

MATHEMATICAL MODELING WITH THE SUPPORT OF TI'S SEQUENCE MODE WITH SUGGESTIONS FOR USING EXCEL

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Many students can find recursive relationships to model patterns such as those associated with Figurative Numbers or the Towers of Hanoi. Until recently this type of modeling was not widely supported by commercial textbooks or school mathematics curricula. Thus when most students arrive at our college mathematics classes they have either forgotten, or never been exposed to recursive modeling. The availability of computational tools such as calculators with a sequence graphing mode and spreadsheets such as Excel enable the numerical, graphical and algebraic exploration of many elementary recursive models. This paper discusses a subset of the models presented in a workshop at the ICTCM conference. The TI-84 is used as the primary computational tool to explore models appropriate for courses including Liberal Arts Mathematics, College Algebra, Pre-Calculus, Finite Mathematics, Discrete Mathematics and courses for future mathematics educators. Recurrence equations are sometimes referred to as difference equations or discrete dynamical systems. The dynamical systems discussed at the conference included those that modeled financial situations, figurative numbers, predator-prey relationships, phone chains and growth and decay problems. Excel was used to model several of those applications. In this paper, suggestions for using Excel are included but with much less detail than provided in the workshop.

Modeling a Phone Chain

The first situation we model is a phone chain that might be used by a business or school to notify employees or families of a delayed opening or closing because of inclement weather or other emergency. The model used is described in paragraph form followed by a diagrammatic representation.

After crunching all the weather reports and forecasts at 6:01 am one person, the boss, is notified by their new automated computer system that the business should not open that day. At 6:02 am she calls the first assistant boss and notifies him that due to the weather the business is closed for the day and at 6:03 she calls her second assistant boss with the same message and goes back to sleep. The first assistant boss also makes two sequential phone calls, the first one at 6:03 and the second one at 6:04, each of which takes him one minute to complete. The second assistant boss does the same thing, making her first call at 6:04. From that point on, each person called then calls two new people and as with the first person, each call takes one minute to complete. This process continues until

everyone is called. For the purposes of this investigation we assume that every person called answers the phone and continues the chain as shown in Figure 1 below.

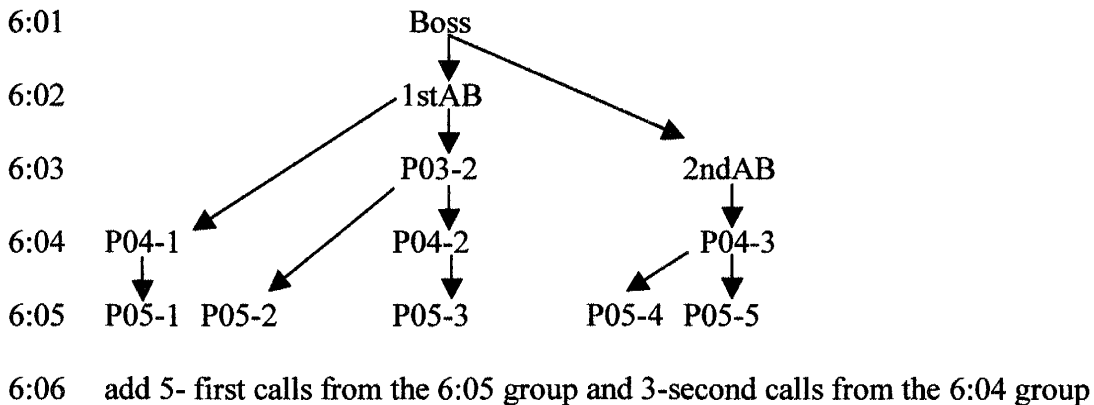


Figure 1. Phone Chain

To model this with the TI-84 press the MODE key and scroll down to the line with FUNC PAR POL SEQ. Right arrow over to SEQ and press the ENTER key. Next press the Y= key. The above exploration justifies entering the following information.

$$\begin{aligned}
 n\text{Min} &= 1 \\
 u(n) &= u(n-1) + u(n-2) & u \text{ is entered by pressing 2ND then 7} \\
 u(n\text{Min}) &= \{1,1\}
 \end{aligned}$$

To see the function values in a table press in succession, the 2ND key and then the WINDOW key. Edit the TABLE SETUP so it is the same as shown in Figure 2. To view the function values press in succession, the 2ND key and then the GRAPH key.

```

TABLE SETUP
TblStart=1
ΔTbl=1
Indent:  Auto Ask
Depend:  Auto Ask
  
```

Figure 2. TABLE SETUP

n	$u(n)$	
1	1	
2	1	
3	2	
4	3	
5	5	
6	8	
7	13	

$n=7$

Figure 3. Phone Chain Values

By this stage of our exploration it is clear that we are generating the Fibonacci numbers and the fact that we are doing it without resorting to some very contrived conditions about the mating and reproduction habits of rabbits may be a relief to some readers. Modifications and extensions that are interesting to consider include changing the number of phone calls some or all of the individuals make and determining how long it would take to notify everyone if there were 100 company employees or 1000 employees and students. This example can be readily modeled with Excel using the “fill” command.

Figurative Numbers

Figurative Numbers are another topic that can be used to enrich students' understanding of mathematics from the upper elementary grades through middle and secondary school, in selected university courses and as a topic for teacher professional development. The triangular numbers, which are encountered in a variety of classical questions such as the handshake and related problems, are often introduced with the pattern of dot triangles shown in Figure 4 below.

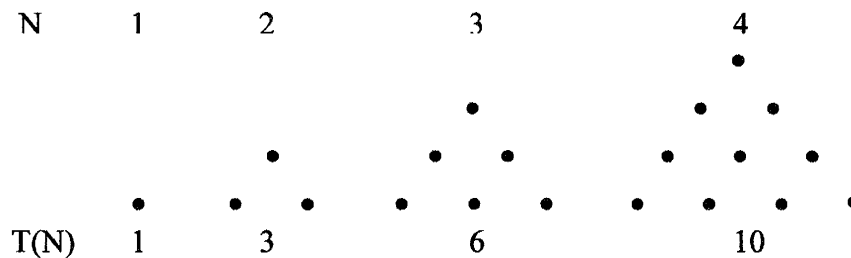


Figure 4. Triangular Numbers

It is instructive, for students and the instructor, to have students spend some time developing a recursive and an explicit formula for these numbers and then to use a calculator or Excel, or both, to create a table of the first 10 or 15 triangular numbers.

Although generating the first 10 or 15 square numbers using an explicit formula is trivial, finding a recursive formula such as $S(N) = S(N - 1) + 2(N - 1) + 1$ often takes longer. Continuing with the pentagonal numbers it is instructive to have students generate a table such as shown in Figure 5 at the right that includes space for the hexagonal, heptagonal and octagonal numbers.

Figurative Numbers						
N	T(N)	S(N)	P(N)	H(N)	h(N)	O(N)
1	1	1	1			
2	3	4	5			
3	6	9	12			
4	10	16	22			
5	15	25	35			
6	21	36	51			
7	28	49	70			
8	36	64	92			
9	45	81	117			
10	55	100	145			
11	66	121	176			

Figure 5.
Partial Table of Figurative Numbers

At this point in the exploration of figurative numbers students should begin to see many patterns, in both the columns and the rows. There are also some very interesting patterns in the both the recursive and explicit formulas for the figurative numbers that can be investigated to a depth that is appropriate for the course and mathematical preparation of the students.

Predator-prey Population Dynamics

There are many examples of the fluctuations in population numbers for two competing groups. The next example is based on the Lotka-Volterra Two Species Model and

notation based on that found in Chapter 6 of the TI-83 family of guidebooks published by Texas Instruments. Additional information about the Lotka-Volterra model can be found in Cowling and Stephens, *Dinner at the Rabbit Café: A Predator-Prey Investigation*.

Consider an environment that contains a population of rabbits and wolves. Let R denote the number of rabbits, $M = .055$ be the growth rate of the rabbit population if there are no wolves present and $K = .0015$ be the death rate (per wolf) of rabbits when wolves are present. Let W denote the number of wolves, $G = .00025$ be the growth rate of the wolf population when there are rabbits present (thus this is per rabbit) and $D = .035$ be the death rate of the wolves when no rabbits are present. The values for the parameters M , K , G , and D are estimates that are usually based on field observations or related studies.

Let n denote time in months. From the above we can write the following two equations. $R(n) = R(n-1)(1 + M - K \cdot W(n-1))$ and $W(n) = W(n-1)(1 - D + G \cdot R(n-1))$. We assume an initial population of 35 wolves and 110 rabbits and use $u(n)$ to represent the size of the rabbit population and $v(n)$ for the wolf population. The predicted population trends for these two interacting species are shown in Figure 6 below. The plot is for 30 years or 360 months. The graphs of u and v were obtained using the Time format option and the window settings shown in the two boxes to the left of the plot in Figure 6.

```
WINDOW
nMin=1
nMax=360
PlotStart=1
PlotStep=1
Xmin=0
Xmax=360
Xscl=15
```

```
WINDOW
↑PlotStep=1
Xmin=0
Xmax=360
Xscl=15
Ymin=0
Ymax=250
Yscl=15
```

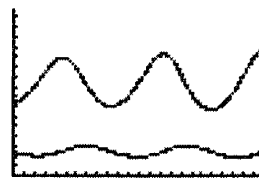


Figure 6. Window data and plot of predator-prey population example.

When using a graphing calculator to view the graphs it is more instructive if you graph them simultaneously. If they are being graphed sequentially you can change to viewing them being graphed simultaneously by selecting that option from the MODE menu.

Chemical Interconversion

An interesting situation involving two or more chemicals is known as interconversion. For example, in an animal's body interconversion occurs when a chemical u is converted into a second chemical v , and at the same time some v is converted into u . Consider two chemicals with an assumed daily interconversion represented by Figure 7 below.

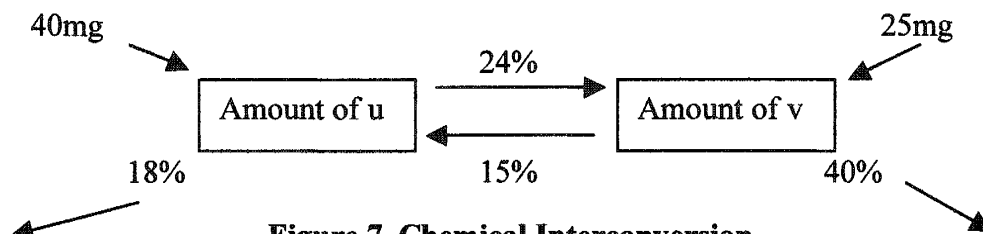


Figure 7. Chemical Interconversion

Using Figure 7, we can write the following two recurrence equations.

$$u(n) = u(n-1) - .18u(n-1) - .24v(n-1) + .15v(n-1) + 40$$

$$v(n) = v(n-1) - .15v(n-1) - .40v(n-1) + .24u(n-1) + 25$$

The values of 40 mg and 25 mg represent the amount of u and v absorbed from the animal's daily diet or supplements. Simplifying the above we get

$$u(n) = .58u(n-1) + .15v(n-1) + 40$$

$$v(n) = .45u(n-1) + .24v(n-1) + 25.$$

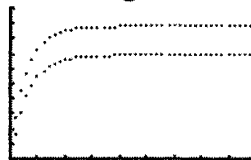


Figure 8.

Chemical Interconversion Equations

Figure 9. Graph of Figure 8 Equations

Using the TI-84 with a viewing window of $1 \leq n \leq 45$, $0 \leq x \leq 45$, $0 \leq y \leq 150$, a graph format of Time with initial values of $u(1) = 40$ and $v(1) = 25$ produces the graph shown in Figure 9 above. The graphs imply that over time the amounts of the two chemicals in the animal body will stabilize. Formal analysis can be used to verify that the limiting values for u and v are approximately 132 and 103 milligrams, respectively.

Summary

Among the tools available to support the teaching and learning of mathematics perhaps the two that are most readily available are the graphing calculator and spreadsheets such as Excel. This paper discussed the use of those tools in connection with the exploration of modeling activities that involve elementary dynamical systems.

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