

## INTERESTING COMPUTER EXPLORATIONS ON DERIVE6

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### Introduction

DERIVE is a user-friendly piece of software that students are able to use independently in a relatively short amount of time. In this article I will discuss how I use DERIVE in multivariate calculus to help students visualize in 3D.

### Making Bagels

One of my favorite activities in multivariate calculus is making bagels with my students. I like this activity because it requires the students to bring together several bits and pieces of information in order to find the correct parametric representation for the bagel. It's really an exercise in constructing a parametric representation for a torus from scratch. In the process the students must remember how to represent a circle parametrically, how to add two vectors, how to represent unit vectors in 2D, how to normalize a vector, and they must be able to see a pattern so they can generalize to get the result. This activity is done as an in-class exercise because the students usually need hints as we go.

We start by parametrizing the circle  $(x - 2)^2 + z^2 = 1$  on the  $\theta = 0$  half-plane. The bagel is formed by rotating this circle around the  $z$ -axis so this circle is the generating curve for the bagel we seek.

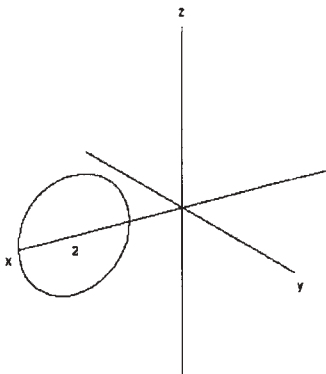


Figure 1: Generating Curve for the Bagel

In general, students find it challenging to work with parametric equations, so to give the students some practice working with the parametric representation of a circle we find the parametrization of this circle when it has been rotated about the  $z$ -axis through the following angles:

- (a)  $\theta = 0$ , (b)  $\theta = \pi/2$ , (c)  $\theta = \pi$ , (d)  $\theta = 3\pi/2$ , and (e)  $\theta = 2\pi$ .

To help the students get the correct position vector and to visualize these circles, I provide the figure below.

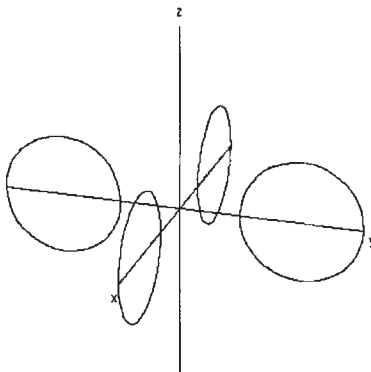


Figure 2: Generating Curve on Coordinate Planes

I usually work with the students to draw in the position vector for each of these circles and get the correct representation in each case. After stepping through these cases, the students are almost ready to generalize, but first we look at one case where the circle is not on a coordinate plane. Specifically, we find the parametrization of the circle in the  $\theta = 3\pi/4$  half-plane. This is a key point in coming up with the parametrization for the bagel! (See below diagram where I have plotted the centerline of the bagel as well as the circle in question.)

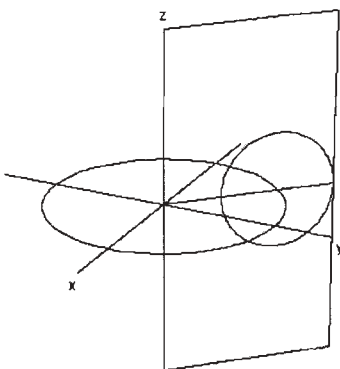


Figure 3: Generating Curve on  $\theta = 3\pi/4$  Half-Plane

Once the correct parametrization for the circle in the  $\theta = 3\pi/4$  half-plane has been obtained, the generalization required to find the circle for any  $\theta$  is found and the parametrization for the bagel is obtained. The parametrization for this bagel is  $[\cos(\theta)(2 + \cos(\alpha)), \sin(\theta)(2 + \cos(\alpha)), \sin(\alpha)]$ . When you Author this vector and plot in the 3D window, you get the bagel below.

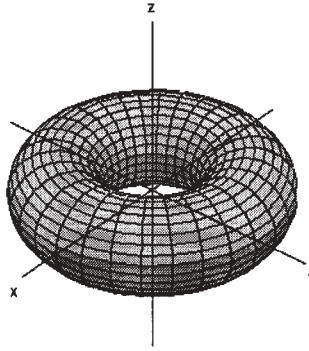


Figure 4: The Bagel

(Credit: This activity was adapted from Group Exercise 2, Section 10.5 of ancillary material to Stewart calculus text, 2nd Ed.)

### Finding a Trace of Lisa

DERIVE is also quite useful in helping students visualize traces of surfaces. I do this by creating a lab exploration for the students to complete outside of class. I usually superimpose a story line onto the lab to make it more “interesting” for the students. In this lab, the story line has an entomological theme with two lightning bugs flying around in 3D. Larry is a *plane* old lightning bug, so he always stays on a plane. Lisa is also a lightning bug, but she’s much too sophisticated to stay on a plane. The lab starts with Lisa flying around on the hyperbolic paraboloid  $z = 2x^2 - y^2 + 1$  and Larry flying around on the  $xy$ -plane and desiring greatly to meet Lisa.

The students are tasked with finding the set of points in 3D that Larry should frequent in the hope of finding a *trace* of Lisa. This requires the students to visualize the hyperbolic paraboloid, the  $z = 0$  plane, and the curve of intersection of these two surfaces. This is where DERIVE comes in handy. The students can quickly plot both surfaces and rotate the graph to see the curve of intersection, which is the  $z = 0$  trace of the hyperbolic paraboloid and Larry’s only hope of meeting Lisa.

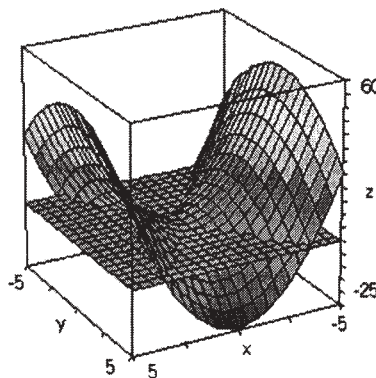


Figure 5: Lisa on Hyperbolic Paraboloid and Larry on  $z = 0$  Plane

Larry flies around all day and makes his way from one coordinate plane to the other and never sees a *trace* of Lisa. To make matters worse, Lisa has now altered her flight patterns in such a way that she will always be somewhere on the surface given by  $9x^2 + 4y^2 = 36$ . DERIVE will not plot this in the 3D window in its current form so the students must convert this equation to its corresponding equation in cylindrical coordinates and then plot. They do this piece by hand. In fact, all of my labs require the students to do something by hand because I don't want them to think that computers can do everything. There's a piece for the computer to do, but there is always a piece for them to do as well. Upon converting to cylindrical coordinates the students find that

Lisa's surface is given by  $r = \frac{6}{\sqrt{9\cos^2(\theta) + 4\sin^2(\theta)}}$ . They then plot this in cylindrical

coordinates followed by plotting each coordinate plane in turn to find out where Larry must go to find a *trace* of Lisa. For each trace, the students include a graph of the surface and the plane, the equation of the trace, and the name of the trace (line, parabola, etc.) I've included one of the traces below.

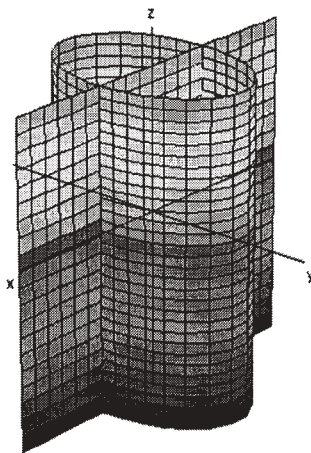


Figure 6: Lisa on Elliptic Cylinder and Larry on  $y = 0$  Plane

Foiled again! Larry flew around all day and did not see a trace of Lisa ... again! Lisa, being the adventurous sort, has decided to alter her flight patterns yet again. Now Lisa is flying around 3D so she always stays on the surface  $-x^2 - y^2 + z^2 = 1$ . Larry is getting more adventurous – or desperate – and decides to venture way out to the  $x = 1$  plane in a last ditch effort to meet Lisa. What is the set of points that Larry should frequent on the  $x = 1$  plane in the hope of finding Lisa?

Once again, DERIVE is unable to plot Lisa's surface in its current form, so the students must use the coordinate transformation equations to convert to spherical coordinates. Upon converting to spherical coordinates the students find that Lisa's surface is given by

$\rho = \frac{1}{\sqrt{\cos(2\phi)}}$ . Plotting the  $x = 1$  plane and Lisa's surface in spherical coordinates yields the graph below.

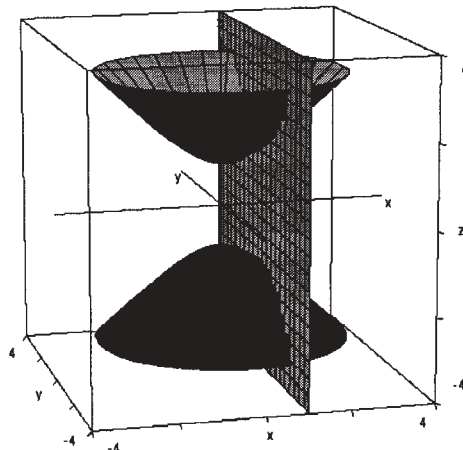


Figure 7: Lisa on Hyperboloid of 2-Sheets and Larry on  $x = 1$  Plane

Success at last! Larry the lonely lightning bug finally met Lisa the loveable lightning bug by going to the highest point below the  $xy$ -plane on the  $x = 1$  trace and hovering. When he saw the light (actually it was a flash) he traveled along the trace to investigate, and sure enough ... it was Lisa! What were the coordinates were Larry hovered? To see more clearly, the  $x = 1$  trace and where Larry hovered, I tell the students to plot the  $x = 1$  trace in the 2D window and label the two axes appropriately. Below is the graph.

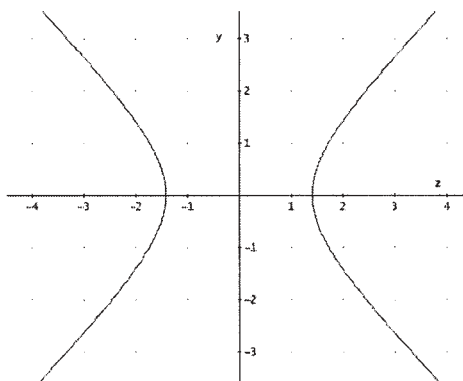


Figure 8: Lisa's Trace on the  $x = 1$  Plane

### Conclusion

DERIVE is a useful tool in multivariate calculus because it helps the students to visualize the symbolic expressions they are working with. It is also a versatile tool for plotting many different kinds of three-dimensional objects. By helping the students to visualize the objects they are working with, DERIVE helps the students to learn the mathematics.