

THE FANTASY OF THE CONSCIENTIOUS INSTRUCTOR: INTELLIGENT PREREQUISITE REVIEW AT ANY INTERMEDIATE STEP

John C. Miller

Professor Emeritus, The City College of C.U.N.Y.
110 Riverside Drive, #14C, New York, NY, 10024
xyalgebra@mindspring.com

A. The Conscientious Instructor

A conscientious full-time instructor has perhaps 100 students each semester, to each of whom are assigned perhaps 1000 homework problems, carefully selected for adequate problem variety and requiring an average of perhaps 5 steps each. The conscientious instructor would love to be available whenever needed for help on each of these 500,000 steps. Indeed, for each step, the conscientious instructor would love to be able to:

1. Confirm its correctness as soon as it is written.
2. Suggest an appropriate next step or type of step as needed.
3. Review the earlier skills needed for that step as appropriate.

The conscientious instructor would especially love to be able to offer such help even to a student who uses a non-standard method of solving a problem and needs help at an intermediate step of an alternate solution method.

B. What Type of Help Is Critical to Success?

Mathematics faculty often must give lip service to the notion that their mission is in significant part to teach "critical thinking" or "higher-order problem solving skills." Typical final exams administered by most mathematics departments, however, rarely contain problems requiring such thinking or skills. As students well know, top grades are routinely available by demonstrating nothing more than the ability to solve predictable types of problems very similar to those assigned as homework.

The most critical kind of help, therefore, is intelligent step-by-step help on routine types of problems whenever a student finds time to work on such problems. Realistically, however, even with aggressive use of available classroom time, office hours and e-mail or telephone availability, an instructor will be unavailable for most of his or her students' solution steps. Only intelligent computer-based instruction offers even the theoretical possibility of such omnipresent help.

C. What Is Available From Commercial Publishers?

Unfortunately, current commercial materials do not come close to the conscientious instructor's ideal. Two obvious limitations are evident in most commercial materials:

1. **Inadequate Problem Variety.** Problems and solutions are often "algorithmically generated," that is, generated from templates. Templates sharply reduce problem variety compared to any carefully written textbook. Worse, the stored solutions available for all problems of a given type are typically generated from a similar template, and thus all consist of exactly the same sequence of steps, differing only by trivial replacement of the constants and variables. See Appendix A.
2. **Inadequate Response to Incorrect Answers.** Prevalent short-answer and multiple-choice solution formats provide the computer little or no useful information about the student's intermediate steps. No help concerning the step or steps on which errors may have occurred, let alone any specific help with such steps, is normally possible. See Appendix B.

D. Next Generation Solutions

Intelligent, step-by-step help is available in a few, typically lesser-known, commercial and free math software packages, available over the Internet rather than from major publishers.

- MathXpert (www.HelpWithMath.com).
 - Student enters "commands" concerning individual steps to be performed.
 - Program performs steps, or helps student to continue using student's method.
- Math Professor (www.mathkal.co.il).
 - Student types steps in simplifying expressions and solving equations.
 - Supports any legitimate solution method.
- xyAlgebra¹ (www.xyalgebra.org).
 - Student types steps in simplifying expressions and solving equations.
 - Supports any legitimate solution method.
 - Promptly reviews the appropriate prerequisite on any error.
 - In verbal problems, changes strategy in response to unexpected correct steps.

See Appendix C.

E. What Can Be Done?

The responsibility lies with the adopters to demand better materials for students.

- When previewing software, ask about:
 - The form of solution entry
 - The variety of problems available.
 - The help available while solving problems.
- Lobby your colleagues!
- Complain to publishers' representatives.
- Raise challenging questions at commercial presentations.

¹ Free instructional algebra package written by the author of this paper. See Appendix C.

Appendix A. The Problem of Problem Variety

A typical textbook problem set on a new topic, such as the "invert and multiply" rule, contains problems whose solutions require a variety of prerequisite skills, as illustrated in Figure 1. Commercial mathematics software all too often relies on so-called "algorithmically generated" lists of problems, such as those in Figure 2. A more appropriate term would be "template generated" problems. The programmer can generate solutions to such problems using a similar template, but the student suffers from the resulting lack of problem variety and practice using various prerequisites.

Typical Textbook Problems and Solutions

$$\frac{\frac{a}{2}}{\frac{b}{3}} = \frac{a}{2} \cdot \frac{3}{b} = \frac{3a}{2b}$$

$$\frac{\frac{c}{3}}{\frac{2}{d}} = \frac{c}{3} \cdot \frac{d}{2} = \frac{cd}{6}$$

$$\frac{\frac{w}{3}}{\frac{x}{9}} = \frac{w}{3} \cdot \frac{9}{x} = \frac{9w}{3x} = \frac{3w}{x}$$

$$\frac{\frac{y}{8}}{\frac{z}{6}} = \frac{y}{8} \cdot \frac{6}{z} = \frac{6y}{8z} = \frac{3y}{4z}$$

Figure 1.

"Template Generated" Problems and Solutions

$$\frac{\frac{a}{2}}{\frac{b}{3}} = \frac{a}{2} \cdot \frac{3}{b} = \frac{3a}{2b}$$

$$\frac{\frac{c}{5}}{\frac{d}{7}} = \frac{c}{5} \cdot \frac{7}{d} = \frac{7c}{5d}$$

$$\frac{\frac{w}{3}}{\frac{x}{7}} = \frac{w}{3} \cdot \frac{7}{x} = \frac{7w}{3x}$$

$$\frac{\frac{y}{3}}{\frac{z}{9}} = \frac{y}{3} \cdot \frac{9}{z} = \frac{9y}{3z}$$

Figure 2.

Adoption decisions involving problem variety for software are much more complicated than for textbooks. The potential textbook adopter can quickly confirm adequate problem variety by spending a few minutes looking at the problems sets of a few selected sections of a proposed textbook. The potential software adopter must typically spend many hours actually solving the problems and keeping careful notes in order to confirm similar variety in a proposed software package.

Appendix B: Short Answer Example from a Commercial Program

The typical verbal problem in Figure 3 accepts only a short answer. The student has found a clever solution (bottom right). It uses a simpler method than the stored solution available from the software. However, the student has made a sign mistake in the last line of the solution, and thus entered an incorrect final answer into the short answer box.

Typical Commercial Software

Steve's age now is 1 year less than twice Leo's age now.
Three years ago, Steve was as old as Leo is now.
How old is Leo now?

Years old.

Done

YOUR ANSWER IS INCORRECT.
Leo is now 4 years old.
Let Leo's age now = x .
Then Steve's age now = $2x - 1$.
Three years ago, Steve's age was $(2x - 1) - 3$.
At that time, Steve was as old as Leo is now.
Steve's age three years ago = Leo's age now.
 $(2x - 1) - 3 = x$
 $2x - 4 = x$
 $x - 4 = 0$
 $x = 4$
So Leo is now 4 years old.

Student's Solution
(on a sheet of paper)

Let x = Leo's age now.
 $x + 3$ = Steve's age now.
 $x + 3 = 2x - 1.$
 $2 = x.$

Figure 3.

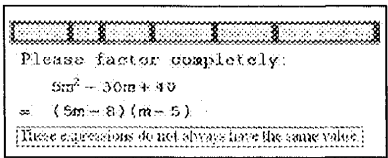
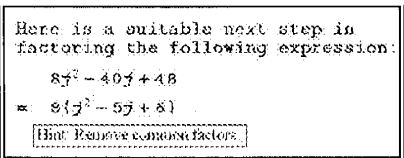
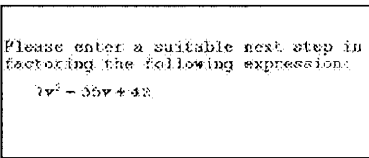
The program's response (bottom left) provides no help to the student in finding the error. Indeed, the program's response strongly implies that the student's solution is totally wrong. A conscientious instructor would have complimented the student for the clever method being used, and then would have gently pointed out the sign mistake.

Appendix C: Instant Review After Any Incorrect Step Using *xyAlgebra*

Upon entering an incorrect intermediate or final step, a student using *xyAlgebra* as a homework tutor benefits from instant review of the appropriate prerequisite skill.

Example. In Figure 4a, a student overlooks the common factor and factors incorrectly. In response *xyAlgebra* generates a similar problem (Figure 4b) and shows how to remove the common factor first. Then it asks the student (Figure 4c) for a step in yet another similar problem and accepts **any** equivalent step "forward" (common factor removed, product of two binomials or completely factored).

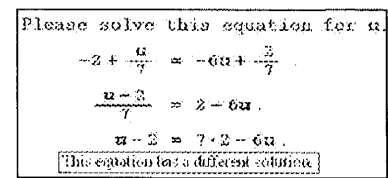
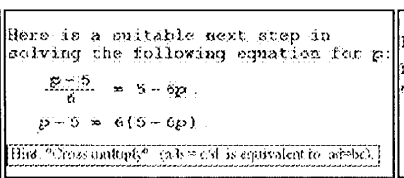
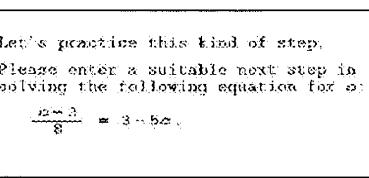
Figure 4. Review of Common Factor Removal Before Trinomial Factoring

 <p>Please factor completely: $5m^2 - 30m + 40$ $= (5m - 8)(m - 5)$ Hint: Remove common factors.</p>	 <p>Here is a suitable next step in factoring the following expression: $8y^2 - 40y + 48$ $= 8(y^2 - 5y + 8)$ Hint: Remove common factors.</p>	 <p>Please enter a suitable next step in factoring the following expression: $7y^2 - 55y + 42$</p>
Figure 4a	Figure 4b	Figure 4c

XyAlgebra offers intelligent help even for unusual solution strategies.

Example. In Figure 5a, a student solves an equation in an unanticipated way by collecting and subtracting the fractions, but then writes the product as if the dot implies parentheses. *XyAlgebra* generates a similar intermediate equation (Figure 5b), expresses the product correctly and reviews the appropriate rule. Then *xyAlgebra* asks the student in Figure 5c for a simpler step in the solution of yet another such equation.

Figure 5. Review of Correct Notation for Expression Multiplication

 <p>Please solve this equation for u. $-2 + \frac{u}{7} = -6u + \frac{5}{7}$ $\frac{u-3}{7} = 2-6u$ $u-3 = 7 \cdot 2 - 6u$ Hint: This equation has a different solution.</p>	 <p>Here is a suitable next step in solving the following equation for p: $\frac{p-3}{8} = 5-6p$ $p-3 = 6(5-6p)$ Hint: Cross multiply! (a/b = c/d is equivalent to ad=bc).</p>	 <p>Let's practice this kind of step. Please enter a suitable next step in solving the following equation for w: $\frac{w-3}{8} = 3-5w$</p>
Figure 5a	Figure 5b	Figure 5c

Daily Homework Assignments Graded by *xyAlgebra*: The Easy Way for Instructors to Use *xyAlgebra* as a Homework Tutor.

1. Ask students to download *xyAlgebra* from www.xyalgebra.com.
2. Run *xyManager* (the instructor's database) each day before class to preview and choose the day's *xyAlgebra* homework items.
3. Ask students to submit periodic *xyAlgebra* printouts showing items completed. The Help File in *xyManager* has more details on the preceding.