The Role of the Graphic Calculator in Mathematical Education

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1. The change direction on Drawing Graph

from the Part to the Whole without calculator is change from the Whole to the Part with it

Drawing the graphs of functions is very important at the mathematical education. The aim of the mathematical education is the drawing the graph of functions. The graph is the useful tool for understanding the mathematics. So we learn the method of the drawing graphs with the differentiation at high school. Now we think that the goal of the mathematical education is the drawing the graph at the high school level. We have the training of the drawing the graph. So students get the method of the drawing the graph with the differentiation. In the textbook they have many problems of the drawing graphs of functions. In Japanese mathematical education we do not use the graphic calculator. So we think that the drawing graph of the function from the part to the whole. We look for special points on the part of the graph.

【Problem　　Draw the graph of the following functions y=x^3 – 3x

【Thinking answer】　graph of the function y=x^3 – 3x

We want to get special points on the graph.

(1) on the x-axes　　x^3 – 3x = 0 then x = 0, -$\sqrt{3}$, $\sqrt{3}$

(2) on the y-axes　　x = 0 then y = 0

(3) making the table with the differentiation

d/dx(x^3 – 3x) = 3x^2 – 3

solving the function 3x^2 – 3 = 0 then x = 1, -1

we can get the fluctuation table of the function

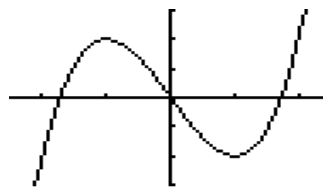| x		-1		1	
y '	+	0	—	0	+
	↗		↘		↗

Seeing the table, we can get this special point and draw this graph. We can make the graph of the function y=x^3 - 3x without the graphic calculator. But we do not use the graph at thinking of mathematics. This problem is the training of drawing the graph of the given functions no using the graphic calculator. In this education, many students cannot understand the meaning of the

graph of functions.
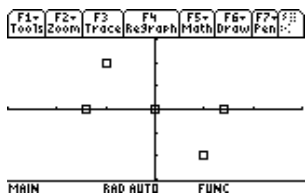
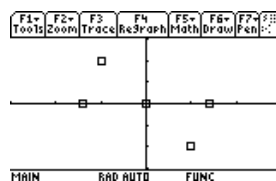

Part: special points of the graph   Whole: the graph of $y=x^3 - 3x$

Drawing the graph without the graphic calculator

We think that we must use the graph in the mathematical education. And we use the graph of the all world. Seeing the graph, we get the wide world of all. We can understand the meaning of the function by seeing the graph. Now we can get the graph of the function $y=x^3 - 3x$ with the graphic calculator.
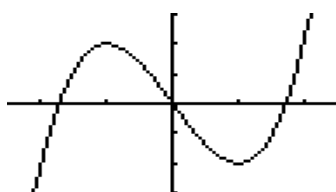
Whole of the graph with tool   Part of special points on the graph

Change the way of the mathematical education with the graphic calculator

The goal of mathematical education is not drawing the graph. When we can see the whole of the graph, we want to look for special points on the graph. We can understand that the graph is very important for mathematical thinking.

2. Example of the very important of the graph

Creative mathematical knowledge with the graph : Making the thinking of the derivable method

We can learn mathematics with tools. In mathematical education the teacher gives every formula to students. Students cannot create the mathematics. We want to change the method of teaching. In the next education system students is the leading actor in the class. The good problem is bringing his good idea.

【Problem】 Ask the maximum of the capacity of the box.   Making the box by the square. This edge of the square is 10. Cut the small four squares on the corner.

【Thinking answer by step and step】

First step   expression mathematical language

We change the language from ordinary to mathematics. We want to express the mathematics.

ordinary language = Mother language (writing problem)

↓

mathematical language $V = x(10-2x)^2$

It is very important to express formula. But for student this step is very difficult without training.

2

Second step   drawing the graph

　　We want to get the solution from the formula $V = x(10-2x)^2$. But we can see the solution with this formula. We want to hint for solving this problem.
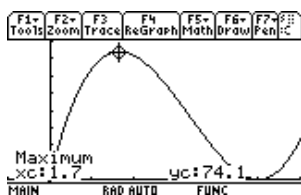
　　　　　the expression $V = x(10-2x)^2$

　　　　　　　　↓

　　　　　the guess the solution

　　　　　　　When x = 0 then V = 0 and x = 5 then V = 0

　　　　　　　So there is the solution between x = 0 and x = 5

But we cannot see the solution from the formula $V = x(10-2x)^2$. We can get the graph, using the graphic calculator. We can get the approximation on the graph. It is very good hint for us. Seeing is good idea. We solve more exact solution.


　　　　seeing the graph　　we get the approximate solution x =1.7, y = 74.1

　　　　　　　　↓

　　　　exact solution?

Thred step　　expand the part of near solution

　　In physics the important thinking is the expanding with the microscope. Physician has the tool. Before mathematician is thinking with the formula, but now we get the expanding tool. We can see the narrow part with the tool. So the computer is very useful tool for mathematical thinking. We can expand the part at the near of the approximate solution.
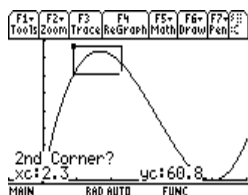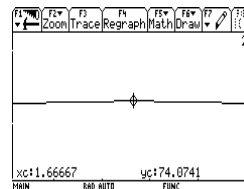
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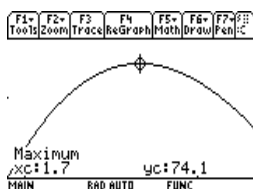
　　the graph of V(x)　　　　　　near the maximum point　　　　more expand at solution

　　　　　　　　using the expand of the part on the graph

In mathematics we cannot answer the exact solution by the expanding at (1.7, 74.0). The formula lim (f(x + h) - f(x)) / h　is the mathematical language of the expression. This formula is abstract for us. We need to training to use the formula. The expanding is very important for people.
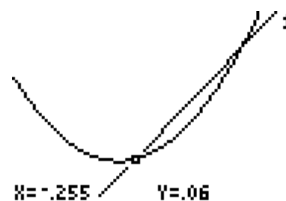
4th step　　creative the conjecture or make the derivable method

　　The solution of the maximum V=V(x) is seeing on the graph. We use the expanding of the part of the drawing graph. The thinking of the expanding is the very important method in the

scientific thinking. In this case, we find the point of the slope = 0 at the part of the graph. This thinking is near differential.


| at point we get the parallel line | lim (f(x + h) - f(x)) / h |
| we draw the tangent line | the point fixed, moving the straight line |

student making the derivable thinking

We think that every student is the mathematician. Students can make the theorems at the mathematical lesson. We say that we have the experience that is the interesting to think mathematics with the good tool. We have the concrete base of mathematics. This base is drawing the graph and understanding the meaning pf graphs of functions.

5th step CAS using of the graphic calculator

More learning mathematics, we use computer algebra system (CAS). Next step is using CAS.
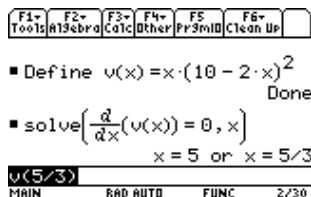
The thinking is very important and is change to like mathematics. We have the base of the thinking, and we think the abstract mathematics easily.

3. The example of the making mathematical theorem with calculator

Student is mathematician, they can make mathematical theorems

In my lesson, students play mathematics with the graphic calculator. Their learning style is near the player of the game. Hey are interesting for the learning mathematics. The graphic calculator is the good tool for learning mathematics for all students. Now we show the theorem that is made by them.

Theorem 1 Integer n is the expression of the sum of continuous integers

(1)given integer N is N=(2p+1)*n, then N is the sum of 2p+1 continuous integers

that is N=n1+n2+····+ n (2p+1) ni : integer

(2)given integer N is N=2p*n+p, then N is the sum of 2p continuous integers

that is N=n1+n2+····+ n 2p ni : integer

**Theorem 2** Integer n is the expression of the product of continuous integers

(1)given integer N is the product of two continuous integers, then N－x＝ x ^{2} has

the solution of the integer.

(2)given integer N is the product of three continuous integers, then $N + x = x^3$ has the solution of the integer.

(3)given integer N is the product of four continuous integers, then $N + 1 = x^2$ and $x+1$ is the product of two integers.

For examples with the graphic calculator TI-89
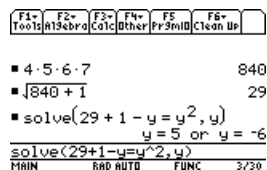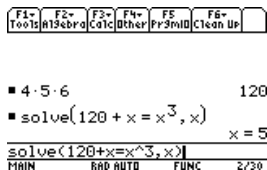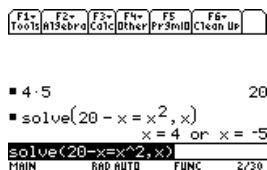
N= 20, then 20=4*5	N=120, then 120=4*5*6	N=840, then 840=4*5*6*7

We think that these theorems are old style without the graphic calculator. They are making them with tools. But at the last expression, they write them without tool. Now we can use the graphic calculator for this problem. We take the answer by dividing or

4. The Role of the Graphic Calculator in Mathematical Education

from the Part to the Whole without calculator is change from the Whole to the Part with it

We have more useful method of mathematical education. We think that the education is change from the Part to the Whole with the graphic calculator. The graphic calculator is very important to the next educational system. In Japan we do not use the tool that is the graphic calculator or the computer. Why do we use it for the education? It is easy to answer this question. We teach the ability of the mathematical skill for the student. And the student wants to learn the systematic mathematical theorems. So in the mathematical lesson, the teacher gives the theorem to the student. The student studies them very hard, but he/she does not think them. The tool is the machine that is no answer of the mathematical problems. If students make the mathematical knowledge in the mathematical classroom, they want to use the tool for thinking or making them. So in Japan the student has not the chance of using tools. They memorize them very good. They think that mathematics is a memory subject. We make the good education system that is "the open end problem" method. If students learn mathematics with "the open end problem", they want to use the tool. And more we know the stile of the modeling method and the problem solving. In Japan teachers try to teach them with the stile of the modeling method and the problem solving. But they cannot evaluate student's abilities of the stile of the modeling method and the problem solving. The next mathematical education is using the tool for thinking or learning the knowledge. And the direction from the part to the whole is changing fro the whole to part. It is very important to change the direction in the next education system. We can see the whole world using the graphic calculator. The role of the graphic calculator is change the thinking direction.