

DESCARTES VISITS WALL STREET AND MEETS THE TI-84+SE:
A FINANCIAL JOURNEY

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Over the past decade, the complex world of finance has been brought into our homes at an ever increasing pace through a variety of media. This workshop explored an array of finance topics beginning with annuities. An annuity is a sequence of periodic payments; the payments may or may not be equal. Typical examples of annuities are mortgage payments and regular deposits to a savings vehicle. We restrict our work to ordinary annuities where periodic payments are made or received at the end of each time period and coincide with the interest conversion period.

Under the annuity umbrella we explored future value and compounding; and present value and discounting. The future value of an annuity is the total accumulated amount of the annuity at the end of its life. In general, this is expressed as the sum of the first n terms of a geometric progression: $FV=R+R(1+i)+R(1+i)^2+\dots+R(1+i)^{n-1}$ where FV represents future value, R is the equal periodic payment, i is the interest rate per compounding period, n is the number of equal periodic payments, and $(1+i)$ is the common ratio. This sum simplifies to: $FV=R\left[\frac{(1+i)^n-1}{i}\right]$.

The present value of an annuity is an amount that must be invested “today” to generate a specified number of periodic payments; it is the value that these future cash flows are worth “today”. In general, present value is expressed as: $PV=R\left[\frac{1-(1+i)^{-n}}{i}\right]$.

Understanding the basics of annuities affords us the opportunity to explore the heart of this paper, which is net present value (NPV) and internal rate of return (IRR).

Net present value and internal rate of return are financial topics used to assist in investing and capital budgeting decisions. It is through exploring the basics of these topics that we find a connection with Descartes and the TI-84+SE. As a quick example into the concept of net present value, suppose that an entrepreneur wanted to open a specific type of small business. After completing some industry research s/he was able to estimate the startup cost, yearly revenue, yearly cost, number of years to run the business, salvage value of the business, and interest rate (here, discount rate) for similar small businesses. The entrepreneur wishes to determine whether or not to undertake this investment. In short, the entrepreneur needs to compare today’s estimated market value with today’s cost of this business venture. To do this, we need to compute net present value which is the

difference between the potential investment's future cash flows in today's dollars and today's cost. Expressing the value of future cash flows in today's dollars is the heart of present value. If this difference, or net present value, is positive, then this is an indication to undertake the investment. If net present value is negative, then the investment should be rejected. If net present value is zero, then the investor is indifferent on an accept/reject decision. It is important to note that net present value is not the same as profit; it is simply net cash returns expressed in today's dollars.

As an example of net present value and the use of the built-in finance applications on the TI-84+SE to assist in solving such problems, suppose an entrepreneur wanted to investigate the possibility of starting a small business. Estimated start-up cost is \$35,000; yearly revenue is projected to be about \$18,000; yearly costs are estimated to be \$11,000; and the salvage value is estimated to be \$3,000 if the business closes in nine years. The discount rate for new ventures similar to this is estimated to be 12%. Should this project be undertaken? The table in Figure 1 below summarizes this information over time.

Time (Years)	0	1	2	3	4	5	6	7	8	9
Initial Cost(-)	\$35K									
Revenue		\$18K	\$18K	\$18K	\$18K	\$18K	\$18K	\$18K	\$18K	\$18K
Costs(-)		-11K	-11K	-11K	-11K	-11K	-11K	-11K	-11K	-11K
Salvage										3K
Net Cash Flow	\$35K	\$7K	\$7K	\$7K	\$7K	\$7K	\$7K	\$7K	\$7K	\$10K

Figure 1: Cash Inflows and Outflows

To find the present value of these cash inflows and outflows, we have two components to deal with. First, we have an annuity present value component consisting of a nine-year annuity of \$7,000 per year; second, we have a single lump-sum inflow of \$3,000 in nine years. We need the present value of these future cash flows at 12%. The present value is

given by: $PV = 7000 \left[\frac{1 - (1 + 0.12)^{-9}}{0.12} \right] + 3000 \left[(1 + 0.12)^{-9} \right]$. Figure 2 below displays this

calculation and result on the TI-84+SE as well as the project's net present value. Figure 3 displays the same results using the built-in net present value formula. Note that results have been set to display two decimal places.

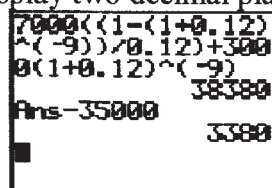


Figure 2: PV and NPV

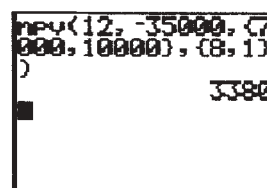


Figure 3: NPV

Since net present value is positive, this indicates that the proposed business project can be undertaken. The format for net present value on the TI-84+SE is: $npv=(\text{interest rate}, \text{initial cash flow}, \{\text{cash flow amounts}\}, \{\text{cash flow frequency}\})$. An entrepreneur may be interested in how varying the discount rate changes the net present value of the proposed project. We can study the relationship between the discount rate and net present value analytically, numerically, and graphically. In fact, the graphical representation of this relationship is called the net present value profile; the discount rate is the independent variable and resulting net present value is the dependent variable. Figures 3 through 6 guide us through these views for the current example.

```

Plot1 Plot2 Plot3
Y1=NPV(X,-35000
,7000,10000),8
,1)
Y2=
Y3=
Y4=
Y5=
  
```

Figure 3: NPV Analytically

X	Y1
7	31000
8	16680
9	6585
10	-746
11	-6202
12	-13380
13	-17584

Figure 4: NPV Numerically

```

WINDOW
Xmin=0
Xmax=30
Xsc1=5
Ymin=-32000
Ymax=32000
Ysc1=2000
Xres=1
  
```

Figure 5: Window for graph

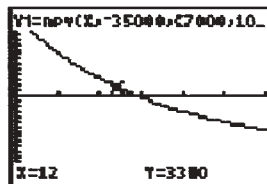


Figure 6: NPV Graphically

In viewing the graph, note that NPV ranges from positive to negative values. Again, note that “accept” project decisions occur where $NPV > 0$ and “reject” project decisions occur where $NPV < 0$. Note that at 12%, $NPV = \$3380$. The discount rate that results in $NPV=0$ is significant and is called the internal rate of return (IRR). Here, the entrepreneur is indifferent about undertaking the proposed project since at this break-even discount rate value is neither added nor lost. Using the CALC ZERO feature of the TI-84+SE, Figure 7 displays an IRR of 14.42% for this example; Figure 8 shows this same result using the built-in IRR formula. The format for IRR on the TI-84+SE is: $irr=(\text{initial cash flow}, \{\text{cash flow list}\}, \{\text{cash flow frequency}\})$. Since we are finding a discount rate, results are displayed to two decimal places.

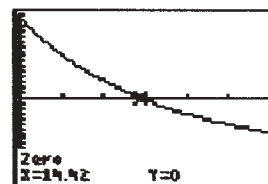


Figure 7: IRR Graphically

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irr(-35000,{7000
,10000},8,1)
14.42
  
```

Figure 8: IRR

Since the discount rate in this context is also considered to be the entrepreneur’s required return, the entrepreneur must determine how high this discount rate or required return would have to be before the proposed project would be rejected. Using the variable x to

represent the entrepreneur's required return and using the NPV profile in Figure 7, we can see that if $x < IRR$ then an "accept" project decision occurs; recall that this coincides with $NPV > 0$. Similarly, if $x > IRR$ then a "reject" project decision occurs; recall that this coincides with $NPV < 0$. This implies that under certain sets of conditions, the NPV rule and IRR rule independently lead to the same decision for a proposed project. One of those sets of conditions is highlighted in the proposed project example we have been investigating. That is, when cash flows are conventional (initial cash flow is negative and remaining cash flows are positive) NPV and IRR rules each lead to the same accept/reject decision for a proposed project. However, when this condition is not met, it turns out that there may be no IRR or more than one IRR since the shape of the NPV profile deviates from that of the conventional shape displayed in Figure 6. In this case, the entrepreneur should consider both NPV and IRR rules together, as well as the required return, to assist in decision making. The question therefore arises about predicting the number of IRRs in such a situation; the answer is found by turning to Descartes' Rule of Signs.

According to Descartes' Rule of Signs, if $f(x) = a_n x^n + \dots + a_0$ is a polynomial of degree n , then the number of positive (*negative*) real zeros of f equals the number of variations in sign of $f(x)$ ($f(-x)$) or that number less some even number. For our IRR situation, it turns out that the maximum number of IRRs is the number of sign changes in the cash flows. The actual number of IRRs either equals this maximum number or differs from it by an even number. For example, suppose a proposed project has a projected initial investment outlay of \$250 (negative), a \$1,425 year one cash inflow (positive), a \$3,030 year two cash outflow (negative), a \$2,855 year three cash inflow (positive), and a \$1005 year four cash outflow (negative). Since this is a non-conventional cash flow situation with four sign changes, Descartes' Rule of Signs predicts a maximum of four IRRs. The actual number of IRRs is either four, two, or zero. Figure 9 displays the TI-84+SE IRR calculation; the TI returns only the smaller (est) IRR value if one exists. Since we found one IRR, we now we know that there are either two or four IRRs. We can look to the NPV profile and table for further information. Figures 10 through 12 display the analytic and numeric representation of the current NPV situation, while Figures 13 and 14 show the WINDOW settings and the NPV profile itself.

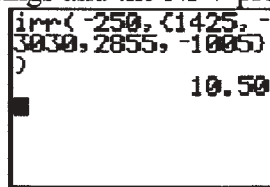


Figure 9: 1st IRR

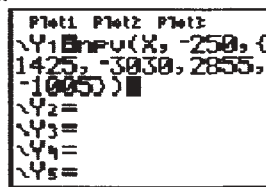


Figure 10: NPV Analytically

X	Y1
0.00	-.50
5.00	-1.72
10.00	-1.10
15.00	.61
20.00	.87
25.00	.74
30.00	.88

X=0

Figure 11: NPV Numerically

X	Y1
65.00	1.68
70.00	1.57
75.00	1.45
80.00	1.32
85.00	1.19
90.00	1.04
95.00	.88

X=95

Figure 12: NPV Numerically Con't

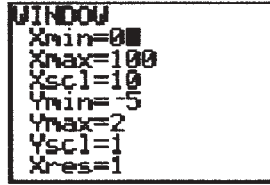


Figure 13: Window for NPV Profile

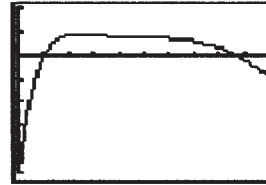


Figure 14: NPV Profile

In the current situation, we can see there are two IRRs to consider; Figure 15 supports the smaller IRR found, and Figure 16 displays the value of the second, larger IRR. Both were found using the TI's CALC ZERO option.

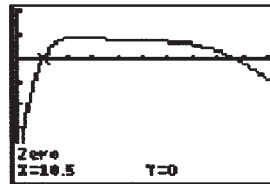


Figure 15: 1st IRR

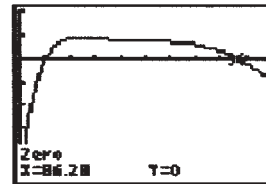


Figure 16: 2nd IRR

Both IRRs are mathematically correct; however we must be cautious with interpretation of results. If we set our required return between 0 and 10.5% exclusive and use the 10.5% IRR value, the IRR rule leads to an “accept” decision; however, $NPV < 0$ in this discount rate range and would imply a “reject” decision. On the other hand, $NPV > 0$ for discount rates between the two IRR values of 10.5% and 86.28%, respectively. If we set our required return between these two IRR values, then we would have to use the IRR of 86.28% in order for the NPV and IRR rules to both lead to an “accept” project decision. Considering both NPV and IRR rules together assist an entrepreneur in making more informed project decisions in a non-conventional cash flow situation than if either rule was used by itself.

Sources

- *Bodie, Z., A. Kane, and A. Marcus. Investments (4E). New York: McGraw-Hill, 1999.
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