

OUTSOURCING INSTRUCTION: THE THREAT AND PROMISE OF INTELLIGENT MATH SOFTWARE

John C. Miller
Professor Emeritus, The City College of C.U.N.Y.
110 Riverside Drive, #14C, New York, NY, 10024
xyalgebra@mindspring.com

A. Background

Secondary mathematics instruction (basic algebra through beginning calculus) costs at least \$50 billion worldwide annually, mostly for salaries and mostly from "public" funds. Major changes may be inevitable if software appears that is provably more effective than current methods of instruction. Existing commercial software, still mostly limited to short answer and multiple choice formats offering little possibility of intelligent feedback, is unlikely to have such an impact. However, various lesser-known programs demonstrate and perhaps anticipate the intelligence and flexibility that may be characteristic of the next generation.

B. Good Practices

The AMATYC "Crossroads Revisited" document is a "best practices" document written by thoughtful and experienced math faculty. The current draft implicitly discusses several difficult issues in ways that raise questions different from those explicitly addressed in the draft itself.

1. The document recommends not relying primarily on a placement test for a student's initial placement in a college mathematics course. In fact, the main problem with initial placement is not the placement test. The main problem is the inevitable "round-off error" incurred when placing a new student, who has some degree of mastery of some topics from high school, into one of a sequence of discrete semester-long prerequisite courses. A properly written self-paced instructional package could initially test and then place each new student tentatively by topic or even subtopic, rather than by course. Then it could continually refine that placement for each student by reviewing forgotten pre-requisite topics and by skipping rapidly through previously mastered topics.
2. The document urges faculty to assume responsibility for understanding individual student learning styles and to implement strategies to maximize learning for each individual student. It recommends that faculty design and implement multiple instructional approaches for students with different learning styles. Unfortunately, a typical full-time developmental math instructor faces a hundred or more unfamiliar students each semester, and is often required to cover topics in less than half the time allotted in typical high school curricula. Careful diagnosis of

individual learning styles and meaningful implementation of multiple instructional approaches in that context is unlikely. However, both of these recommendations can be implemented relatively easily in instructional software that can monitor each student's responses to various types of presentations of mathematical material and problems and modify its presentations and explanations accordingly thereafter.

C. The Threat/Promise

Sooner or later, instructional software will probably appear that covers the entire secondary math curriculum, and offers all or most of the following:

- Student's choice of language.
- Initial placement by subtopic, not by course.
- Individual pacing.
- Intelligent step-by-step help on each problem.
- Review of prerequisites as necessary.
- Support for each student's preferred learning style.
- Optimal individualized instructional strategies.

Such software may prove to be more effective than current instructional methods. It will probably be marketed to state university chancellors and state education commissioners, who may find the prospect of improved results and large savings irresistible.

D. Programs Available Today.

The following three programs were written by individuals who distribute them over the Internet rather than via traditional publishers. All offer more of the features listed in the previous section than most of the current offerings of traditional publishers. Although none appears likely to have a huge impact, they may point the way to more impressive efforts to come.

- MathXpert (www.HelpWithMath.com).
 - Student enters "commands" concerning individual steps to be performed.
 - Program performs steps, or helps student to continue using student's method.
- Math Professor (www.mathkal.co.il).
 - Student types steps in simplifying expressions and solving equations.
 - Supports any legitimate solution method.
- xyAlgebra¹ (www.xyalgebra.org).
 - Student types steps in simplifying expressions and solving equations.
 - Supports any legitimate solution method.
 - Promptly reviews the appropriate prerequisite on any error.
 - In verbal problems, changes strategy in response to unexpected correct steps.

See the Appendix for more information on xyAlgebra.

¹ Free instructional algebra package written by the author of this paper. See Appendix.

E. Implementation Scenarios

The obvious implementation scenario is the "top-down" scenario. In this scenario, software of the sort described above is imposed as a cost-saving measure by deans, provosts, college presidents or university chancellors. These administrators are under constant financial pressure from their funding sources, most of which are ultimately using public funds. They know how much their basic skills mathematics instruction costs their institutions, and are likely to be receptive to plausible and skillfully promoted efforts to reduce those costs.

However, there is another scenario, better described as "bottom-up." Students may gradually find more and more resources available over the Internet, either free or available at moderate cost, providing better tools and resources for practice and skill development than the available classroom environments. Since most colleges offering high school level mathematics courses give examinations requiring nothing more than successful solution of very predictable types of problems, increasing numbers of students may simply stop showing up for traditional classes in which they are nominally enrolled. This could be the basis of a major scandal ("Student Preferring to Learn On-Line Perform Better Than Students Regularly Attending University Math Classes!") if software that really gives student the support they need becomes available.

F. The Big Question

If provably superior instructional software eventually appears, what should be – and what will be – the profession's response:

- Welcome and endorse whatever works best for our students?
- Protect our teaching jobs at any cost?

Many current faculty may confront this issue at some point in their careers.

Appendix: Instant Review After Any Incorrect Step Using *xyAlgebra*

Upon entering an incorrect intermediate or final step, a student using *xyAlgebra* as a homework tutor benefits from instant review of the appropriate prerequisite skill.

Example. In Figure 4a, a student overlooks the common factor and factors incorrectly. In response *xyAlgebra* generates a similar problem (Figure 4b) and shows how to remove the common factor first. Then it asks the student (Figure 4c) for a step in yet another similar problem and accepts **any** equivalent step "forward" (common factor removed, product of two binomials or completely factored).

Figure 4. Review of Common Factor Removal Before Trinomial Factoring

<p>Please factor completely:</p> $9m^2 - 30m + 40$ $= (5m - 8)(m - 5)$ <p>These expressions do not always have the same value.</p>	<p>Here is a suitable next step in factoring the following expression:</p> $8j^2 - 40j + 48$ $= 8(j^2 - 5j + 6)$ <p>[Hint: Remove common factors.]</p>	<p>Please enter a suitable next step in factoring the following expression:</p> $7v^2 - 35v + 42$
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Figure 4a

Figure 4b

Figure 4c

XyAlgebra offers intelligent help even for unusual solution strategies.

Example. In Figure 5a, a student solves an equation in an unanticipated way by collecting and subtracting the fractions, but then writes the product as if the dot implies parentheses. *XyAlgebra* generates a similar intermediate equation (Figure 5b), expresses the product correctly and reviews the appropriate rule. Then *xyAlgebra* asks the student in Figure 5c for a simpler step in the solution of yet another such equation.

Figure 5. Review of Correct Notation for Expression Multiplication

<p>Please solve this equation for x:</p> $-2 + \frac{x}{7} = -6x + \frac{2}{7}$ $\frac{x-2}{7} = 2 - 6x$ $x-2 = 7 \cdot 2 - 6x$ <p>This equation has a different solution.</p>	<p>Here is a suitable next step in solving the following equation for p:</p> $\frac{p-5}{6} = 5 - 6p$ $p-5 = 6(5-6p)$ <p>[Hint: "Cross-multiply" (at = cd is equivalent to ad=bc)]</p>	<p>Let's practice this kind of step.</p> <p>Please enter a suitable next step in solving the following equation for c:</p> $\frac{c-3}{8} = 3 - 5c$
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Figure 5a

Figure 5b

Figure 5c

Daily Homework Assignments Graded by *xyAlgebra*: The Easy Way for Instructors to Use *xyAlgebra* as a Homework Tutor.

1. Ask students to download *xyAlgebra* from www.xyalgebra.com.
 2. Run *xyManager* (the instructor's database) each day before class to preview and choose the day's *xyAlgebra* homework items.
 3. Ask students to submit periodic *xyAlgebra* printouts showing items completed.
- The Help File in *xyManager* has more details on the preceding.