UNDERDAMPED MOTION : STRUCTURED MAPLE PROJECTS FOR DIFFERENTIAL EQUATIONS

Joseph Fadyn
Southern Polytechnic State University
1100 South Marietta Parkway
Marietta, Georgia 30060
jfadyn@spsu.edu

INTRODUCTION

I have been writing and assigning group Maple projects in my Differential Equations courses for a number of years now. I refer to these projects as "structured" in the sense that they are written in the form of a Maple worksheet which guides students through completing the project. This guidance allows for a minimal knowledge of Maple by the student since in many (although not all) cases, examples of the Maple syntax are blended into the presentation and precede the questions asked in the project. Other questions asked in these projects might, for example, produce a plot or an animation and require detailed explanations of these by the student. The idea is that students should learn some Maple, but not be overburdened by the Maple syntax since this would certainly take away from the main idea of using the technology to aid in understanding the mathematics involved.

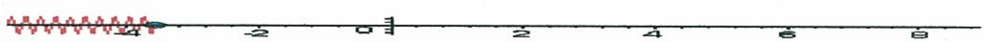
The projects I will be discussing all relate to the underdamped motion of a simple mass spring system. The disk which I will be distributing (described below) was produced in Maple 10 (the "classic worksheet" version) and represents a substantial revision and improvement of earlier work done in Maple 9.

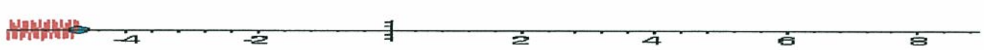
THE DISK

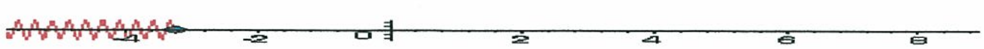
During my presentation at ICTCM19, I will be distributing a disk which contains a set of two structured Maple projects for differential equations (Underdamped Motion Projects I and II), as well as complete solutions to each project. Although I retain the copyright to this material, I hope that those who receive the material will be able to use some of it in their differential equations classes. I would ask, however, that you do not provide students copies of the project solutions or publish any of these solutions to the web. Each of these projects relates the the following problem:

THE UNDERDAMPED MOTION PROBLEM

We consider a simple mass-spring system which is set into a back and forth motion by stretching or compressing the spring from equilibrium and perhaps giving the mass an initial velocity to the right (positive) or to the left (negative). We will assume that no outside forces act on this system--that is, the motion is "free":







If k is the spring constant and if we assume that resistance is proportional to the velocity with a constant of proportionality c (sometimes called the damping constant), then if x represents the position of the mass m at time t, we have:

$m\left(\frac{d^2}{dt^2}x(t)\right)+c\left(\frac{d}{dt}x(t)\right)+k\,x(t)=0$. To simplify the form of the solution of this differential equation we will make a temporary change of variables.

Let $p=\frac{c}{2\,m}$ and $\omega_0=\sqrt{\frac{k}{m}}$. Then our differential equation can be written as: $\left(\frac{d^2}{dt^2}x(t)\right)+2\,p\left(\frac{d}{dt}x(t)\right)+\omega_0^{\;2}\,x(t)=0$. This equation is second-order linear and homogeneous. The auxiliary equation is:

$m^2+2\,p\,m+\omega_0^{\;2}=0$. We can use Maple to help us find the solution of this differential equation. We find that the roots of the auxiliary equation are:

$$-p+\sqrt{p^2-\omega_0^{\;2}},\; -p-\sqrt{p^2-\omega_0^{\;2}}$$

In this project we will consider *only* the case where $p^2-\omega_0^{\;2}<0$. Observe that since:

$p^2-\omega_0^{\;2}=\frac{c^2}{4\,m^2}-\frac{k}{m}\;=\frac{c^2-4\,k\,m}{4\,m^2}$, this is equivalent to the case where $c^2-4\,k\,m<0$. In this case the roots are the complex conjugates $-p+i\sqrt{\omega_0^{\;2}-p^2}$ and $-p-i\sqrt{\omega_0^{\;2}-p^2}$. If we use the notation: $\omega_1=\sqrt{\omega_0^{\;2}-p^2}\;=\frac{\sqrt{4\,k\,m-c^2}}{2\,m}$,

then the solution to the differential equation can be written in the compact form:

$$x:=t\rightarrow \mathrm{e}^{(-p\,t)}(A\cos(\omega_1\,t)+B\sin(\omega_1\,t))$$

Observe that the Maple statement above makes the position x a function of time t. This solution represents exponentially damped motion about equilibrium and is sometimes called "damped harmonic motion". Of course A and B are constants which can be determined if we are given appropriate initial conditions. So we next determine the general solution to the underdamped problem if we have the initial position (alpha) and the initial velocity (beta). That is: x(0) = α and x ' (0) = β . Using Maple, it is not hard to show that the solution of this initial value problem is:

$$x=\;\mathrm{e}^{\left(-\frac{c\,t}{2\,m}\right)}\left(\alpha\cos\left(\frac{\sqrt{4\,k\,m-c^2}\;t}{2\,m}\right)+\frac{2\left(\frac{c\,\alpha}{2\,m}+\beta\right)m\sin\left(\frac{\sqrt{4\,k\,m-c^2}\;t}{2\,m}\right)}{\sqrt{4\,k\,m-c^2}}\right).$$

A brief description of each of the the projects follows.

1. FREE UNDERDAMPED MOTION I: A detailed analysis of the free, underdamped motion of a mass-spring system. We consider a simple mass-spring system which is set into a back and forth motion by stretching or compressing the spring from equilibrium and perhaps giving the mass an initial velocity to the right (positive) or to the left (negative). We will assume that no outside forces act on this system--that is, the motion is "free". This project develops general formulas for the pseudoperiod, times the mass passes through equilibrium, the time between successive extrema, and the familiar "cosine" form of the solution. Some analysis of three-dimensional surfaces is required by the student.

2. FREE UNDERDAMPED MOTION II: Refer to 1 above for the basic problem of free underdamped motion of a mass-spring system. In Part I we developed formulas for the solution of the free underdamped motion of a mass on a spring and various quantities related to this motion. In this project (which is completely self-contained) we continue this investigation. However, in this project, many of the required formulas from Part I will be stated without proof or derivation. If you require more details about these formulas, you might want to refer to Part I. However, this project is completely independent of Part I. After some preliminaries and an example, our main focus in this project is to investigate the effect of holding all parameters in the solution constant except one. We will concentrate on the three cases of : (i) varying the mass m; (ii) varying the damping coefficient c; (iii) varying the spring constant k.

THE WEB SITE

The web site given below contains a sample of the differential equations Maple projects as well as some additional Maple worksheets. Please feel free to access this material and to use whatever you find appropriate in your own classes. The copyright on this material is, however, retained by myself.

http://www2.SPSU.edu/math/fadyn/index.html

The material can also be found by beginning at the Southern Polytechnic State University home page and then navigating to Professor Fadyn's home page. The home page for Southern Polytechnic State University is located at: http://www.spsu.edu/

HYBRID INTERMEDIATE ALGEBRA AND HYBRID COLLEGE ALGEBRA: A SUCCESSFUL PILOT

Sonia Ford
Midland College
Department of Mathematics and Sciences
3600 N. Garfield Street
Midland, Texas 79705
sford@midland.edu

The goal of the presentation and paper is to present a successful pilot of courses offered during the Spring 2006 semester and again offered during the Fall 2006 semester in the Midland College Mathematics Department.

To begin, let me give a brief history of placement and course offerings from the Midland College Mathematics Department. At Midland College, students are enrolled into mathematics courses based upon their scores on THEA (Texas Higher Education Assessment), the SAT/ACT, or COMPASS placement test scores. A majority of students who need a mathematics course will meet with a counselor and will be advised to take the COMPASS placement test before they register, especially if there has been at least one semester since exams or previous math courses were taken. After the placement test is taken by a student, he or she will meet again with a counselor and are advised as to which mathematics course to enroll based upon set placement scores. At Midland College we offer Basic Mathematics (Math 0389), Introductory Algebra (Math 0390), and Intermediate Algebra (Math 0391) in our developmental sequence. After successful completion of Intermediate Algebra with a grade of "C" or better, students may register for College Algebra (Math 1314), and after successful completion of Intermediate Algebra with a grade of "B" or better, students may register for Statistics (Math 1342) or Math for Business and Social Sciences I (Math 1324).

Our department has found that many students who are placed in Developmental Mathematics based solely on placement exam scores only require a refresher course before moving on to transfer level courses. In addition, it is discouraging to students who are placed into developmental mathematics to realize that they have one or two semesters of coursework to complete before they begin taking classes for transfer credit. To address these issues, I developed an accelerated hybrid format for Intermediate Algebra and College Algebra. In this hybrid, accelerated course format, students are able to complete both Intermediate Algebra and College Algebra within a 16-week time frame. During the first 8-weeks of the semester students complete Intermediate Algebra and then complete College Algebra during the last 8-weeks of the semester. It is important to note that each course is taught separately.