JUST IN TIME – FOR WHAT?

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Abstract

"Just-in-time" teaching of advanced applied mathematics such as Calculus II or Differential Equations means that students take these math courses in the same semester as they take science courses (usually physics) in which they would use the mathematical concepts for mathematical modeling. While this simultaneous offering of courses may not be a problem at large universities, small colleges face the problem that suitable science courses may not be scheduled in the same semester as the math courses, or may not be offered at all. This report describes the experience of including science experiments in a differential equations course.

The experiments included damped and undamped harmonic motion, Newton's Law of Cooling, and resonance in LCR circuits. Students were asked to prepare for each experiment by setting up a differential equation that would model each experiment. They were asked to use the model to qualitatively predict the behavior of the experiment. Students performed to actual experiments in small groups: Measurements were taken using computer-interfaced devices. Students used the software to compare the data obtained from the experiment with anticipated solution.

Introduction

The just-in-time approach - sometimes, more appropriately, called the *integrated* approach - refers to the timing of teaching differential equations (or another suitable mathematics course) with applied physics or engineering courses, so that students who would encounter a differential equation in their math class would be able to use it right away (just in time) in their physics class. This approach has been successfully used at several large universities to increase persistence and student success [2, 4]. Some researchers have reported success in synchronizing up to four interdisciplinary courses and applying what was learned in one course in the others [3]. Clearly, a just-in-time approach would give students plenty of opportunity to use and appreciate the usefulness of differential equations.

Indiana University East is a small regional campus with 2500 commuter students. Most of these students are enrolled in Business, Nursing and Education, with only few in the Arts and Sciences. The most advanced physics course offered at Indiana University East only requires Trigonometry. This makes it impossible to deliver a just-in-time approach together with a suitable physics course. One approach to overcome this is to provide students with data-sets, that the instructor has previously collected, or with other data that is readily available from the internet. Students could then use this data to verify that it indeed fits a particular model. While researchers report that this kind of modeling approach positively affects students' attitudes and learning [1], this approach denies students the hands-on activity of actually collecting the data themselves.

In order to give students the hands-on experience of actually collecting the data, experiments were created and included in the course. These experiments let students apply differential equations in the context of the mathematical modeling process and use the data they collected in order to verify the adequacy of the model. This approach is intended to mimic the experience students would gain if they indeed were part of a true just-in-time learning community. The goal in the design of this class format was to let students experience the processes of conjecturing a model to verifying its truth with experimental data.

The Course Model

Of course, accommodating any new material in a course is always a challenge. In order to be able to maintain the rigor and the required amount of material that must be taught, some topics were slightly curtailed. This was justified because none of the students in the class were science or engineering majors. Indeed, most students were pre-service teachers, whose professional career would probably not suffer if they were not exposed to the Frobenius series solution near regular singular points. In addition, the final exam was replaced with a project in which students had to report on the lab experiments. This freed up enough class to accommodate three class periods in which experiments could be performed.

The slight reorganization of topics permitted to gear the entire course toward the cumulative experience of using differential equations to model behavior in the natural sciences.

One of the experiments used was Harmonic motion. Students know from experience that a mass that is bobbing up and down on a spring will move in a periodic motion, and that friction will, over time, reduce the amplitude of the motion. Experience also tells that amplitude and period of the motion are influence by the mass and by the stiffness of the spring.

The modeling process begins by using laws of physics to obtain a differential equation that models the motion of the spring. In the case of Harmonic motion, one obtains a second order linear differential equation. The laws of physics used in obtaining the differential equation were chosen so that they could be reasonably expected to be within reach of the audience. For example, the modeling for the Harmonic Motion differential equation requires such facts as Newton's Second Law of motion ($F = mx''$), Hooke's Law ($F = -kx$), dampening ($F = -cx'$) and the principle of conservation of force. The process of obtaining the differential equation was left to the students. Each student was given the task to prepare a presentation in which they explained how a differential equation models a given situation; Harmonic motion was the topic of one of these presentations. The differential equation obtained for harmonic motion is $mx'' + cx' + kx = 0$, where m is the mass of the object, k is the stiffness of the spring, and c is a dampening coefficient and x is the position of the object.

Modeling a phenomenon, like Harmonic Motion, preceded the discussion of methods by which differential equations of this sort can be solved. As such, the students' presentations served as the motivating example for the discussion of how certain classes of differential equations can be solved. In this example, the differential equation served to motivate how to solve linear differential equations with constant coefficients.

The methods for solving this class of differential equations were discussed in a in a relatively traditional class-format, in which lecture and guided group practice were interlaced.

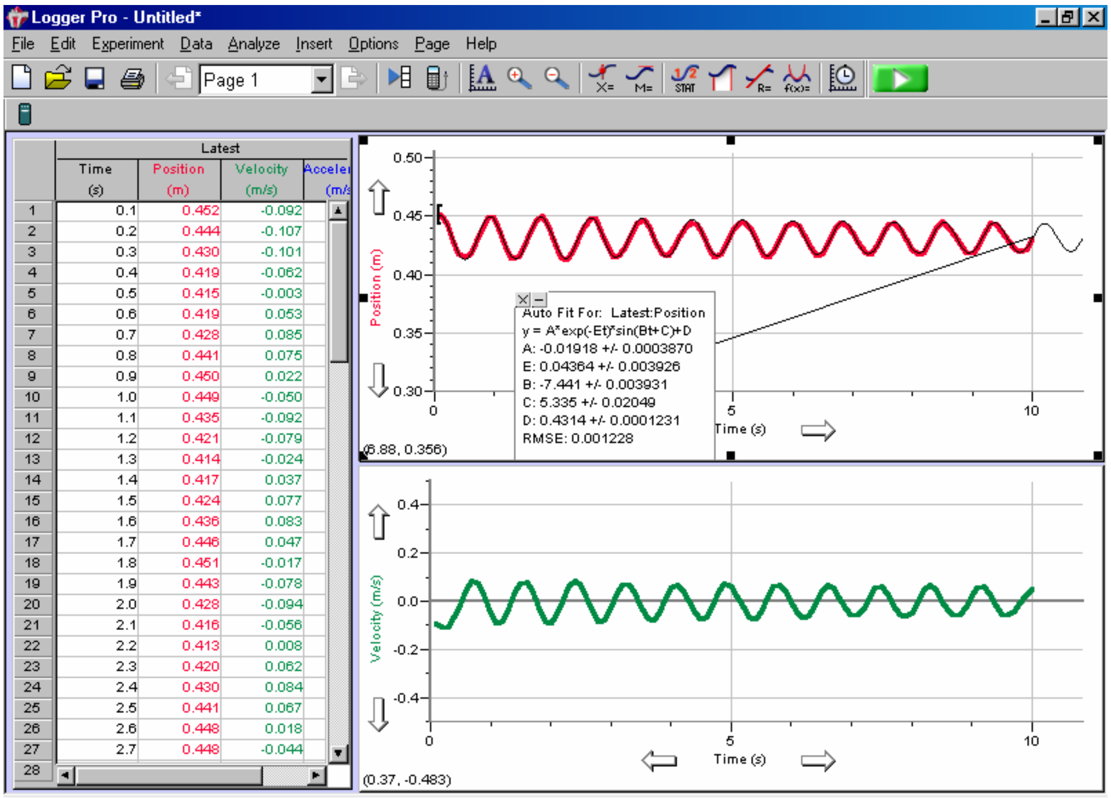
Once the theoretic background for the solution-process was introduced the original problem was discussed anew, this time emphasizing the possible solutions. The solution to the differential equation above can have several qualitatively different solutions: If there is no dampening, the mass will continue to bob up and down forever. If dampening is small, compared to the mass and stiffness of the spring, the mass will move up and down, but the amount by which it moves will decrease over time. Finally, if the friction is large, the mass will return toward its rest position without oscillating. While these phenomena are clearly common knowledge, differential equations are needed to predict which of the three scenarios will occur. In addition, the solution to the differential equation will predict the period of the oscillation for undamped or underdamped motion and also the amount by which the amplitude of underdamped motion decreases over time.

Until this stage, students had discussed the applications of differential equations only from a theoretical point of view. The depth of the discussion up to this point may be expected in any other differential equations course. In addition to considering the applications of differential equations from a theoretic point of view, three class periods in the semester were included during which students performed experiments and verified that their solution to the initial problem does indeed model the situation.

In the example of the Harmonic Motion, students began by determining the stiffness of the spring *k*, and the effective moving mass *m*. The actual experiment consisted of letting the mass swing on the spring. When letting the mass swing in air, there is almost no friction and this case can be considered as the undampend case. To obtain an example of a dampened case, the moving mass was suspended first in water and then in engine oil. Students collected the data electronically, and used the computer to compare their data with the prediction made by the theoretical consideration.

In all experiments, students used equipment from the physics department in order to collect and analyze the data. In the example of Harmonic Motion, the displacement of the mass was collected as a function of time. The software displayed the data graphically and also performed an analysis that provided students with the observed amplitude and period of the motion. The setup of the experiment for Harmonic Motion is in the adjacent picture.

The software used to collect that data was Logger Pro, with suitable probes for sensing motion, temperature and electrical current for the various experiments. An example of a screen plot of the collected data is below.



Besides displaying the collected data in tabular and graphical form, the program provides a curve-fitting feature that allows students to determine the best fit curve. This feature allowed students to determine the parameters of their curve and compare those with the parameters that one would expect based on the solution to the differential equation and the parameters of the experiment (e.g., the mass and the stiffness of the spring).

The cumulative final exam asked students to reflect on the material. This report should include how the differential equation is obtained from the modeling process, how it is solved analytically. Students were also asked to provide a qualitative analysis of the solution. Finally, students were asked to describe the experiment they conducted, and to compare the actual data from the experiment with the expected data.

Other Experiments Used

Other experiments conducted for this course were the following:

Differential Equation Topic	Experiment(s)
Separation of variables	Newton's Law of Cooling Toricelli's Law
Second Order Linear Differential Equations with constant coefficients	Harmonic motion (damped and undamped)
Resonance	LRC circuit with sine-wave of varying frequency
Fourier Transforms	LRC circuit with square wave

Because this course had less than 10 students enrolled, no qualitative data is available to document the effect of the changed course format. Anecdotal evidence suggests that students were at first weary of having to perform experiments in a math class. Students' comments after the completion of the semester suggested, however, that they enjoyed the class.

References

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