EXPLORING SOLUTIONS OF A DYNAMICAL SYSTEM WITH MAPLE.

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The differential equations modeling the forced motion of a spring mass system are non-homogeneous and can be modeled by the non-homogeneous differential equation of the form

$m\frac{d^2x}{dt^2} = -kx - c\frac{dx}{dt} + f(t)$, where $x(t)$ denotes the displacement of the mass at time t., kx represents the spring force, the k being the spring constant, $c\frac{dx}{dt}$ represents the damping force and $f(t)$ represents some external driving force. When $c = 0$ the motion is said to be un-damped and when $f(t) = 0$ the motion is un-forced. We consider some special cases of un-damped forced motion for our purpose.

Since the motion is a forced motion the differential equation takes the form $m\frac{d^2x}{dt^2} + kx = f(t)$. In solving this non-homogeneous differential equation we need to use the method of variation of parameters or the method of undetermined coefficients.

With Maple we will solve some un-damped forced motions of spring mass systems and investigate the solutions for various external forces. For convenience we will consider the case with $m = 1, k = 4$ and vary the external force "$f(t)$". We will solve the non-homogeneous system using the "dsolve" command of the "DEtools" directory of Maple 10.

For instance the cases $f(t) = 1, -1, \cos(t)$ with some given initial conditions can be solved and the solution can be plotted using Maple as follows. See the Maple segments attached herewith for details.

By visually looking at the solution curve we can see that when the forcing function is a positive(negative) constant function the mass never moves above(below) the equilibrium position and when $f(t) = \cos(t)$ it moves below equilibrium as well as above the equilibrium. We can similarly investigate the behavior of solutions by solving the differential equation for various forcing functions. The "dsolve" command in the "DEtools" directory is used for solving the differential equations and plotting the solution is done via the plot command of Maple occurring in the "plots" directory.

MAPLE SEGMENTS for the cases $f(t) = 1, -1, \cos(t)$

> $ode1 := (D@@2)(x)(t) + 4 \cdot x(t) = 1;$

$$ode1 := ((D^{(2)})(x))(t) + 4\,x(t) = 1$$

> $IC := x(0) = 0, D(x)(0) = 0;$

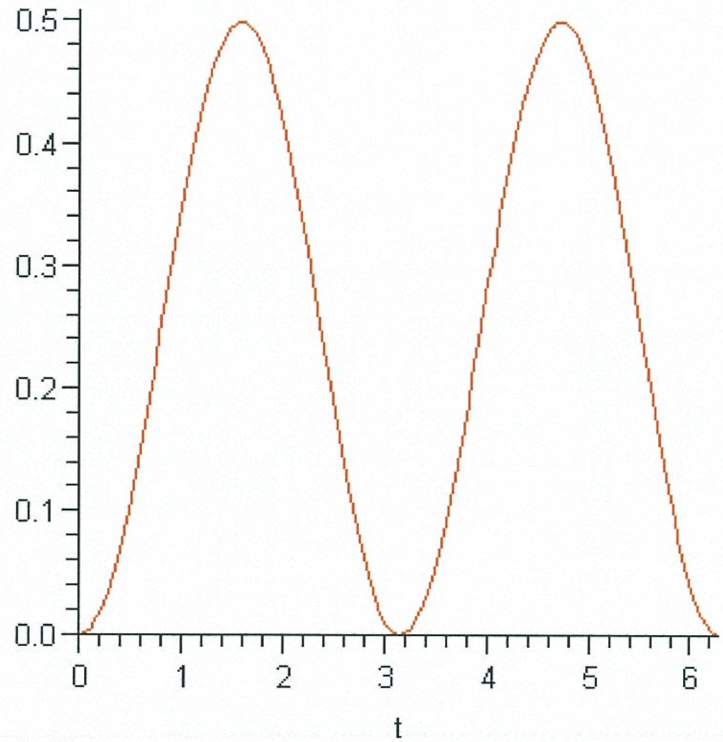
$$IC := x(0) = 0,\ (D(x))(0) = 0$$

> $dsolve(\{ode1, IC\}, x(t));$

$$x(t) = \frac{1}{4} - \frac{1}{4}\cos(2\,t)$$

>

Figure1(solution curve when forcing function = 1 unit)



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$$ode2 := (D@@2)(x)(t) + 4 \cdot x(t) = -1;$$

$$ode2 := ((D^{(2)})(x))(t) + 4\,x(t) = -1$$

> $IC := x(0) = 0,\ D(x)(0) = 0;$

$$IC := x(0) = 0,\ (D(x))(0) = 0$$

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> dsolve({ode2,IC},x(t));
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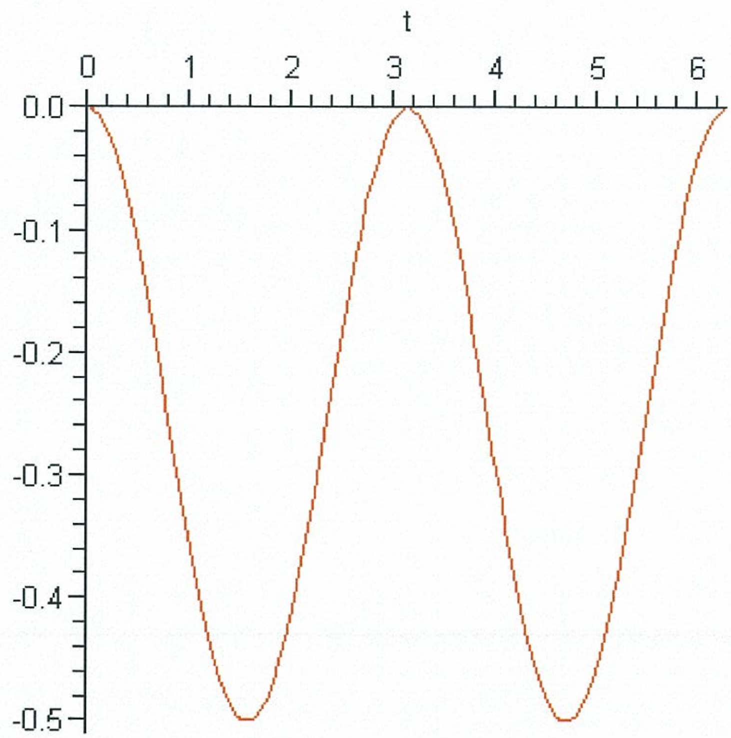
$$x(t) = -\frac{1}{4} + \frac{1}{4}\cos(2\,t)$$

> $plot\left(-\frac{1}{4} + \frac{1}{4}\cos(2\,t),\ t = 0..2\cdot\pi\right);$

Figure2(solution curve when forcing function = -1 unit)



> $ode1 := (D@@2)(x)(t) + 4 \cdot x(t) = cos(t);$

$$ode1 := ((D^{(2)})(x))(t) + 4\,x(t) = \cos(t)$$

> $IC := x(0) = 0, D(x)(0) = 0;$

$$IC := x(0) = 0,\ (D(x))(0) = 0$$

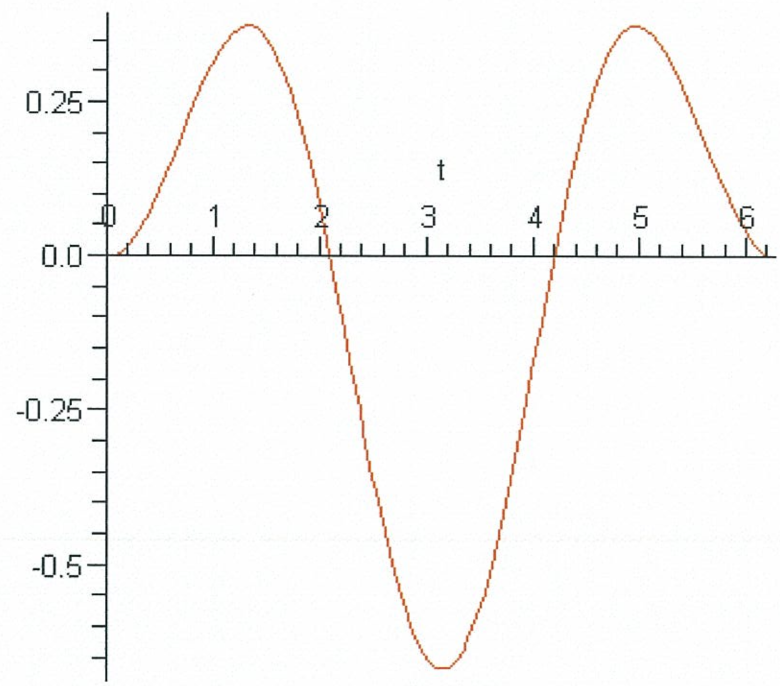
> $dsolve(\{ode1, IC\}, x(t));$

$$x(t) = -\frac{1}{3}\cos(2\,t) + \frac{1}{3}\cos(t)$$

$$plot\left(-\frac{1}{3}\cos(2\,t)+\frac{1}{3}\cos(t),\ t=0\,..2\cdot\,\pi\right);$$

>

Figure3(solution curve when forcing function = cos(t))



Conclusion: Maple gives us the convenience in computation and visualization of solutions of various dynamical systems of our interest.

References:

(1) Soma Velummylum .Maple can Solve Initial Value Problems. Students do not have to memorize some of the Mathematical Formulas, Maple Summer Workshop Proceedings, University of Waterloo, Ontario, Canada, July 25-28(2002)

(2) Abell & Braselton, Modern Differential Equations, Second Edition, HarCourt College Publishers.