TANKS THE FINAL TIME : STRUCTURED MAPLE PROJECTS FOR DIFFERENTIAL EQUATIONS

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INTRODUCTION

I have been writing and assigning group Maple projects in my Differential Equations courses for a number of years now. I refer to these projects as "structured" in the sense that they are written in the form of a Maple worksheet which guides students through completing the project. This guidance allows for a minimal knowledge of Maple by the student since in many (although not all) cases, examples of the Maple syntax are blended into the presentation and precede the questions asked in the project. Other questions asked in these projects might, for example, produce a plot or an animation and require detailed explanations of these by the student. The idea is that students should learn some Maple, but not be overburdened by the Maple syntax since this would certainly take away from the main idea of using the technology to aid in understanding the mathematics involved.

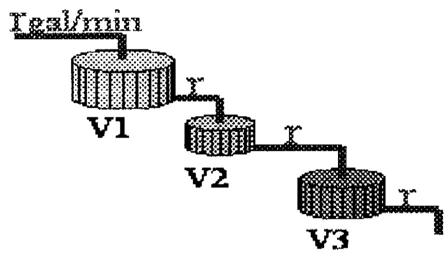
The projects I will be discussing all relate to a cascade of three brine tanks. These projects were originally written in Maple 9.5. I have also tested them in Maple 10 (the "classic worksheet" version and they ran just fine). The disk which I will be distributing (described below) was produced in Maple 9.5.

THE DISK

During my presentation at ICTCM18, I will be distributing a disk which contains a set of three structured Maple projects for differential equations (Tank Time Projects III, IV and V), as well as complete solutions to each project. Tank Time Projects I and II which are described below are not included on this disk, as they were presented at an earlier ICTCM meeting. Tank Time Project I can be found at the author's web site. Although I retain the copyright to this material, I hope that those who receive the material will be able to use some of it in their differential equations classes. I would ask, however, that you do not provide students copies of the project solutions or publish any of these solutions to the web. Each of these projects relates the the following problem:

THE TANK TIME PROBLEM

Let's consider a system of three brine tanks containing V_1, V_2, and V_3 gallons of brine, respectively. A brine solution containing k pounds of salt per gallon flows into tank 1 at a rate of r gallons per minute, while mixed brine flows from tank 1 into tank 2, then from tank 2 into tank 3, and then out of tank 3, all at the same flow rate of r gallons per minute. Assume initially that tanks 1, 2 and 3 contain a, b, and c pounds of salt respectively. Let $x_i(t)$ represent the amount (in pounds) of salt in tank i at time t, for i = 1, 2, 3. We'll be interested in finding $x_i(t)$, of course. Let's see a graphic of this situation:



If we let: $\frac{r}{V_1} = \alpha$, $\frac{r}{V_2} = \beta$, $\frac{r}{V_3} = \delta$, and $k\,r = p$, then we obtain the system of three differential equations:

$$
\begin{aligned}
\frac{d}{dt}x_1(t) &= -\alpha\, x_1(t) && + p \\
\frac{d}{dt}x_2(t) &= \alpha\, x_1(t) - \beta\, x_2(t) \\
\frac{d}{dt}x_3(t) &= \beta\, x_2(t) - \delta\, x_3(t)
\end{aligned}
$$

A brief description of each of the these projects follows.

1. TANK TIME PROJECT I: We discuss some problems related to a cascade system of three brine tanks. This investigation leads immediately to a system of first-order linear equations. The theory of linear first-order systems and Maple provide us with a number of ways to solve these systems. One objective of this project is to show students how to apply some of these methods using Maple. We will actually use five different methods of solving the appropriate system of differential equations. The basic problem which will consider can be found in the text Differential Equations and Linear Algebra by Edwards and Penney as Example 2 on page 493; however, we state the problem in the more general form . First we discuss various ways to use Maple to help us solve the basic problem as it appears in Edwards and Penney. Later, we try to answer some deeper questions about a more general form of the problem using Maple to help us. The focus of the latter part of this project is obtaining a complete answer to the following question: at any given time t0 > 0 and any salt content c0 what conditions on t0 and c0 are necessary to make it possible for all three tanks have content c0 at time t0 by appropriate choices for the initial salt contents a, b, and c of the three tanks?

2. TANK TIME PROJECT II: As in Tank Time Project I (see 1 above), we consider a cascade of three brine tanks. To be more specific, we consider the following problem: Consider a system of brine tanks containing V_1, V_2 , and V_3 gallons of brine, respectively. A brine solution containing k pounds of salt per gallon flows into tank 1 at a rate of r gallons per minute, while mixed brine flows from tank 1 into tank 2 , then from tank 2 into tank 3, and then out of tank 3, all at the same flow rate of r gallons per minute. Assume initially that tanks 1, 2 and 3 contain a, b, and c pounds of salt respectively. Let $x_i(t)$; represent the amount (in pounds) of salt in tank i at time t, for i = 1, 2, 3. Find $x_i(t)$, for i = 1, 2 , 3. In this project, we fix the initial salt contents a, b, and c of the three tanks and the flow rate r. We allow k, V_1, V_2, and V_3 to vary and attempt to find conditions on these four variables which will give us 50 lb of salt in tank 1 after 1 minute, 100 lb of salt in tank 2 after 2 minutes and 150 lb of salt in tank 3 after 3 minutes. This project is completely independent of the project Tank Time Project I.

3. TANK TIME PROJECT III: In the solution of the problem described above in Tank Time Project II there are nine variables to contend with in our solution to the general problem: t , r , k , a , b , c , V1 , V2 , and V3. Clearly there are too many to deal with all at once! We will fix the values of seven of the nine variables and investigate the results one at a time. The variable t will never be fixed, so that means we will fix in turn seven of the variables r, k, a, b, c, V1, V2, and V3 while keeping the eighth one variable. The "fixed" values we will be using are: r = 10 gallons/minute , k = 2 pounds/gallon , a = 25 pounds, b = 50 pounds, c = 75 pounds, V1 = 100 gallons, V2 = 200 gallons and V3 = 300 gallons. We'll be interested in seeing the effect on our solutions of changing that eighth variable. Specifically, in each instance we will be

interested in finding the values of those variables and the times t for which the salt contents of two or more tanks are equal. The amount of analysis required to complete this investigation is too much for a single project. So we will begin the investigation here but it will require two additional projects to complete it. These additional projects are described below. In tank time project III we consider first the problem of varying the flow rate r. After this analysis, we consider the problem of varying the concentration of the incoming brine k. This project is completely independent of the projects Tank Time Projects I and II.

4. TANK TIME PROJECT IV: In this project we consider the effects of varying the initial salt contents of the three tanks: a, b, and c. See the description of Tank Time Project III above for further details. This project is independent of all of the other Tank Time projects.

5. TANK TIME PROJECT V: In this project we consider the effects of varying the volumes V_1, V_2, and V_3 of the three tanks. See the description of Tank Time Project III above for further details. This project is independent of all of the other Tank Time projects.

THE WEB SITE

The web site given below contains a sample of the differential equations Maple projects as well as some additional Maple worksheets. Please feel free to access this material and to use whatever you find appropriate in your own classes. The copyright on this material is, however, retained by myself.

http://www2.SPSU.edu/math/fadyn/index.html

The material can also be found by beginning at the Southern Polytechnic State University home page and then navigating to Professor Fadyn's home page. The home page for Southern Polytechnic State University is located at: http://www.spsu.edu/