Calculus Computer Laboratory: Experience Guiding Current Practice

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Valparaiso University has more than three decades' experience with using computers in our calculus classes. Over the years, assignments have changed dramatically as the technology available to students has increased in power and sophistication. Today Calculus I and II are taught in a 3+2 hour lecture/laboratory course, with the laboratory assignments designed to complement and extend the work done in class.

This long experience has made us aware of the challenges there are in creating a successful laboratory assignment. The challenges are for an assignment to be:

1. Something that could not be done without the computer;
2. Worth doing, not just busy work – don't waste student time;
3. Challenging enough to maintain interest, but doable;
4. Broad enough in scope to provide opportunities for students to think;
5. Focussed enough to be feasible;
6. Structured to stress communication of results, not just a mess of printouts.

The overall goal is to get the students to perceive the computer as a benefit and not merely an additional burden.

Preliminary Laboratory Assignments in Preparation for the Consulting Example

The laboratory assignments form a progression from direct, "just follow the instructions" form (in which they become familiar with the software) to more open-ended, "think before you start typing" form (which meet our goal of teaching problem solving and logical thinking). The relevant preliminary assignments were:

1. Riemann sums - Left- and right-hand endpoints, midpoints. This includes error bounds - graphically estimating bounds on nth derivatives makes this a relatively painless process, and clearly exhibits the relative accuracy of different methods.

2. Trapezoidal rule and Simpson's rule - again there is an impressive difference in the number of intervals required for equal accuracy in the different methods.

3. Applications of integration - arc length, volumes and surface areas of revolution. They use the functions and methods derived in the first two labs.

These assignments shared a common student perception that they were abstract, not "real-world", and perhaps even irrelevant symbol manipulation only. To place these assignments into a more concrete setting I had them estimate the volume and surface area of the Saint Louis Arch (see below), and then the Consulting Assignment (which follows it).

The report had to be in the form of a formal proposal, with grade dependent in part on style of presentation. So how do you grade a report that has words and not just formulas and algebra and numbers? I have found a grading template (see below) helps both in my evaluation and in the students' understanding of their grades. Style especially is a continuum rather than discrete steps, so that is how I mark them. Points are then assigned, with an "Average" presentation getting full marks, and "Superlative" performance getting extra points. Extra credit for me is superior work, not merely an additional quantity of mediocre work.

A significant part of the process is the post-mortem, when papers are handed back. The estimates have generally gone from 50% too low to an order of magnitude too high. This gives the opportunity to discuss reasonableness of solutions, how to recognize when answers don't make sense, and how to go about determining where problems lie. It also helps answer the question, "Why do I have to do this (whatever the current assignment is) when a computer (or increasingly, a calculator) can do it all more easily?"

Saint Louis Arch Laboratory Assignment

Time shift: Imagine it is some years ago and the St. Louis Arch is still in the planning stage. You have been hired to write up an analysis of the proposed design. It is expected that your report will be neat, thorough and understandable. You should present enough details and discussion so that even your boss can follow what you do and be convinced that your answers are correct. Graphs or diagrams can be very helpful. What you hand in should be ready for publication with your name on it. Your pay for the report (i.e., points awarded for it) will depend on the thoroughness and the professional level of your presentation. The assignment sheet should **not** be part of your report.

Report Format: Start with a discussion of the problem, then a section describing the method of solution. Follow this with your results, then a section of conclusions and observations. Additional data and graphs can be included in an appendix.

Integrals should be approximated using your function for Simpson's Rule. Decide an acceptable error for your approximation and use this to determine n, the number of intervals to use, for each integral. Document this in your report. The actual determination of n, graphs or equations used, should be in an appendix rather than in the main report.

Information Provided: The equation used to construct the Gateway Arch in St. Louis will be

$$y = 693.8597 - 68.7672 \cosh(.0100333\, x), \qquad -299.2239 \geq x \geq 299.2239$$

where x and y are measured in feet. Cross sections of the arch are equilateral triangles, point inward, and (x,y) traces the path through the centers of mass of the cross-sectional triangles. For each value of x, the area of the cross-sectional triangle is

$$A(x) = 125.1406 \cosh(.0100333\, x).$$

Information Not Provided: How to find the height and length of side of an equilateral triangle of given area, and where its center of mass is located. You will need all of these for this assignment.

Discussion: Include at least answers to the following:

a) What is the highest point on the curve through the centers of mass?

b) What is the height of the highest point of the arch?

c) How wide is the arch at ground level?

d) What is the volume of the arch? **(N.B.** Integrating A(x) dx is a quick and easy answer to this question. It's major drawback is that it is wrong.)

e) What is the length of the curve through the centers of mass?

f) What is the surface area of the arch?

g) Anything else you might consider interesting. You might, for instance, look at what the shape would have been if the curve had been a parabola or ellipse instead of a catenary.

Consulting Laboratory Assignment

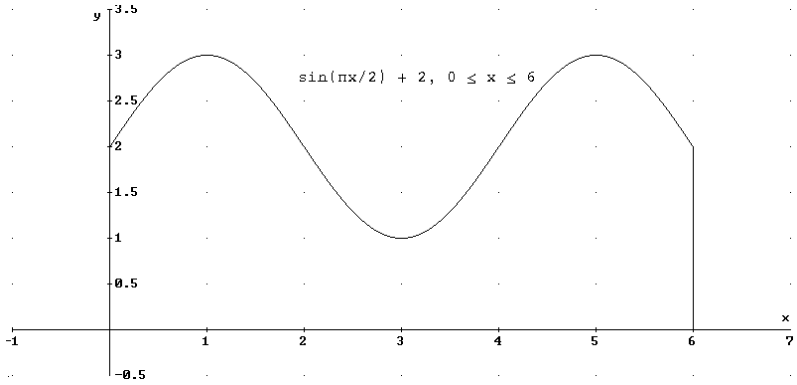
If You Don't Read This You Will Fail the Assignment

The important part of this assignment is the thought you put into it, the analysis you do and the presentation of your solution, not the answers themselves. The "right" answers presented in an unsubstantiated or sloppy manner will be worth very little. If you are unconvincing you have failed. Points will be added (or subtracted) for style.

The Situation

Having just graduated you have started your own consulting company, Don't Panic Dot Calm, whose motto is "There's No Problem so Big We Won't Take Your Money to Attempt a Solution". Both rent and loan payments are due so you can't be picky and take anything that comes along in answer to your newspaper ad: "Desperate Problems? Deadline Tomorrow? Just my thing - I'm a student."

The local millionaire, Mor Dahlarzan Senz (known as M.D. or Doc) has decided to expand his mansion. He plans to put a monumental railing around the top of it, supported by 200 pillars each 6 feet tall. Being somewhat eccentric (if he didn't have money he would be just crazy) he has decided they should have a sinusoidal shape, generated by revolving the region bounded by $y = \sin\left(\frac{\pi x}{2}\right) + 2$, x = 0, x = 6 and the x-axis around the x-axis (see picture).



His contractor is trying to work out an estimate of materials needed, and you are being hired as high priced consultant to the project. In meeting with the contractor you learned the following:

1. The pillars will be molded concrete, made at the site. How much concrete will be needed?
2. The pillars are finished by painting them with a protective resin, .2 inches thick. How much resin will be needed?
3. 6 decorative braids will be applied to each pillar, running along it from top to bottom. What length of braid will be needed?
4. Concrete costs $ 76.50 per yard, resin $12.95 per gallon (7.48 gallons per cubic foot), and the decorative braid runs $1.07 per linear foot. What is your estimate for the cost of materials?
5. Being eccentric, Doc wants the estimates in 1 to 3 to be within a tolerance of .001 .

Stating the obvious: Reports done with word processors are more impressive. Your report should include a description of the problem, the method(s) used, the results, and any observations or conclusions. Include appropriate graphs or tables, clearly labeled, and DERIVE calculations as an appendix. Clear explanations are easier to comprehend than just final answers. If your roommate can't understand your report without knowing the assignment your report isn't adequate.

Grading Template

Contractor's Evaluation of the Consultants' Report from

Don't Panic Dot Calm

"There's No Problem So Big We Won't Take Your Money to Attempt a Solution"

1. (5) Discussion of the Problem: ___ None ___ Duplicate assignment ___ Expanded

If Expanded:

	Underwhelming	Average	Superlative		Underwhelming	Average	Superlative
Clarity				Grammar			
Presentation				Content			

2. (15) Derivation of Integrals:

		Underwhelming	Average	Superlative	
Explain	Method				_____ No Explanation

	Correct Equation			Presentation			
	No	Almost	Yes	Underwhelming	Average	Superlative	
Volume							
Surface Area							
Arc Length							

3. (10) Within tolerance:

	Graph $f^{(n)}(x)$			k_i			n			value		
	No	Almost	Yes	No	Almost	Yes	No	Almost	Yes	No	Almost	Yes
Volume												
Surface Area												
Arc Length												

4. (15) Cost

	Correct Value per pillar			Correct Value for 200		
	No	Almost	Yes	No	Almost	Yes
Concrete						
Resin						
Braid						
Total						

5. (5) General Style

Underwhelming	Average	Superlative