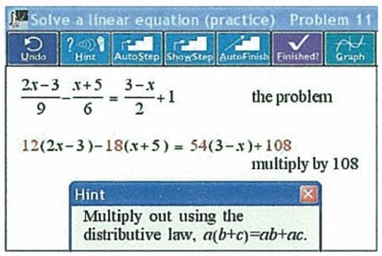

Figure 2b. MathXpert

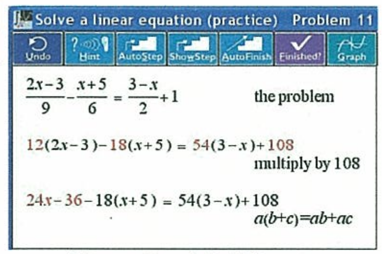

Figure 2c. MathXpert

Math Professor (www.mathkal.co.il)

Figure 3a below shows Math Professor rejecting an incorrect step. It suggests using the distributive law, and reviews the statement of that law. In Figure 3b, Math Professor accepts the student's next step and then, when asked for a hint, suggests the next appropriate operation. Although this is a simple example, taken from the short demo version of Math Professor, it shows the general approach of the program.

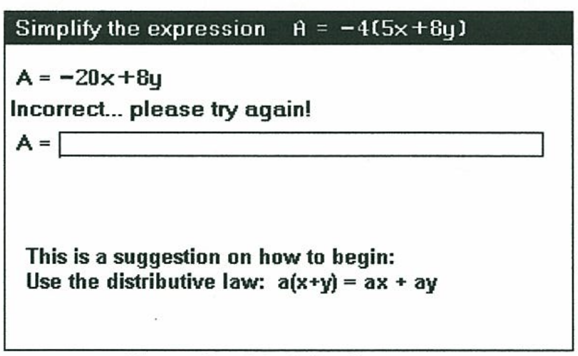

Figure 3a. Math Professor

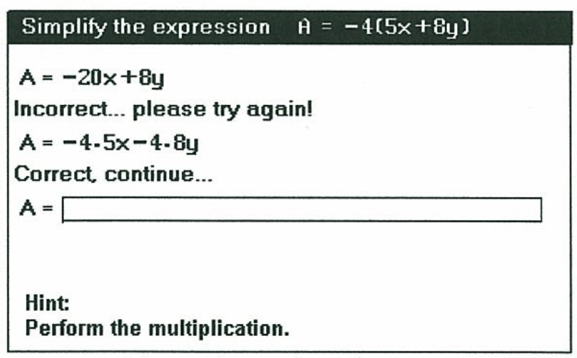

Figure 3b. Math Professor

xyAlgebra (www.xyalgebra.org; a free program written by the author of this paper)

In Figure 4a below, an xyAlgebra student uses a mildly unusual strategy of "collecting fractions." XyAlgebra accepts the student's first two steps, but detects an error when the student tries to cross multiply. XyAlgebra emulates an instructor by opening a sub-window (Figure 4b) and varying the coefficients to generate a similar step also requiring cross multiplication. Then it shows and describes an appropriate next step saying, in effect, "Here's how we do that." In Figure 4c, xyAlgebra implicitly says, "Now you try one." This repeats until the student enters a correct step. Then the sub-window closes and the student resumes the original problem.

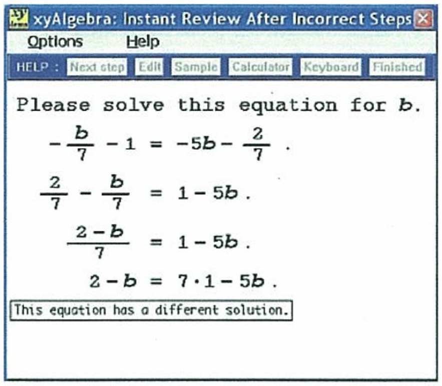

Figure 4a xyAlgebra.

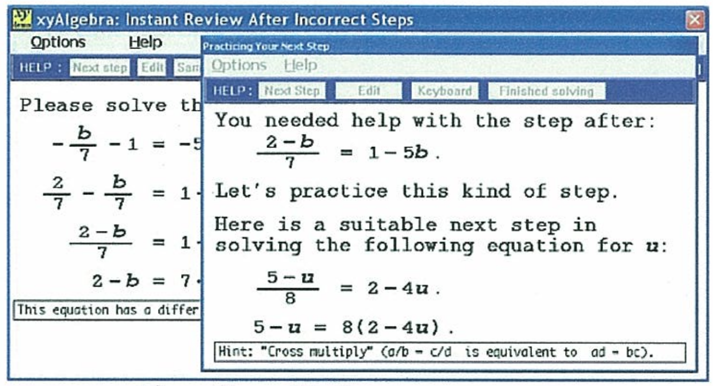

Figure 4b. xyAlgebra

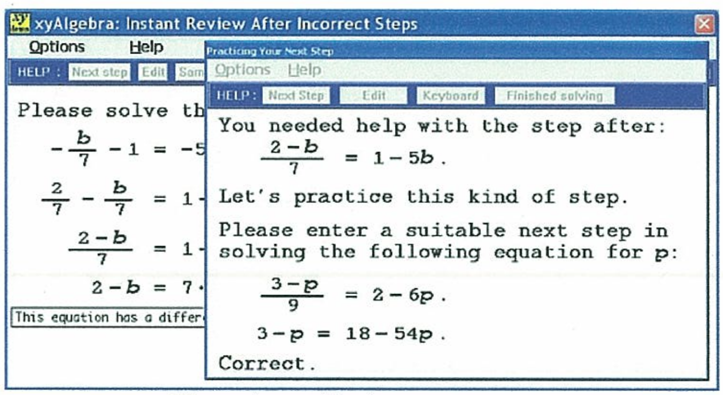

Figure 4c. xyAlgebra

Relative Strengths and Weaknesses of These Intelligent Programs.

MathXpert has many problems on many topics, and also allows students to enter their own problems. However, it performs by itself the steps requested by its students, denying them the chance to make and learn from many common types of mistakes.

Math Professor appears to respond well to both correct and incorrect steps entered by the student. Its problem variety and topic coverage are difficult to assess from the limited demo version available.

XyAlgebra has very good problem variety, and excels in its review of prerequisites and practice offered when students make errors. However, it covers only the standard first algebra course, from signed numbers and expression evaluation through simplification, factoring, rational expressions, linear equations, graphing and verbal problems.[2]

[2] To see how xyAlgebra handles verbal problems, download and run the Windows demo program: www.xyalgebra.org/xyDemo52.exe