USING CALCTOOL TO TEACH CALCULUS

James S. Rolf, PhD
United States Air Force Academy
Department of Mathematical Sciences
2354 Fairchild Hall
USAF Academy, CO 80840
jim.rolf@usafa.edu

Introduction

In the fall of 2002, the Department of Mathematical Sciences at the United States Air Force Academy began to integrate technology into our core mathematics courses. Initially, 'technology' consisted of a laptop computer, Excel spreadsheets, a computer algebra system, and java applets supplied by our textbook publisher. However, students the java applets the most useful of the software options at their disposal. This was , to a large degree, due to the relatively easy-to-learn syntax and the interactivity that such applets provided. So in an effort to provide users with a consistent look-and-feel and to free ourselves from textbook publishers, we developed two suites of java applets to be used in Pre-Calculus, Differential Calculus, and Integral Calculus. The most recent versions of these applets-- CalcTool 1 and CalcTool 2-- can be freely used and downloaded from http://www.jimrolf.com/calcTool.htm. The remainder of this paper will give an overview of each of these tools.

CalcTool 1

CalcTool 1 allows the student to visualize or 'discover' ideas commonly discussed in Pre-Calculus and Differential Calculus courses. Students have the ability to graph functions, explore function properties (including trigonometric functions), visualize the ideas behind a delta-epsilon proof of the limit of a function, discover the relationship between slopes of secant and tangent lines along with the derivative operation, compute symbolic derivatives, visualize graphs of parametric functions, and explore certain vector properties.

The Graphing panel (see Figure 1) provides much of the functionality of a graphing calculator. It allows the user to easily input up to three functions and visualize their graphs. The user can zoom in on a selected point by clicking on the graph. Additionally, the user can evaluate functions at selected points or find roots of functions by using the 'Solve' option.

The Comp panel (see Figure 2) allows the user to use a slider to change parameters that affect properties of user-defined functions. Students can quickly visualize the algebraic relationship between multiplying the independent variable by a constant and how this scales the resulting graph of the function. Students can similarly visualize how a function must be altered to translate it along the x-axis. In short, this panel provides the student an interactive tool that allows visual learners to discover relationships between algebraic manipulations of functions and the effects on their graphs. The Trig panel specifically

examines trigonometric functions and allows students to change to change parameters that affect frequency, amplitude, and phase shifts.

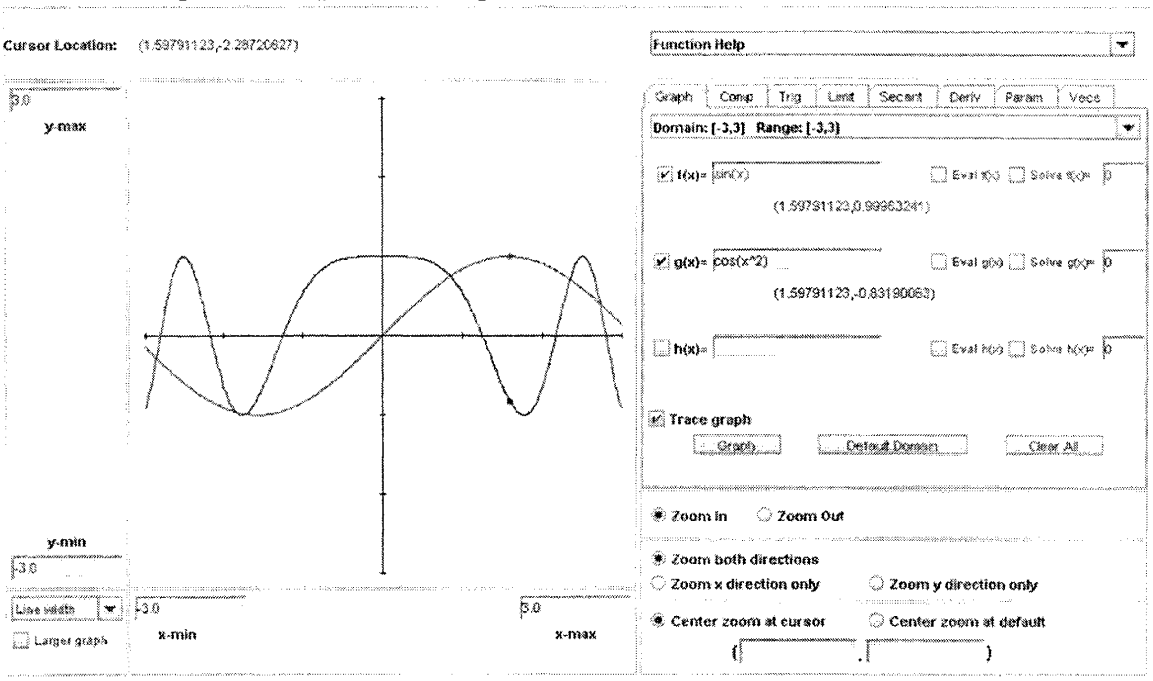

Figure 1: Graphing Panel of CalcTool 1

The Limit panel provides a visual interpretation of traditional epsilon-delta proofs of the limit of a function, $\lim_{x \to c} f(x) = L$ The user specifies values for L, c, and epsilon. This generates the horizontal lines representing $(L-\varepsilon, L+\varepsilon)$ in the range of the function (see Figure 3). The user may then use a slider to change the value of delta to alter the radius of the vertical lines in Figure 3 in order to determine which delta is required to force the function $f(x)$ to be completely contained within the $(L-\varepsilon, L+\varepsilon)$ range represented by the horizontal lines. This panel allows us to give the students an intuitive idea of what a limit means without suffering the difficulty of algebraic manipulations required to precisely 'prove' this notion.

The Secant panel allows the student to click on the graph of a function to produce a tangent line and a secant line. The student can then move one point on the secant line closer to the tangent point and observe how the slope of the secant line approaches the slope of the tangent line. This sort of functionality is common in other applets on the internet. The benefit of including it in this suite is that the user has a consistent look-and-feel across applets, which lowers the learning barrier needed from applet to applet.

The Derivative panel symbolically computes up to two derivatives for user-defined functions. Students find this panel useful to graph a function along with its first two derivatives and explore the relationships between the roots of derivatives, concavity, and the increasing/decreasing nature of the original function.

The Vector panel enables students to easily graph vectors and move them around in order to visualize vector addition/subtraction. Again, this provides manner to visualize an algebraic process and can easily be combined with exploratory activities that allow students to 'discover' the geometric interpretation of vector addition/subtraction.

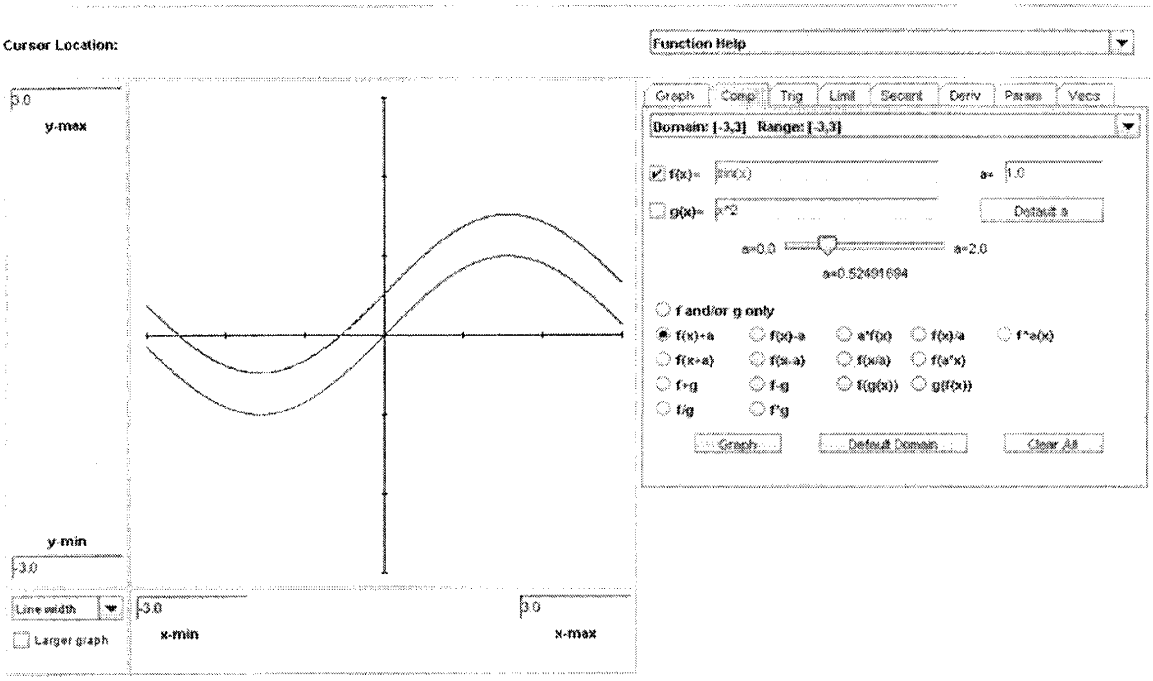

Figure 2: Comp Panel of CalcTool 1

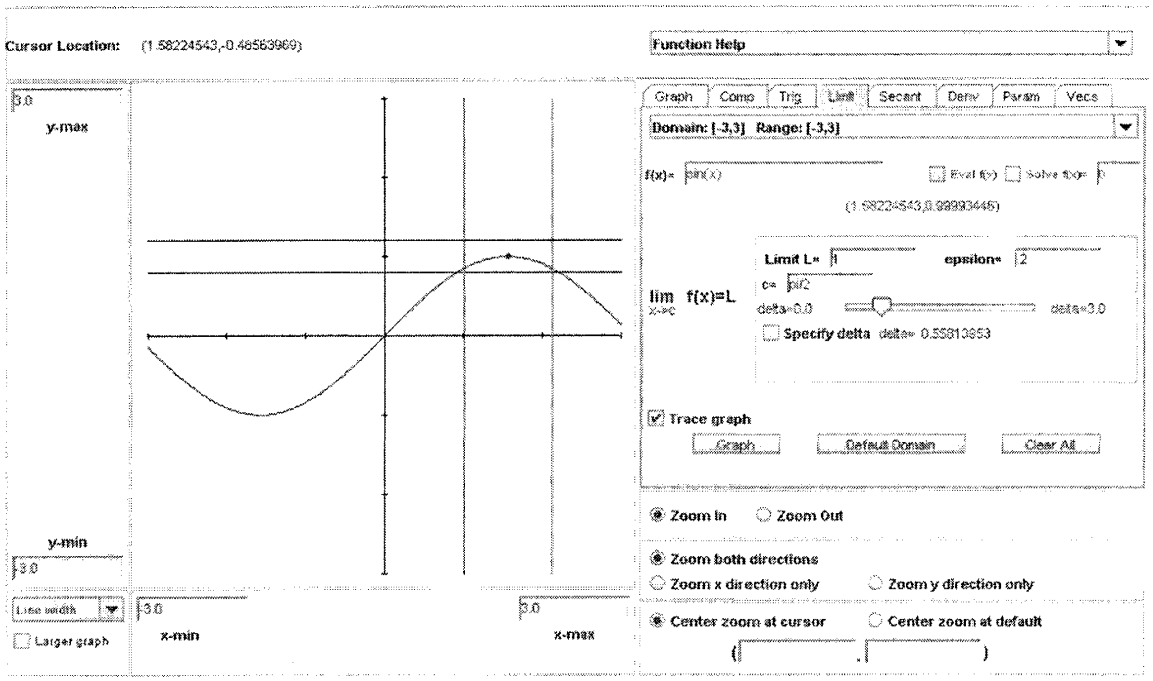

Figure 3: Limit Panel of CalcTool 1

CalcTool 2

CalcTool 2 covers topics typically found in an Integral Calculus course. The same Graph panel and Derivative panel found in CalcTool 1 are also included in this suite of applets. Students can additionally construct Riemann sums, graph numerical approximations to anti-derivatives, visually explore the Fundamental Theorem of Calculus, construct Taylor Series, power series, and generalized function series, graph slope fields and solutions to systems of differential equations, and visualize Euler's method. The look-and-feel of CalcTool 2 is the same as CalcTool 1, so students need not learn any new syntax when utilizing this tool.

The Riemann panel allows student to input an arbitrary function and graph Riemann sum approximations to the area under this curve (see Figure 4). The user may choose to approximate with the Left endpoint rule, the Right endpoint rule, or the Midpoint rule. Or he/she can use a slider to utilize any point in between the Left and Right endpoints. The user can easily increment the number of rectangles by clicking a button and quickly see how the approximation of the area improves. Finally, the user can utilize trapezoids for the approximation of the area if he/she chooses. This kind of applet allows the student to see quickly and easily the important ideas behind the construction of a Riemann sum before dealing with messy algebra.

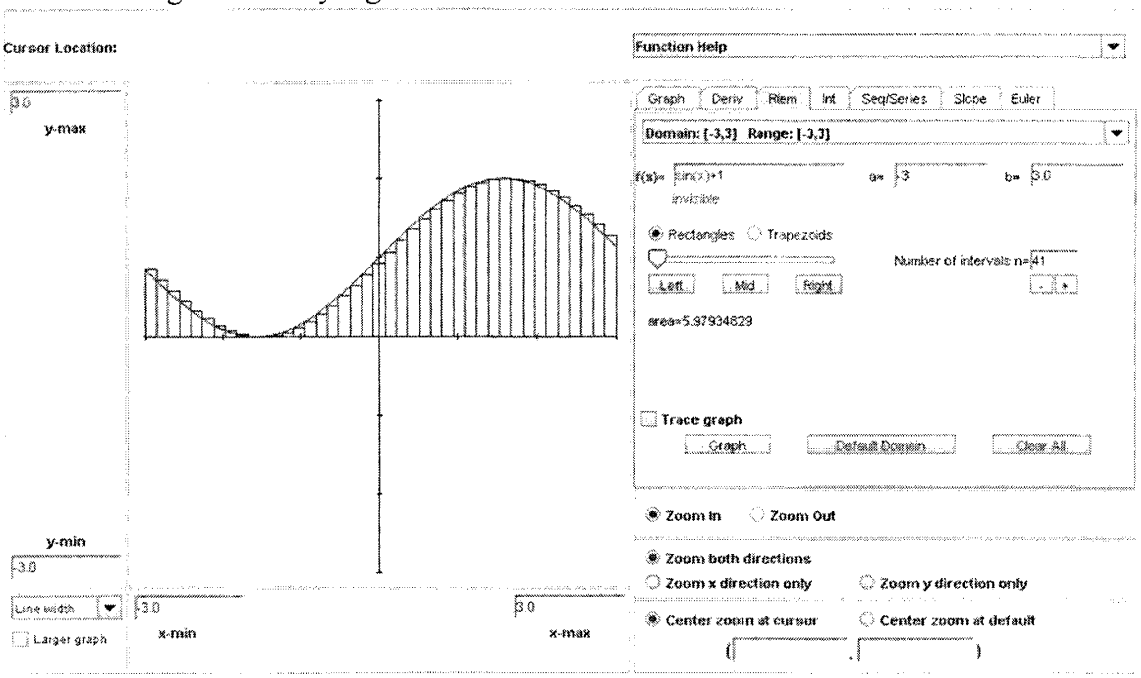

Figure 4: Riemann panel of CalcTool 2

The Seq/Series panel allows a student to quickly construct series approximations of functions. Figure 4 illustrates Taylor Series approximation to $f(x) = \sin(x) + 1$ using a fourth order polynomial centered at $x = 1$. This panel has the ability to graph general

power series of the form $\sum_{k=0}^{N} c_k (x-a)^k$ with user-supplied definitions of c_k and a. If desired, students may also graph generalized function series of the form $\sum_{k=0}^{N} a_k(x)$.

The Slope panel (see Figure 5) graphs slope fields for both a single differential equation and a system of differential equations. The user simply clicks on an initial point on the slope field and the applet graphs numerical solutions to the differential equation(s). This same idea is used in the Euler panel, except that consecutive clicks on the graph reproduce Euler approximations to the solution.

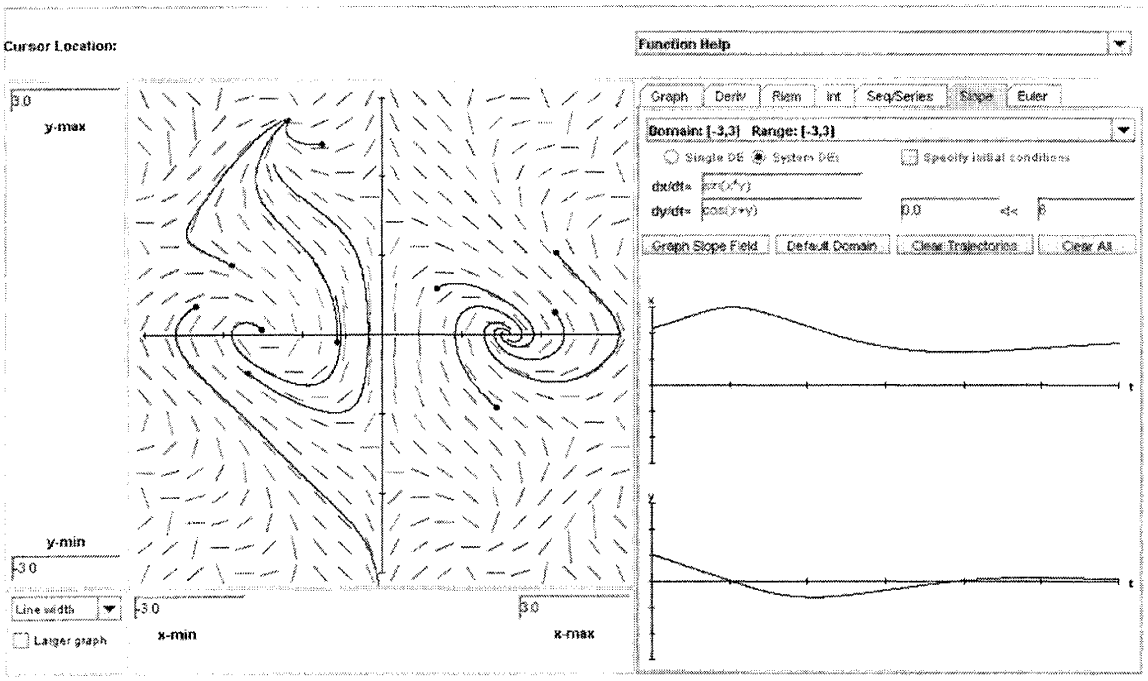

Figure 5: Slope panel of CalcTool 2

Conclusion

CalcTool 1 and 2 are two suites of applets freely available at http://www.jimrolf.com/calcTool.htm for use and download. Many of the ideas contained in these applets are not new. However, the collection of all of these ideas into one place allows the user to have a consistent experience and reduces the overhead necessary to install and learn how to use applets that are collected from the far reaches of the internet. These applets further allow the user to interact with important mathematical concepts while minimizing algebraic computations that sometimes get in the way of students understanding the big picture ideas at work behind the scenes. Finally, many of the applets in these suites can easily be used in conjunction with exploratory activities in class so that the student can discovery for him/herself important mathematical constructs.