PERSONALIZING DERIVATIVES AND ANTIDERIVATIVES WITH GSP, A CBR, AND A TI-84

Judy O'Neal
North Georgia College & State University
Department of Mathematics and Computer Science
Dahlonega, GA 30597
joneal@ngcsu.edu

Beginning calculus students frequently have only a minimal understanding of derivatives and antiderivatives. Despite our best efforts to stress meaning over procedure as the concepts are developed, students emerge with a tenuous, if not outright shaky, ability to convey the meaning of a derivative or antiderivative. In an effort to deepen students' conceptual understanding of such foundational concepts, a multi-phase technology-based project incorporating *The Geometer's Sketchpad (GSP),* a Calculator-Based Ranger (CBR), and a TI-84 (or TI-83 Plus) has been developed and utilized with first semester calculus students. This paper illustrates how motion, velocity, and distance traveled are personalized for students through their involvement in a small-group setting that applies a kinesthetic approach that activates students via movement.

PHASE 1 – WALKING A PATH

The Calculator-Based Ranger (CBR) and the TI-84 (or TI-83 Plus) are used to collect students' walking data. Prior to students beginning this phase of the project, they have had experience with walking back and forth in front of the CBR and examining the resulting graph on the TI-84. Students are instructed to walk slowly away from the CBR for 2.5 seconds, and then to walk more quickly toward the CBR for 1.5 seconds. Prior to the actual walking, students are asked to predict how the resulting graph should look. After walking and once they are satisfied with their resulting graph, the group works together to determine which type of curve most accurately fits their data (Figures 1 and 2). Instructions for determining a regression equation using the STAT-CALC feature are provided on the printed project. The appropriate regression equation is stored using VARS and YVARS in one of the Y= locations and then graphed on the scatter plot of the walking data. Students are then asked to describe how they think their walking rate (velocity) relates to the steepness and direction of the graph resulting from their walk (Figure 3).

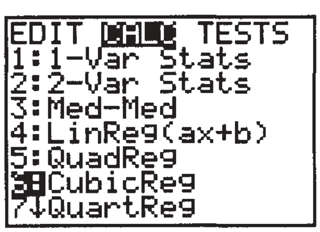

Figure 1

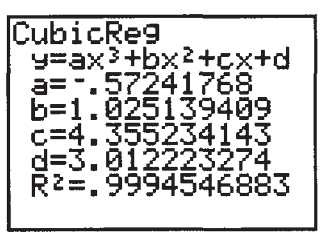

Figure 2

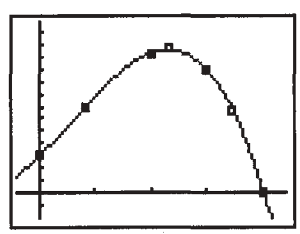

Figure 3

PHASE 2 – THE PATH AND ITS DERIVATIVE

The Geometer's Sketchpad is an effective tool for students to use in determining the accuracy of their conclusion about how their walking velocity affects the shape of the graph. A representation of a tangent line to the curve, where the x-distance between the two given points on the actual secant line drawn on the curve is very small, dynamically illustrates how the velocity changes over a given time interval. Students use the Plot New Function feature in *GSP* to graph the best-fit curve (Figure 4). They construct a line $\overleftrightarrow{AB}$ with points A and B on the curve, determine the slope of $\overleftrightarrow{AB}$ and the abscissa of point A using the Measure menu, plot the point (x_A, m_{AB}) using the Graph - Plot as (x, y) feature, and trace the plot of (x_A, m_{AB}) using the animation of (x_A, m_{AB}) (Figure 5). Watching the secant line move while the graph of the derivative appears as a trace is fascinating and extremely convincing evidence for students.

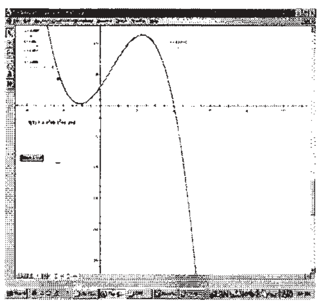

Figure 4

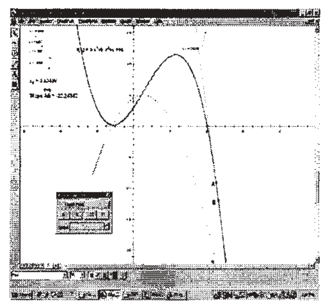

Figure 5

Students discover that the resulting trace is that of a parabola. The quadratic equation has positive values between the local minimum indicated on the left concave upward portion of the graph and the local maximum on the right concave downward portion of the graph. Negative values of the quadratic equation are found where the cubic curve is decreasing.

The next task within this phase is analyzing the derivative trace. Students determine the zeroes, local/relative extrema, domain, magnitude, and sign of the derivative. They then use these results to determine what relationship exists between the walk function and the velocity (derivative) function. Students are asked to revisit their conclusion drawn in Phase 1 regarding the relationship of their walking rate (velocity) to the steepness and direction of the graph resulting from their walk and to compare it to their findings from Phase 2.

PHASE 3 – DISTANCE TRAVELED VIA POLYGON AREA

The concept of distance traveled is developed through the use of the students' walking data. The graph of their movement is used as the curve for which the concept of an antiderivative and its relationship to area under its corresponding curve is developed. Using rectangles as the polygon of choice for filling the area under the students' walking graph, they identify their beginning and ending times, determine elapsed time, select the number of rectangles they wish to use, and then identify the rectangle height and width. Rectangles are constructed using either Left, Right, or Midpoint Rectangles (Figure 6). A Trapezoid Custom tool is available for those who have shied away from this polygon choice because of its more computationally- intensive nature, which is now alleviated with the use of *GSP*. A Left Sum, Right Sum, or Midpoint Sum is calculated. Most students eagerly employ all three choices since the graphs and computations are quickly accomplished, and a determination of the closeness of the results becomes a topic for group discussion. At the completion of this phase, students are ready to find the exact distance traveled during their initial walk. This is translated to finding the exact area under a curve.

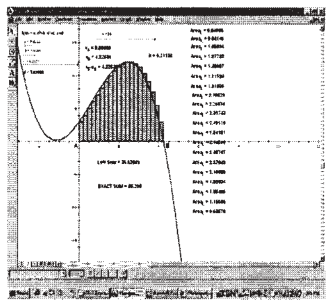

Figure 6

EVALUATION

At the completion of Phase 1 students create a report that includes screen captures showing the lists with their walking data, the regression calculation, a Stat Plot of their walking data, and a graph of their regression equation superimposed on their Stat Plot. They use *TI-Connect* software and a TI-Graph Link cable to capture their calculator images. In addition a written description of the relationships among path, velocity, and slope is included.

For Phase 2 students submit either a *GSP* file containing a page for each requirement or a printed *GSP* document. Requirements include their regression equation graphed in *GSP*, the derivative trace, and a Motion Controller (Animation) of the tangent (secant) line. Written responses to questions related to the derivative's zeros, relative/local maximum and minimum points, domain, magnitude, and sign are to be completed within *GSP* using the Text Box.

The assessment of Phase 3, as in Phase 2, requires the submission of either a *GSP* file containing a page for each requirement or a printed *GSP* document. Students must submit graphs illustrating Left, Right, or Midpoint Rectangles; rectangle areas; and Left Sum, Right Sum, or Midpoint Sum calculation. An error analysis comparing the exact area to the rectangle approximation method that includes numerical and descriptive components is required. Finally, a written description completed within *GSP* of the relationship between distance traveled and velocity is submitted.

CONCLUSION

This multi-phase project that spans a semester has proven to be an effective vehicle for motivating beginning calculus students to become active participants in learning and applying calculus concepts. Although skill development is not stressed within the three phases of the project, students emerge with a clearer understanding of how everyday events such as walking are related to the mathematics of change. This understanding is evident in class discussions and student performance on tests and the final exam when questions asking them to explain relationships between velocity, slope, and distance are posed. Positive student reaction to this type of kinesthetic and technology-focused project appears regularly on the student comments section of the course/instructor evaluation instrument. Obviously, there is no project or activity that will transform all beginning calculus students to "A" students; however, their positive attitude toward learning calculus does make teaching an extremely rewarding and enjoyable experience.

REFERENCES

Clements, C., Pantozzi, R. & Steketee, S. (2002). *Exploring Calculus with The Geometer's Sketchpad.* Emeryville, CA: Key Curriculum Press.

TI-Connect Software – Downloadable from http://education.ti.com/us/product/accessory/connectivity/down/download.html