EXPLORING TRANSFORMATIONS OF THE PLANE VIA GEOMETERS SKETCHPAD

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In this session we will discuss two computer labs that I have written for our College Geometry course which is taken primarily by secondary math majors.

In the first lab five pairs of specified transformation are given. Students are expected to use the properties of the various types of transformations and Sketchpad to construct a single transformation that is equivalent to the composition. They are also asked to determine if their construction is robust. That is, does their constructed transformation remain equal to the composition if the starting triangle and/or other elements are moved?

The following are the pairs of transformations they have to work with. In each case they are to transform an arbitrary triangle.

a) Two reflections.
b) Two rotations.
c) Two translations.
d) A reflection and a rotation about a point on the line of reflection.
e) A reflection and a rotation about a point not on the line of reflection.

The solutions to each of the above pairs of transformations are as follows. In the explanations below A is a vertex of the original triangle and A" is its image under the composition.

a) The solution for two reflections depends on whether the two lines of reflection are i) parallel or ii) intersecting. In case i) the composition is a translation in the direction of the vector that is determined by the points A and A". In case ii) the composition is a rotation about the point of intersection, P, of the two lines of symmetry through an angle equal to the negative of the angle determined by the points A, P, & A".

b) For two rotations the solution is found as follows. Construct the segments determined by A & A" and B & B" and then construct the perpendicular bisectors of them. The intersection of the bisectors, P, will be the new center of rotation and the angle of rotation is the negative of the angle formed by APA".

c) The vector sum of the two translations will determine the translation that is equal to the composition.

d) For the composition of a reflection and a rotation about a point on the line of reflection we do the following construction. Construct the perpendicular bisector of the segment determined by A & A". This will be the line of symmetry for the reflection that is equal to the composition.

e) For the composition of a reflection and a rotation about a point not on the line of reflection we do the following construction. Find the midpoint of the segment determined by A & A" and the midpoint of the segment determined by B & B". The line determined by these two midpoints will be the line of symmetry for a glide-reflection that is equal to the composition.

In the second lab students worked with four classes of transformations. The classes were reflections, rotations, translations, and dilations. For each class of transformation they had to select an arbitrary member of the class and form the composition of that transformation with itself to see if it was equivalent to the identity map. If their particular composition was equivalent to the identity map, they were asked if this would be the case for every member of that class. If their particular composition wasn't equivalent to the identity map, they were asked whether the composition of some other member of the class with itself would be equivalent to the identity map or if no member of that class had that property.

This lab is quite straight forward. The conclusions that students are expected to reach are as follows.

1) No member is self-inverse for the classes of translations and dilations.
2) Every member of the class of reflections is self-inverse.
3) Only some rotations are self-inverse; namely those whose angle of rotation is an odd integer multiple of 180 degrees.