USING MAPLE IN AN "ANALYSIS FOR TEACHERS" GRADUATE COURSE

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Abstract

We will discuss how Maple can be used to enhance a masters level "Analysis for Teachers" course. Both aspects will be considered: Maple for the student's use and Maple for the student's student use.

Introduction

More students take introductory calculus in high schools than in post-secondary institutions. Many mathematics secondary education programs do not require an advanced calculus course. These two observations led us to include an "Analysis for Teachers" course in our masters program for secondary teachers. Our rationale being that those who were likely to teach calculus needed a deeper understanding. We've designed a 4 credit course, required in the program, to meet this need.

The course objectives listed in the syllabus are:

- *Connect analysis and calculus at an advanced level.*
  Each of us has had a calculus sequence and an undergraduate analysis or advanced calculus course. We will look at the major topics of analysis to gain a deeper understanding & appreciation and be better able to teach calculus.

- *Investigate concepts and theory through problems.*
  Summer sessions are very short; we will use a modified approach to study analysis. We will use problems to bring out the topics, both conceptually and theoretically, to better understand advanced calculus.

- *Synthesize topics through history and readings.*
  History gives perspective that helps us understand the intricacies of the development of analysis. We will look at founders of the theory such as Fermat, Newton & Leibniz, Cauchy, Euler, and Riemann. Students will make presentations on their readings.

Our current text is *Resources for Learning Real Analysis* by Brabenec, published by the MAA. We use graphing calculators and Maple extensively throughout the course.
North Carolina's Dep't of Public Instruction mandated a graduate degree program for educators titled an "advanced masters" with several requirements for a larger amount of

content v. pedagogy courses. We included "Analysis for Teachers" as a required course paralleling "Linear Algebra for Teachers". These two courses give 8 credit hours towards the content needed in the 36 credit hour masters program. The prerequisite is admission to the program, essentially, a BS in mathematics education or mathematics and secondary teaching experience. Because of the large diversity in major requirements at different schools, we cannot assume an undergraduate advanced calculus background — making teaching the class a challenge. Since the course is designed for in-service teachers, we offer it in a 4 week summer session meeting 2 hours and 40 minutes per day, 5 days a week.

Course Design

As opposed to trying to cover a significant number of analysis topics with thorough lectures in such a short period, we use problems to bring out concepts and give concise, encapsulated presentations on the topics. The first week of the course is spent looking at a standard calculus sequence that would be taught in an AP Calculus-BC class. We select the topics, again brought out through problems in class (sometimes through group work, other times individual) and in homework that will lead us to analysis and deeper insights. Typically, we look at limits with ϵ-δ arguments, derivatives via limits of difference quotients, definite integrals via summations, and sequences & series. We also look at several sample AP tests to ground the teachers with problems they will face in their classes.

We move to analysis in the second week of class by considering the same problems in deeper ways, looking at deeper problems, and examining counter-examples. We begin by revisiting limits and doing careful ϵ-δ proofs. Continuity and Dirichlet's functions make for interesting discussions. (See Figure 1.) All through our work, we use graphing calculators and Maple to demonstrate and explore. After going through derivatives and integrals, we look at sequence and series convergence modes and tests.

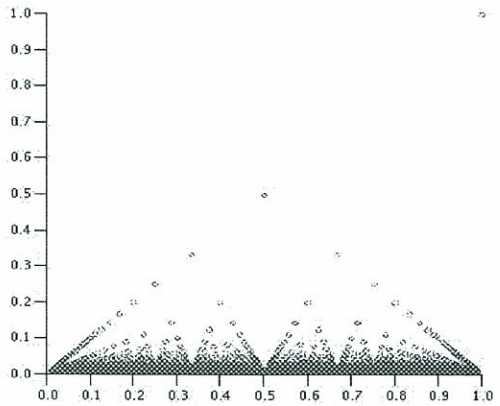

Figure 1: Maple Plot of "Salt & Pepper."

The last week focuses on readings, finishing projects, presentations on my favourite topics (usually having something to do with special functions), and student presentations. This past summer, a student staged a skit—something I've never had in class before—on the Newton-Leibniz controversy. The skit was very clever, quite instructional, and well suited for his calculus class this fall.

Students have 3 projects to complete. Each does an individual paper and presentation on a historical topic in analysis; two class projects are to create an annotated bibliography and draw a concept map for analysis.

Maple in the Course

In the calculus section of the class, we use Maple in the standard way for demonstrations, graphics, and investigations. Maple worksheets can be prepared before sessions. Figure 2 shows a Maple document with component plots and sliders embedded in a table, all used to investigate limits.

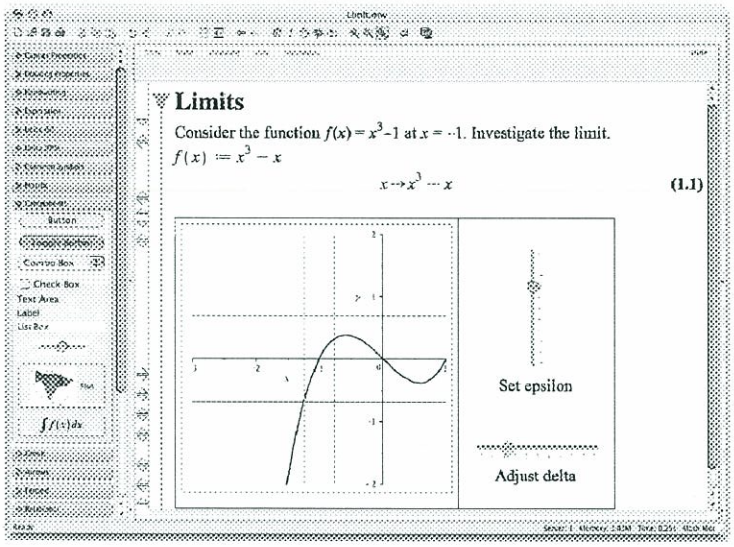

Figure 2: Limits via Components.

Worksheets can have embedded commands that are automatically executed to set up the components to make "stand-alone" environments for students. Assignments to create worksheets illustrating concepts are also profitable. We can spend a little time learning to add components to make worksheets for the students' students.

One of the newer aspects of the software, Maple Tutors, provides a nice interface that helps reduce, or even eliminate, the need to learn syntax. Figure 3 shows Maple's tutor for multivariate Taylor polynomials. The tutors are Maplets designed for student explorations.

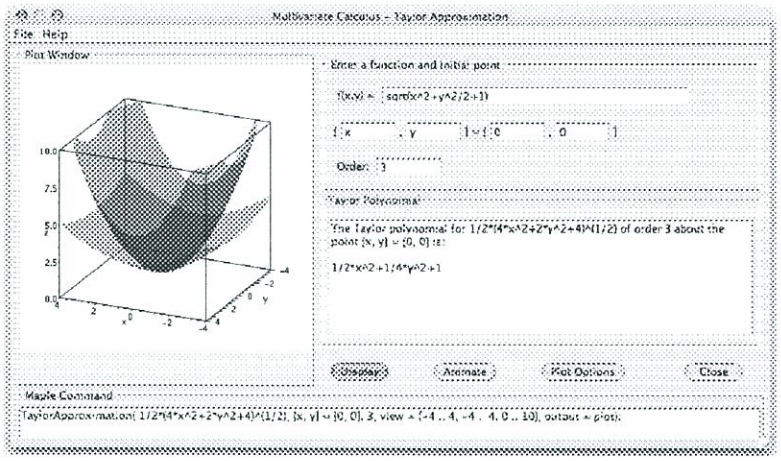

Figure 3: Multivariate Taylor Polynomial Tutor.

The tutorial Maplets from the *Student package* are listed and categorized in the *Tools* menu under the submenus *Calculus - Multi-variable, Calculus - Single Variable, Differential Equations, Linear Algebra, Precalculus,* and *Vector Calculus.* For an overview, enter `?tutors`. We have two foci when studying these: the teachers investigate concepts, and then, they look at the tutors from the perspective of possible use in their classes. Maple also includes a maplet in the menu *Tools/Assistants/Maplet Builder* for constructing an interface for a maplet. (See `?Student` and `?maplet`.)

A nice topic for students to investigate using Maple is standard induction proofs. We can let

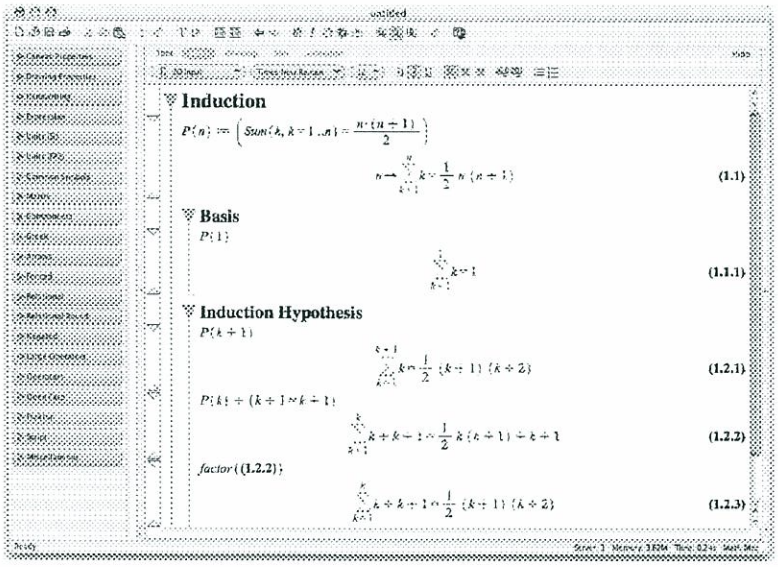

Figure 4: Induction and Sums.

Maple handle the algebra and introduce students to using computer algebra to support proofs. Essentially all the teachers in the class are surprised that a Maple function can return a proposition, rather that simply a value. This is a good lesson for the teachers that extends their concept of function.

Another topic that is quite fun and goes much more smoothly in the accelerated time-frame of our course by using Maple is series convergence.

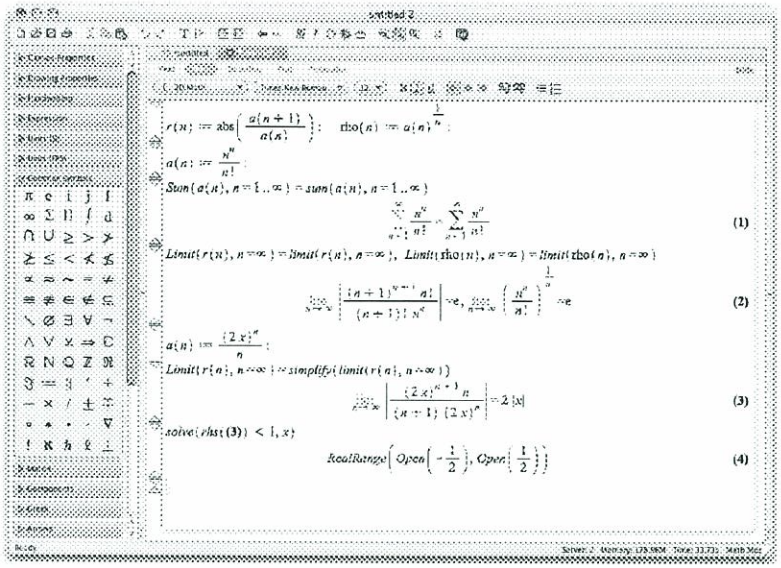

Figure 5: Ratio and Root Tests.

We can quickly check a number of series for convergence and determine the radius and interval of convergence for power series with Maple

Conclusion

Enhancing in-service teachers' content knowledge in a graduate *Analysis for Teachers* course is made much easier by using Maple. More depth in our investigations is possible in spite of the terribly short time frame of our summer 4-week course. And it's a lot of fun.

Web Links

Course materials and ICTCM 19 presentation slides are available at:

- Course materials: http://www.mathsci.appstate.edu/~wmcb/Class/archive.html
- ICTCM slides (pdf): http://www.mathsci.appstate.edu/~wmcb/ICTCM19/