DESIGNING LEARNING ACTIVITIES WITH DYNAMIC GEOMETRY AND COMPUTER ALGEBRA SYSTEMS: THE ELLIPSE CASE.

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Introduction

The aim of this paper is to explore the role that dynamic geometry and computer algebra systems can play in the organization of a didactic sequence that has by objective the study of the conic sections in a course of Analytic Geometry. The possibility of didactic designs different from the habitual ones, considering the use of the symbolic calculator Voyage 200™ of Texas Instruments™ as a didactic resource, is taken in account. In particular, in this paper the study of the ellipse through different representations is approached, taking advantage of the different capabilities from the symbolic calculator: dynamic geometry, symbolic calculation, graphs, tables, programing, thus creating means in which the student can explore, conjecture, analyze, verify ideas, etc.

Theoretical aspects

Within the framework of R. Duval's (1998) theory on registers of semiotic representation, it is considered that there is no knowledge that can be mobilized by an individual without a representation activity and that the use of several systems of representation is essential for the exercise and the development of the fundamental cognitive activities. Duval defines semiotic representations as productions constituted by the use of signs and that belongs to a representation system, which has its own limitations of meaning and operation. A semiotic system is a register of representation, if it allows three fundamental cognitive activities: *the formation* of an identifiable representation within a given register; *the treatment* of a representation, that is the transformation of this representation in the same register where it has been formed; and *the conversion* of a representation, that is the transformation of this representation in a representation within another register.

The ellipse as a teaching object

At first, it is required to produce a structured and systematic description of the ellipse from the didactic perspective focusing attention on the use and coordination of multiple representations. It is possible to identify four relevant registers of representation for his description, each one of which activates different cognitive processes: the one of natural language, algebraic, graphical, and numerical.

The register of the natural language is used for presenting problemic situations, making descriptions, propositions or nominal designations, for argumentation, for introducing the definition of the ellipse (Lehmann, 1997), for the coordination of the different representations, etc. In the algebraic register, we consider in this paper the ordinary equations $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$, $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$. These expressions include several parameters that determine particular characteristics of the ellipse. In the graphical register of representation, the characteristics of the ellipse become patent. Its diverse elements are visualized (points of intersection with the axes, center lines, position in the plane, concavity, curvature, length of the axis, etc.). It is also possible to appreciate characteristics of the ellipse from the perspective of his geometric construction. In the numerical register, it is possible to appreciate some of the characteristics and elements identified in the algebraic and graphical representations. Through the use of the symbolic calculator as didactic resource, the forming and transforming of the different representations can be facilitated. It is possible to work with the graphical representation through the Cabrí-Geometry application, the `Y= Editor` and the `Graph Editor`; to give treatment to the algebraic representation through its powerful computer algebra system (CAS); and to work with numerical representations through the `Data/Matrix Editor`.

Linguistic or verbal representation

The idea is to start with a particular problem, and later to study the kind of problems that are generated, when varying systematically some of the components of this problem. For example, the following situation could be considered: *Find the locus of the points of the plane such that the sum of the distances from the points* (-3, 0) *and* (3, 0) *is equal to* 10, *and the equation that represents it.* Once the problem is approached of graphical, numerical, algebraic form, one could explore the effect of varying the position of both points (conserving the fact that they are on *x*-axis and they are placed symmetrically with respect to the origin); or varying the sum of the distances. This could take us to one first generalization of the problem: *Find the locus of the points of the plane such that the sum of the distances from the points* (*-c*, 0) *and* (*c*, 0) *is equal to a given constant, and the equation that represents it.* This way the student can advance in a gradual way toward the definition of the ellipse.

Graphic representation: Use of Cabri-Geometry

First, it will be considered a way to approach the problem from the perspective of the geometric construction, using the Cabri-Geometry application of the symbolic calculator `Voyage 200™` of Texas Instruments™. The details of construction are not given here.

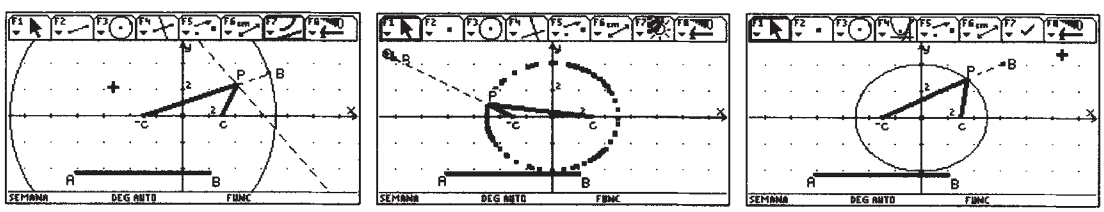

Fig. 1. Ellipse construction from its definition.

The images in this document are static, but the great potential of this representation in the calculator, is its dynamic quality. In this phase, the student can manipulate point B on the circumference to change the position of the segment and to observe how point P moves consequently. This point P is the one that satisfies the geometric conditions of the problem. It is possible to activate the `F7:2:Trace On/Off` or `F4:A:Locus` tool so that the software shows the locus of the point P (See **Fig. 1**). This locus changes dynamically when modifying some of its constituent elements as the position of the foci or the length of the segment. In a later stage, this file can be used so that the students explore the effect of changing the position of the points while the length of the reference segment stays fixed; and vice versa, holding the position of the points fixed while the length of the segment is varied. It is important also to explore until what extent these variations can be carried out without loosing the locus.

An associated numerical representation to the graphic representation.

It is possible to obtain numeric data from a Cabri file of the calculator and to transfer it to the `Data/Matrix Editor`, with which we will be connecting this graphic representation to a representation of another register: the numerical one. Unfortunately, there is no place to give the details here. The process is carried out several times to form a table, as shown in **Fig. 2**. The first column of the table corresponds to the x coordinate of the point P, and the second column corresponds to the y coordinate. In the third column it is introduced the formula to compute the distance `d1` from the point P to the focus (-3,0), and in column 4, the distance `d2` from P to the focus (3,0). Finally, in the column 5 the formula is introduced to add the two computed distances, `d=d1+d2`. Then it is observed the constant amount 10, that represents the sum of the distances from the point P to the ellipse foci (See **Fig. 2**).

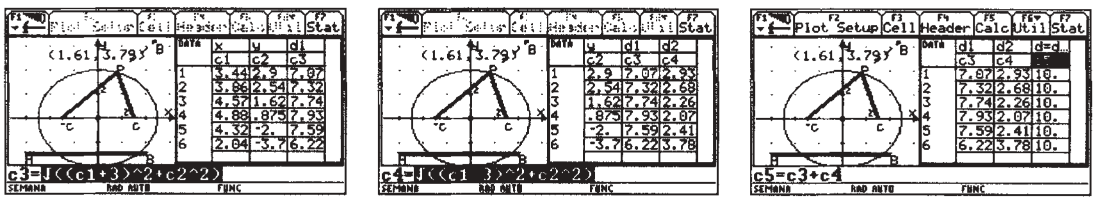

Fig. 2. Using Data/Matrix Editor

An algebraic representation: the equation of the locus.

When approaching the problem of finding an equation that represents the constructed locus, we look for connecting the previous representations, with a representation in the algebraic register. For it, laborious algebraic manipulations are required and, therefore, the computer algebra system incorporated in the `Voyage 200™` calculator will be used. We will try to relate the contents of columns 3, 4 and 5 for obtaining an analytical expression. (See **Fig 3**). Thus, an equation is obtained, step by step, that analytically expresses the geometric conditions that each point (x, y) in the locus satisfies the property: the sum of the distances from the point (x, y) to the points (-3, 0) and to (3, 0) is equal to 10.

$$\sqrt{(x+3)^2+y^2}+\sqrt{(x-3)^2+y^2}=10$$

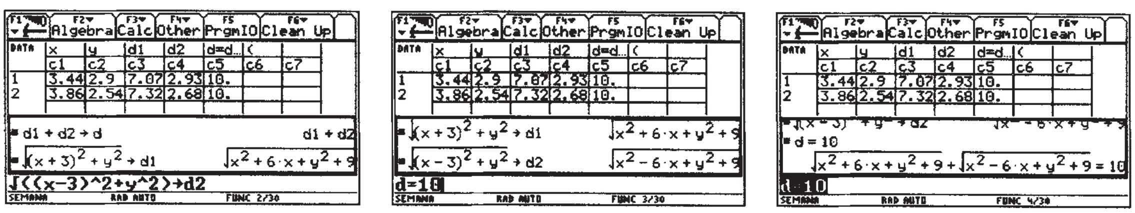

Fig. 3. Algebraic representation

Through a sequence of transformations that the student must be proposing, the expression is simplified until obtaining its canonical form. In Analytic Geometry textbooks, the algebraic development is made only once, considering the general case where the foci have coordinates (-c, 0) and (c, 0) and the fixed distance is considered to be 2*a*. The transformations to simplify the equation are carried out in a very special way to facilitate as far as possible, the algebraic pencil and paper manipulation. With the symbolic calculator this procedure, besides to be carried out in different ways, can be executed several times for different tactical values. It can also be carried out, considering the parameters in a general way. (See **Fig. 4.**)

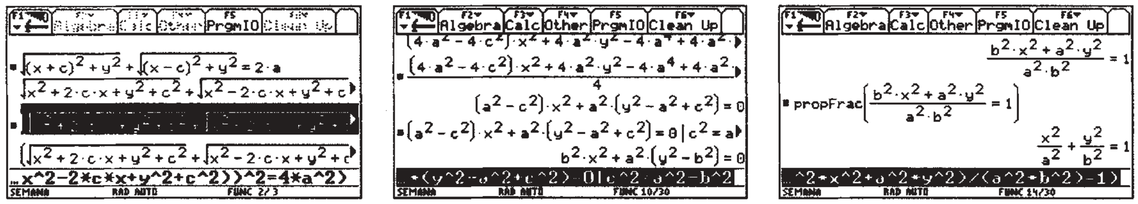

Fig. 4. A canonical form of the ellipse equation.

Another graphical representation

Once obtained the equation for the locus in consideration, it is possible to use the `Y=` `Editor` and `Graph Editor` of the symbolic calculator, as a mean to verify the obtained results. In order to graph, it is necessary to express *y* as a function of *x*, so, successive transformations to the equation can be applied. It is possible to use the Solve command of the Home application for solving for *y*. This option is ilustrated in **Fig.5**.

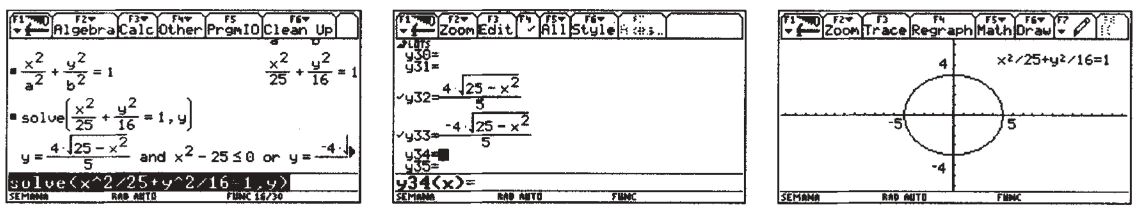

Fig. 5. Y= Editor and Graph Editor

Representations dynamically connected

Up to here, it has been ilustrated the use of the symbolic calculator to support the cognitive activities of formation and treatment of representations. Now, we will try to illustrate a way to promote the cognitive activity of convertion, using the capacities of programming of the computer `Voyage 200`™ of Texas Instruments™. It is assumed the fact that, through different representations dynamically connected, the convertion between representations can be favored. For such effect, a program that tries to promote the coordination of the natural language, algebraic, and graphical registers has been designed. In this program graphical and algebraic representations of the family of curves $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ or $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$ are handled simultaneously, that is to say, they are dynamically connected. With this correspondence of the graphical and algebraic representations we want to promote the coordination of both registers. Also, through the user menus that allow the operation of the program, we try to establish the terminology that will allow the suitable coordination of the register of the natural language to the operations of reconfiguration in the graphical register and to the variations of the parameters in the algebraic register.

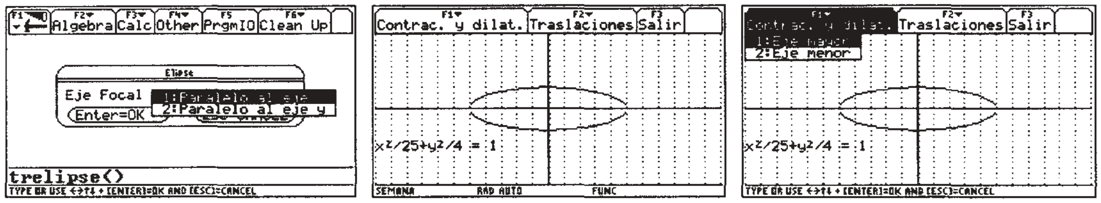

Fig. 6. `trelipse()` program

After selecting the option `Major Axis` or `Minor Axis`, an animation will appear in the screen into which the graph is transformed in a cyclical way, according to the selected option, a determined number of times, with its corresponding analytical expression. In this way, the students could be asked to relate the parameters in the canonical form of the equation of the ellipse, with the semimajor and semiminor axes, and the interceptions of the graph with the coordinates axes. In **Fig. 7** they appear, of static way of course, some of the images used in the animation in which the major axis is varied.

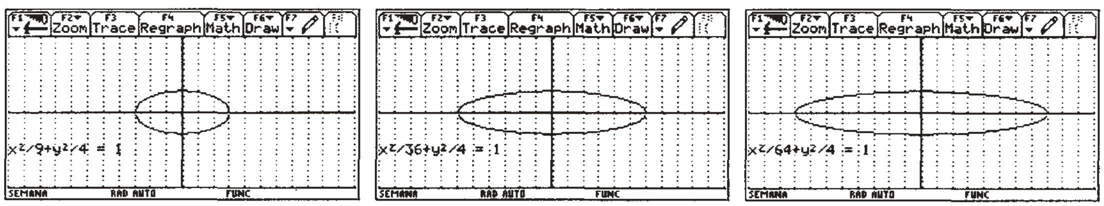

Fig. 7. Graphs for the animation of expansion and contraction of major axis when this is on *x*-axis.

In order to organize the transit towards the ordinary equations of the ellipse $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ or $\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$, the student will relate the role of the parameters in this algebraic representation to the transformations done to the ellipse, when its center is considered outside the origin. For it, there is an option in the user's menu to transfer the ellipse in vertical, horizontal or in diagonal direction.

Final considerations

What it has been presented here is only one small sample of which it can become in the classroom of mathematics when this type of instruments are used as a didactic resource. Of course, the problems of teaching and learning mathematics are not solved with the use of this type of devices, but it is doubtless that the opportunities for students to explore, to conjecture, to analyze, to verify ideas are increased.

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